1 **a** $3x = 2x + 1$
$x = 1$
$\therefore\ x = 1, y = 3$

b $x - 6 = \frac{1}{2}x - 4$
$x = 4$
$\therefore\ x = 4, y = -2$

c $2x + 6 = 3 - 4x$
$x = -\frac{1}{2}$
$\therefore\ x = -\frac{1}{2}, y = 5$

d subtracting
$y + 4 = 0$
$y = -4$
$\therefore\ x = 7, y = -4$

e $2x + 4y + 22 = 0$
$2x - 3y + 1 = 0$
subtracting
$7y + 21 = 0$
$y = -3$
$\therefore\ x = -5, y = -3$

f $6x + 6y + 8 = 0$
$15x - 6y - 15 = 0$
adding
$21x - 7 = 0$
$x = \frac{1}{3}$
$\therefore\ x = \frac{1}{3}, y = -\frac{5}{3}$

2 **a** $x + 2 = x^2 - 4$
$x^2 - x - 6 = 0$
$(x + 2)(x - 3) = 0$
$x = -2$ or 3
$\therefore$ $(-2, 0)$ and $(3, 5)$

b $4x + 11 = x^2 + 3x - 1$
$x^2 - x - 12 = 0$
$(x + 3)(x - 4) = 0$
$x = -3$ or 4
$\therefore$ $(-3, -1)$ and $(4, 27)$

c $2x - 1 = 2x^2 + 3x - 7$
$2x^2 + x - 6 = 0$
$(2x - 3)(x + 2) = 0$
$x = -2$ or $\frac{3}{2}$
$\therefore$ $(-2, -5)$ and $(\frac{3}{2}, 2)$

3 **a** subtracting
$x^2 - x - 2 = 0$
$(x + 1)(x - 2) = 0$
$x = -1$ or 2
$\therefore\quad x = -1, y = 4$
or $\quad x = 2, y = 7$

b adding
$2x^2 - 7x + 3 = 0$
$(2x - 1)(x - 3) = 0$
$x = \frac{1}{2}$ or 3
$\therefore\quad x = \frac{1}{2}, y = -\frac{7}{2}$
or $\quad x = 3, y = -6$

c $y = 2x - 5$
sub
$x^2 + (2x - 5)^2 = 25$
$x^2 - 4x = 0$
$x(x - 4) = 0$
$x = 0$ or 4
$\therefore\quad x = 0, y = -5$
or $\quad x = 4, y = 3$

d $y = 2x + 10$
sub.
$x^2 + 2x(2x + 10) + 15 = 0$
$x^2 + 4x + 3 = 0$
$(x + 3)(x + 1) = 0$
$x = -3$ or -1
$\therefore\quad x = -3, y = 4$
or $\quad x = -1, y = 8$

e $y = 1 - x$
sub.
$x^2 - 2x(1 - x) - (1 - x)^2 = 7$
$x^2 = 4$
$x = \pm 2$
$\therefore\quad x = -2, y = 3$
or $\quad x = 2, y = -1$

f $y = 1 - x$
sub.
$3x^2 - x - (1 - x)^2 = 0$
$2x^2 + x - 1 = 0$
$(2x - 1)(x + 1) = 0$
$x = -1$ or $\frac{1}{2}$
$\therefore\quad x = -1, y = 2$
or $\quad x = \frac{1}{2}, y = \frac{1}{2}$

g $y = 4 - x$
sub.
$2x^2 + x(4 - x) + (4 - x)^2 = 22$
$x^2 - 2x - 3 = 0$
$(x + 1)(x - 3) = 0$
$x = -1$ or 3
$\therefore\quad x = -1, y = 5$
or $\quad x = 3, y = 1$

h $x = 2y$
sub.
$(2y)^2 - 4y - y^2 = 0$
$3y^2 - 4y = 0$
$y(3y - 4) = 0$
$y = 0$ or $\frac{4}{3}$
$\therefore\quad x = 0, y = 0$
or $\quad x = \frac{8}{3}, y = \frac{4}{3}$

i $y = 3 - \frac{3}{2}x$
sub.
$x^2 + x(3 - \frac{3}{2}x) = 4$
$x^2 - 6x + 8 = 0$
$(x - 2)(x - 4) = 0$
$x = 2$ or 4
$\therefore\quad x = 2, y = 0$
or $\quad x = 4, y = -3$

