

Name:.....

Total Marks:.....

GCSE (9-1) Grade 7

Composite and Inverse Functions



Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name.
- Answer **all** questions.
- Answer the questions in the spaces provided
 - there may be more space than you need.
- **Show all your working out**

Information

- The total mark for this paper is 148.
- The marks for **each** question are shown in brackets.
 - use this as a guide as to how much time to spend on each question.
- Questions labelled with an **asterisk** (*) are ones where the quality of your written communication will be assessed

Advice

- Read each question carefully before you start to answer it
- Attempt every question
- Check your answers if you have time at the end



1. Functions f and g is such that $f(x) = 2x - 1$ and $g(x) = \frac{3}{x}$

(a) Find the value of

(i) $f(3)$

..... (1)

(ii) $fg(6)$

..... (2)

(b) $f^{-1}(x) =$

..... (2)

(c) $gf(x) =$

..... (2)

2. The function f is such that $f(x) = 4x - 1$

(a) Find $f^{-1}(x)$

..... (2)

The function g is such that $g(x) = kx^2$ where k is a constant.

(b) Given that $fg(2) = 12$, work out the value of k

..... (2)



3. Functions f and g are such that $f(x) = 3(x - 4)$ and $g(x) = \frac{x}{5} + 1$

(a) Find the value of $f(10)$

..... (1)

(b) Find $g^{-1}(x)$

..... (2)

(c) Show that $ff(x) = 9x - 48$

..... (2)

4. Given that $f(x) = x^2$ and $g(x) = x - 6$, solve the equation $fg(x) = g^{-1}(x)$

..... (3)



5. f and g are functions such that $f(x) = 2x - 3$ and $g(x) = 1 + \sqrt{x}$

(a) Calculate $f(-4)$

..... (1)

(b) Given that $f(a) = 5$, find the value of a .

..... (2)

(c) Calculate $gf(6)$.

..... (2)

(d) Find the inverse function $g^{-1}(x)$.

..... (2)



6. Functions f and g are such that

$$f(x) = \frac{1}{x+2} \text{ and } g(x) = \sqrt{x-1}$$

(a) Calculate $fg(10)$

..... (2)

(b) Find the inverse function $g^{-1}(x)$.

..... (2)



7. Functions f and g are such that
 $f(x) = 2x + 2$ and $g(x) = 2x - 5$

(a) Find the composite function fg .
Give your answer as simply as possible.

..... (2)

(b) Find the inverse function $f^{-1}(x)$.

..... (2)

(c) Hence, or otherwise, solve $f^{-1}(x) = g^{-1}(x)$.

..... (3)



8. The function f is such that $f(x) = \frac{1}{x+3}$

(a) Find the value of $f(2)$

..... (1)

(b) Given that $f(a) = \frac{1}{10}$, find the value of a .

..... (2)

The function g is such that $g(x) = x + 2$

(c) Find the function gf .

Give your answer as a single algebraic fraction in its simplest form.

..... (2)



9. Functions f and g are such that $f(x) = x^2$ and $g(x) = x - 3$

(a) Find $gf(x)$.

..... (2)

(b) Find the inverse function $g^{-1}(x)$.

..... (2)

(c) Solve the equation $gf(x) = g^{-1}(x)$.

..... (3)



10. The function f is such that $f(x) = (x - 1)^2$

(a) Find $f(8)$

..... (1)

The function g is such that $g(x) = \frac{x}{x - 1}$

(b) Solve the equation $g(x) = 1.2$

..... (3)

(c) (i) Express the inverse function g^{-1} in the form $g^{-1}(x) =$

..... (2)

(ii) Hence write down $gg(x)$ in terms of x .

..... (2)



11. f is a function such that $f(x) = \frac{1}{x^2 + 1}$

(a) Find $f(\frac{1}{2})$

..... (1)

g is a function such that $g(x) = \sqrt{x-1}$, $x \geq 1$

(b) Find $fg(x)$

Give your answer as simply as possible.

..... (3)



12. The function f is such that $f(x) = \frac{x-6}{2}$

(a) Find $f(8)$

..... (1)

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) =$

..... (2)

The function g is such that $g(x) = \sqrt{x-4}$

(c) Express the function gf in the form $gf(x) =$
Give your answer as simply as possible.

..... (3)



13. Functions f and g are such that $f(x) = 3x - 2$ and $g(x) = \frac{10}{x + 2}$
- (a) Express the inverse function f^{-1} in the form $f^{-1}(x) =$

..... (2)

- (b) Find $gf(x)$
Simplify your answer.

..... (2)



14. Functions f and g are such that $f(x) = \frac{2}{x}$ and $g(x) = \frac{x+1}{x}$

(a) Solve $gf(a) = 3$

..... (3)

(b) Express the inverse function g^{-1} in the form $g^{-1}(x) =$

..... (3)



15. Functions g and h are such that $g(x) = \frac{x}{2x-5}$ and $h(x) = x + 4$

(a) Find the value of $g(1)$

..... (1)

(b) Find $gh(x)$
Simplify your answer.

..... (2)

(c) Express the inverse function g^{-1} in the form $g^{-1}(x) =$

..... (3)



16. The function f is such that $f(x) = \frac{1}{x+4}$, $x \neq -4$.

Evaluate $f^{-1}(3)$.

..... (3)

17. Given that $f(x) = 3x - 1$, $g(x) = x^2 + 4$ and $fg(x) = gf(x)$,

show that $x^2 - x - 1 = 0$

..... (5)



18. Functions f and g are such that

$$f(x) = 3x + 2$$

$$g(x) = x^2 + 1$$

Find an expression for $(fg)^{-1}(x)$

..... (3)

19. The function f is such that $f(x) = \frac{8}{x+2}$

(a) Find $f^{-1}(x)$.

..... (2)

(b) Solve the equation $f^{-1}(x) = f(x)$

..... (3)



20. The function g is such that $g(x) = \frac{1}{1-x}$ for $x \neq 1$

(a) Prove that $gg(x) = \frac{x-1}{x}$

..... (3)

(b) Find $ggg(3)$

..... (3)



21. Functions f , g and h are such that $f(x) = 3 - x$, $g(x) = x^2 - 14$ and $h(x) = x - 2$
Given that $f(x) = gfh(x)$, find the values of x .

..... (5)

22. The function f is defined by $f(x) = \frac{x-1}{x}$, $x \neq 0$
Solve $ff(x) = -2$

..... (4)



23. $f(x) = \frac{x}{x+3}, \quad x \in \mathbb{R}, \quad x \neq -3$

(a) If $f^{-1}(x) = -5$, find the value of x .

..... (3)

(b) Show that $ff^{-1}(x) = x$

..... (3)



24. $f(x) = \frac{x}{x+3}$, $x \in \mathbb{R}$, $x \neq -3$

(a) If $f^{-1}(x) = -5$, find the value of x .

..... (3)

(b) Show that $ff^{-1}(x) = x$

..... (3)



25. $f(x) = 2x + c$

$$g(x) = cx + 5$$

$$fg(x) = 6x + d$$

c and d are constants.

Work out the value of d .

..... (3)

26. $f(x) = 3x^2 - 2x - 8$

Express $f(x + 2)$ in the form $ax^2 + bx$

..... (3)



27. $f(x) = x^2 - 2x - 4$

Express $f(2x - 1)$ in the form $ax^2 + bx + c$

..... (3)

28. $f(x) = x^2 + 3x + 4$

Show that $f(x - 2) - f(x) = -4x - 2$

..... (3)

TOTAL FOR PAPER: 148 MARKS