j $y = 2x - 3$
sub.
$2x^2 + (2x-3) - (2x-3)^2 = 8$
$x^2 - 7x + 10 = 0$
$(x-2)(x-5) = 0$
$x = 2$ or 5
$\therefore \quad x = 2, y = 1$
or $\quad x = 5, y = 7$

k $y = 2x - 7$
sub.
$x^2 - x(2x-7) + (2x-7)^2 = 13$
$x^2 - 7x + 12 = 0$
$(x-3)(x-4) = 0$
$x = 3$ or 4
$\therefore \quad x = 3, y = -1$
or $\quad x = 4, y = 1$

l $y = 5 - 3x$
sub.
$x^2 - 5x + (5-3x)^2 = 0$
$2x^2 - 7x + 5 = 0$
$(2x-5)(x-1) = 0$
$x = 1$ or $\frac{5}{2}$
$\therefore \quad x = 1, y = 2$
or $\quad x = \frac{5}{2}, y = -\frac{5}{2}$

m $x = 2y + 10$
sub.
$3(2y+10)^2 - y(2y+10) + y^2 = 36$
$y^2 + 10y + 24 = 0$
$(y+6)(y+4) = 0$
$y = -6$ or -4
$\therefore \quad x = -2, y = -6$
or $\quad x = 2, y = -4$

n $y = \frac{3}{2}x - 2$
sub.
$2x^2 + x - 4(\frac{3}{2}x - 2) = 6$
$2x^2 - 5x + 2 = 0$
$(2x-1)(x-2) = 0$
$x = \frac{1}{2}$ or 2
$\therefore \quad x = \frac{1}{2}, y = -\frac{5}{4}$
or $\quad x = 2, y = 1$

o $x = 3y - 17$
sub.
$(3y-17)^2 + (3y-17) + 2y^2 - 52 = 0$
$y^2 - 9y + 20 = 0$
$(y-4)(y-5) = 0$
$y = 4$ or 5
$\therefore \quad x = -5, y = 4$
or $\quad x = -2, y = 5$

4 **a** subtracting
$-\frac{1}{y} + 2y + 1 = 0$
$-1 + 2y^2 + y = 0$
$2y^2 + y - 1 = 0$
$(2y-1)(y+1) = 0$
$y = -1$ or $\frac{1}{2}$
$\therefore \quad x = -5, y = -1$
or $\quad x = 4, y = \frac{1}{2}$

b $y = x - 5$
sub.
$x(x-5) = 6$
$x^2 - 5x - 6 = 0$
$(x+1)(x-6) = 0$
$x = -1$ or 6
$\therefore \quad x = -1, y = -6$
or $\quad x = 6, y = 1$

c $y = 7 - 4x$
sub.
$\frac{3}{x} - 2(7 - 4x) + 4 = 0$
$3 - 2x(7 - 4x) + 4x = 0$
$8x^2 - 10x + 3 = 0$
$(4x-3)(2x-1) = 0$
$x = \frac{1}{2}$ or $\frac{3}{4}$
$\therefore \quad x = \frac{1}{2}, y = 5$
or $\quad x = \frac{3}{4}, y = 4$

5 $5 - x = x^2 - 3x + 2$
$x^2 - 2x - 3 = 0$
$(x+1)(x-3) = 0$
$x = -1$ or 3
P and Q are the points $(-1, 6)$ and $(3, 2)$
$PQ^2 = (3+1)^2 + (2-6)^2$
$PQ = \sqrt{32} = 4\sqrt{2}$

6 $3^{x-1} = (3^2)^{2y} \quad \therefore \; x - 1 = 4y$
$(2^3)^{x-2} = (2^2)^{1+y} \quad \therefore \; 3x - 6 = 2 + 2y$
$6x - 16 = 4y$
$\Rightarrow \quad 6x - 16 = x - 1$
$x = 3$
$\therefore \quad x = 3, y = \frac{1}{2}$

7 $AB - A\sqrt{3} + 2B\sqrt{3} - 6 \equiv 9\sqrt{3} - 1$
A and B integers $\therefore \quad AB - 6 = -1 \quad (1)$ and $-A + 2B = 9 \quad (2)$
(2) $\Rightarrow \quad A = 2B - 9$
sub. (1) $\quad B(2B-9) - 6 = -1 \quad \Rightarrow \quad 2B^2 - 9B - 5 = 0$
$(2B+1)(B-5) = 0$
$B = -\frac{1}{2}$ or 5
B integer $\therefore \; B = 5 \quad \Rightarrow \quad A = 1, B = 5$

1 **a** $2x < 6$, $x < 3$ **b** $3x \geq 21$, $x \geq 7$ **c** $2x > 8$, $x > 4$ **d** $3x \leq 36$, $x \leq 12$

e $5x \geq -15$, $x \geq -3$ **f** $\frac{1}{3}x < 1$, $x < 3$ **g** $9x \geq 54$, $x \geq 6$ **h** $3x < -4$, $x < -\frac{4}{3}$

i $x < 14$ **j** $4x \leq -10$, $x \leq -\frac{5}{2}$ **k** $2 < 3x$, $x > \frac{2}{3}$ **l** $5 \geq \frac{1}{2}x$, $x \leq 10$

2 **a** $y > 7$ **b** $4p \leq 2$, $p \leq \frac{1}{2}$ **c** $6 < 2x$, $x > 3$

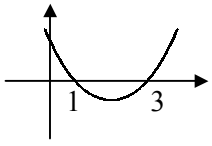
d $2a \geq 4$, $a \geq 2$ **e** $15 < 3u$, $u > 5$ **f** $2b \geq 9$, $b \geq \frac{9}{2}$

g $3x < -18$, $x < -6$ **h** $y \geq -13$ **i** $-20 \leq 4p$, $p \geq -5$

j $r - 2 > 6$, $r > 8$ **k** $3 - 6t \leq t - 4$, $7 \leq 7t$, $t \geq 1$ **l** $6 + 2x \geq 24 - 4x$, $6x \geq 18$, $x \geq 3$

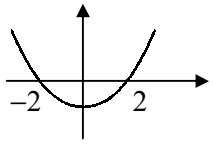
m $7y + 21 - 6y + 2 < 0$, $y < -23$ **n** $20 - 8x > 21 - 6x$, $-1 > 2x$, $x < -\frac{1}{2}$ **o** $12u - 3 - 5u + 15 < 9$, $7u < -3$, $u < -\frac{3}{7}$

3 **a** $(x - 1)(x - 3) < 0$



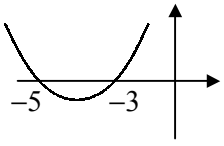
$\therefore 1 < x < 3$

b $(x + 2)(x - 2) \leq 0$



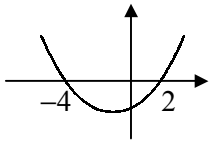
$\therefore -2 \leq x \leq 2$

c $(x + 5)(x + 3) < 0$



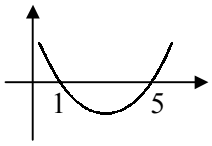
$\therefore -5 < x < -3$

d $x^2 + 2x - 8 \leq 0$, $(x + 4)(x - 2) \leq 0$



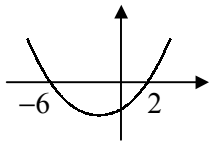
$\therefore -4 \leq x \leq 2$

e $(x - 1)(x - 5) > 0$



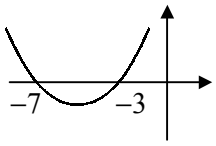
$\therefore x < 1$ or $x > 5$

f $x^2 + 4x - 12 > 0$, $(x + 6)(x - 2) > 0$



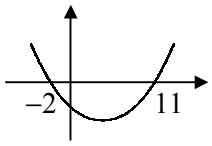
$\therefore x < -6$ or $x > 2$

g $(x + 7)(x + 3) \geq 0$



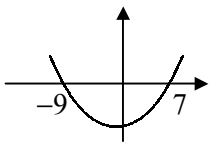
$\therefore x \leq -7$ or $x \geq -3$

h $x^2 - 9x - 22 < 0$, $(x + 2)(x - 11) < 0$



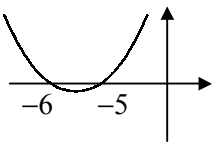
$\therefore -2 < x < 11$

i $x^2 + 2x - 63 \geq 0$, $(x + 9)(x - 7) \geq 0$



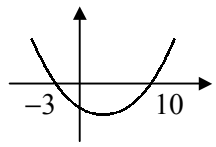
$\therefore x \leq -9$ or $x \geq 7$

j $(x + 6)(x + 5) > 0$



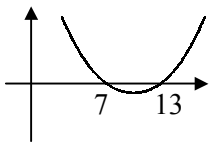
$\therefore x < -6$ or $x > -5$

k $x^2 - 7x - 30 < 0$, $(x + 3)(x - 10) < 0$



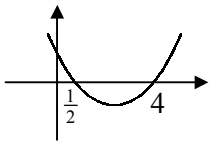
$\therefore -3 < x < 10$

l $x^2 - 20x + 91 \geq 0$, $(x - 7)(x - 13) \geq 0$



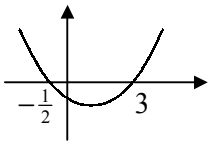
$\therefore x \leq 7$ or $x \geq 13$

4 **a** $(2x - 1)(x - 4) \leq 0$



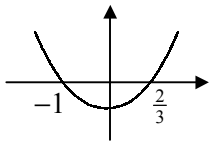
$\therefore \frac{1}{2} \leq x \leq 4$

b $(2r + 1)(r - 3) < 0$



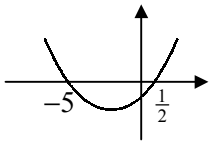
$\therefore -\frac{1}{2} < r < 3$

c $3p^2 + p - 2 \leq 0$
$(3p - 2)(p + 1) \leq 0$



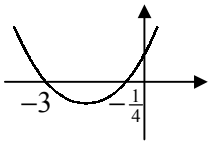
$\therefore -1 \leq p \leq \frac{2}{3}$

d $(2y - 1)(y + 5) > 0$



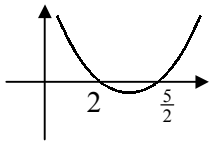
$\therefore y < -5$ or $y > \frac{1}{2}$

e $(4m + 1)(m + 3) < 0$



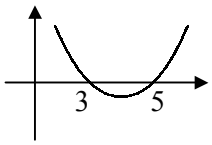
$\therefore -3 < m < -\frac{1}{4}$

f $2x^2 - 9x + 10 \geq 0$
$(2x - 5)(x - 2) \geq 0$



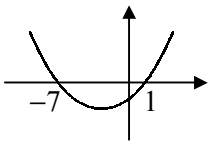
$\therefore x \leq 2$ or $x \geq \frac{5}{2}$

g $a^2 - 8a + 15 < 0$
$(a - 3)(a - 5) < 0$



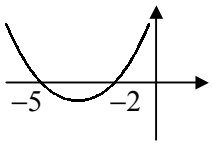
$\therefore 3 < a < 5$

h $x^2 + 4x \leq 7 - 2x$
$x^2 + 6x - 7 \leq 0$
$(x + 7)(x - 1) \leq 0$



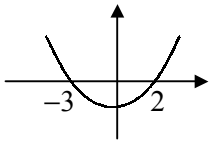
$\therefore -7 \leq x \leq 1$

i $y^2 + 9y > 2y - 10$
$y^2 + 7y + 10 > 0$
$(y + 5)(y + 2) > 0$



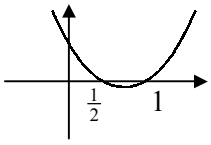
$\therefore y < -5$ or $y > -2$

j $2x^2 + x > x^2 + 6$
$x^2 + x - 6 > 0$
$(x + 3)(x - 2) < 0$



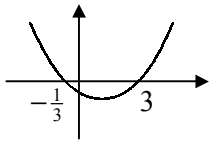
$\therefore -3 < x < -2$

k $5u - 6u^2 < 3 - 4u$
$2u^2 - 3u + 1 > 0$
$(2u - 1)(u - 1) > 0$



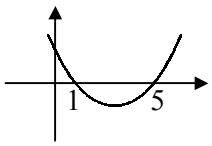
$\therefore u < \frac{1}{2}$ or $u > 1$

l $2t + 3 \geq 3t^2 - 6t$
$3t^2 - 8t - 3 \leq 0$
$(3t + 1)(t - 3) \leq 0$



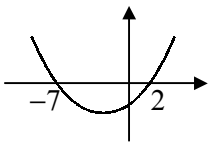
$\therefore -\frac{1}{3} \leq t \leq 3$

m $y^2 - 4y + 4 \leq 2y - 1$
$y^2 - 6y + 5 \leq 0$
$(y - 1)(y - 5) \leq 0$



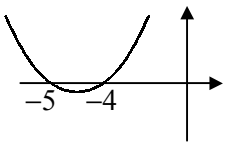
$\therefore 1 \leq y \leq 5$

n $p^2 + 5p + 6 \geq 20$
$p^2 + 5p - 14 \geq 0$
$(p + 7)(p - 2) \geq 0$



$\therefore p \leq -7$ or $p \geq 2$

o $26 + 4x < 6 - 5x - x^2$
$x^2 + 9x + 20 < 0$
$(x + 5)(x + 4) < 0$



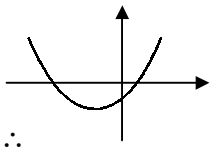
$\therefore -5 < x < -4$

5 **a** for critical values

$x = \dfrac{-2 \pm \sqrt{4+4}}{2}$

$x = \dfrac{-2 \pm 2\sqrt{2}}{2}$

$x = -1 \pm \sqrt{2}$



$\therefore$

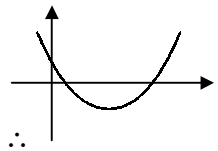
$-1-\sqrt{2} < x < -1+\sqrt{2}$

b for critical values

$x = \dfrac{6 \pm \sqrt{36-16}}{2}$

$x = \dfrac{6 \pm 2\sqrt{5}}{2}$

$x = 3 \pm \sqrt{5}$



$\therefore$

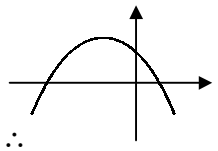
$x < 3-\sqrt{5}$ or $x > 3+\sqrt{5}$

c for critical values

$x = \dfrac{6 \pm \sqrt{36+44}}{-2}$

$x = \dfrac{6 \pm 4\sqrt{5}}{-2}$

$x = -3 \pm 2\sqrt{5}$



$\therefore$

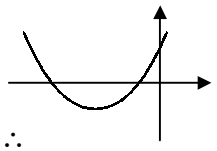
$-3-2\sqrt{5} < x < -3+2\sqrt{5}$

d for critical values

$x = \dfrac{-4 \pm \sqrt{16-4}}{2}$

$x = \dfrac{-4 \pm 2\sqrt{3}}{2}$

$x = -2 \pm \sqrt{3}$



$\therefore$

$x \leq -2-\sqrt{3}$ or $x \geq -2+\sqrt{3}$

6 **a** equal roots

$\therefore\ b^2 - 4ac = 0$

$36 - 4k = 0$

$k = 9$

b real and distinct roots

$\therefore\ b^2 - 4ac > 0$

$4 - 4k > 0$

$4 > 4k$

$k < 1$

c no real roots

$\therefore\ b^2 - 4ac < 0$

$9 - 4k < 0$

$9 < 4k$

$k > \frac{9}{4}$

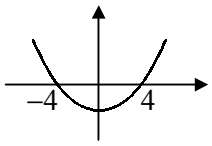
d real roots

$\therefore\ b^2 - 4ac \geq 0$

$k^2 - 16 \geq 0$

$(k+4)(k-4) \geq 0$

$k \leq -4$ or $k \geq 4$



e equal roots

$\therefore\ b^2 - 4ac = 0$

$1 + 4k = 0$

$k = -\frac{1}{4}$

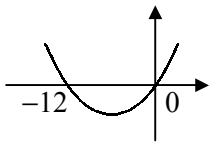
f no real roots

$\therefore\ b^2 - 4ac < 0$

$k^2 + 12k < 0$

$k(k+12) < 0$

$-12 < k < 0$



g real and distinct roots

$\therefore\ b^2 - 4ac > 0$

$4 - 4(k-2) > 0$

$12 > 4k$

$k < 3$

h equal roots

$\therefore\ b^2 - 4ac = 0$

$k^2 - 8k = 0$

$k(k-8) = 0$

$k = 0$ or 8

i no real roots

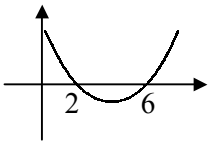
$\therefore\ b^2 - 4ac < 0$

$k^2 - 4(2k-3) < 0$

$k^2 - 8k + 12 < 0$

$(k-2)(k-6) < 0$

$2 < k < 6$



j real roots

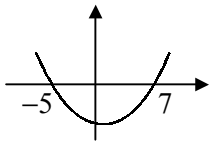
$\therefore\ b^2 - 4ac \geq 0$

$(k-1)^2 - 36 \geq 0$

$k^2 - 2k - 35 \geq 0$

$(k+5)(k-7) \geq 0$

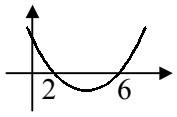
$k \leq -5$ or $k \geq 7$



1 **a** $4 > \frac{3}{2}y$

$y < \frac{8}{3}$

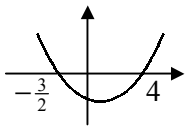
b $(x-2)(x-6) \geq 0$



$\therefore x \leq 2$ or $x \geq 6$

2 $2n^2 - 5n - 12 < 0$

$(2n+3)(n-4) < 0$



$-\frac{3}{2} < n < 4$

n integer $\therefore$ $n = -1, 0, 1, 2, 3$

3 **a** $(x+8) \geq 1.5 \times x$

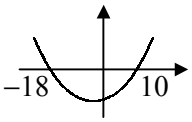
$8 \geq 0.5x$

$x \leq 16$

b $x(x+8) \geq 180$

$x^2 + 8x - 180 \geq 0$

$(x+18)(x-10) \geq 0$



$x \leq -18$ or $x \geq 10$

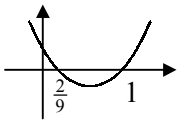
but $x > 0$ (width > 0)

and $x \leq 16$ $\therefore$ $10 \leq x \leq 16$

4 $9x^2 - 6x + 1 < 5x - 1$

$9x^2 - 11x + 2 < 0$

$(9x-2)(x-1) < 0$



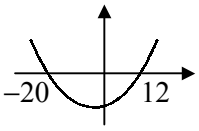
$\frac{2}{9} < x < 1$

5 $x = y + 8$

sub. $y(y+8) \leq 240$

$y^2 + 8y - 240 \leq 0$

$(y+20)(y-12) \leq 0$



$-20 \leq y \leq 12$

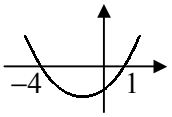
$x + y = y + 8 + y = 2y + 8$

$\therefore$ max value of $(x+y) = 2(12) + 8 = 32$

6 $3t^2 - 11t - 4 \geq 2t^2 - 14t$

$t^2 + 3t - 4 \geq 0$

$(t+4)(t-1) \geq 0$



$t \leq -4$ or $t \geq 1$

7 **a** $2x^2 + 2x - kx + 8 = 0$

real and distinct roots

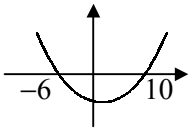
$\therefore$ $b^2 - 4ac > 0$

$(2-k)^2 - 64 > 0$

$4 - 4k + k^2 - 64 > 0$

$k^2 - 4k - 60 > 0$

b $(k+6)(k-10) > 0$



$k < -6$ or $k > 10$

8 let height be h $\therefore$ $h^2 = (3r-4)^2 - r^2$

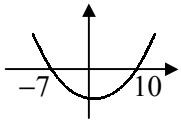
but $h \leq 24$

$\therefore$ $h^2 \leq 24^2$

$(3r-4)^2 - r^2 \leq 576$

$r^2 - 3r - 70 \leq 0$

$(r+7)(r-10) \leq 0$



$-7 \leq r \leq 10$

$\therefore$ maximum value of $r = 10$