Constant Acceleration : Vertical Motion - SOLUTIONS

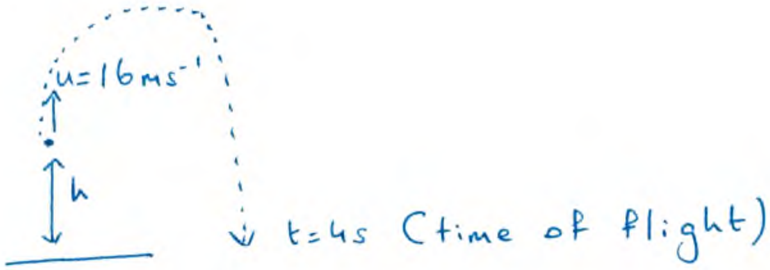
1. A stone is thrown vertically upwards with speed 16 m s^{-1} from a point h metres above the ground. The stone hits the ground 4 s later. Find

(*a*) the value of h, **(3)**

(*b*) the speed of the stone as it hits the ground. **(3)**

Jan 06 Q1

a)



+ve ↑

$s = -h$ (↓)
$u = 16$
v
$a = -9.8$ (↓)
$t = 4$

(↑) $s = ut + \frac{1}{2}at^2$

$-h = 16(4) + \frac{1}{2}(-9.8)(4^2)$

$-h = 64 - 4.9(16)$

$-h = 14.4$

$\therefore h = 14.4$ m

b) +ve ↑

$s = -h$ (↓)
$u = 16$
$v = ?$
$a = -9.8$(↓)
$t = 4$

(↑) $v = u + at$

$= 16 + (-9.8)(4)$

$= (-)23.2$ ms^{-1}

negative as direction is opposite to +ve direction

$\therefore$ speed of stone $= 23.2$ ms^{-1}

Alternative method

+ve (↑) $v^2 = u^2 + 2as$

$= 16^2 + 2(-9.8)(-14.4)$

$v^2 = 538.24$

$v = \sqrt{538.24} = 23.2$ ms^{-1}

2. A ball is projected vertically upwards with speed 21 m s^{-1} from a point A, which is 1.5 m above the ground. After projection, the ball moves freely under gravity until it reaches the ground. Modelling the ball as a particle, find

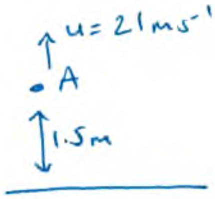
(*a*) the greatest height above A reached by the ball, **(3)**

(*b*) the speed of the ball as it reaches the ground, **(3)**

(*c*) the time between the instant when the ball is projected from A and the instant when the ball reaches the ground. **(4)**

Jan 07 Q5

a,



+ve ↑

$s = ?$
$u = 21$
$v = 0$
$a = -9.8$ (↓)
t

At maximum height, $v = 0$

(↑) Using $v^2 = u^2 + 2as$

$0 = 21^2 + 2(-9.8)s$

$s = \frac{441}{19.6}$

$= 22.5\text{ m}$

b, +ve ↑

$s = -1.5$ (↓)
$u = 21$
$v = ?$
$a = -9.8$ (↓)
$t =$

(↑) $v^2 = u^2 + 2as$

$v^2 = 21^2 + 2(-9.8)(-1.5)$

$v^2 = 470.4$

$v = \sqrt{470.4} = 21.68\ldots$

$= 22\text{ ms}^{-1}$ (2 S.F)

c) +ve ↑

$s = -1.5$ (↓)
$u = 21$
v
$a = -9.8$ (↓)
$t = ?$

(↑) $s = ut + \frac{1}{2}at^2$

$-1.5 = 21t + \frac{1}{2}(-9.8)t^2$

$-1.5 = 21t - 4.9t^2$

$4.9t^2 - 21t - 1.5 = 0$

$t = \frac{21 \pm \sqrt{(-21)^2 - 4(4.9)(-1.5)}}{2(4.9)}$

$= 4.355\ldots$

$= 4.4\text{ s}$ (2 s.f)

Alternative method

+ve ↑ $v = u + at$

$-\sqrt{470.4} = 21 + (-9.8)t$

$t = 4.4\text{ s}$

3. A firework rocket starts from rest at ground level and moves vertically. In the first 3 s of its motion, the rocket rises 27 m. The rocket is modelled as a particle moving with constant acceleration a m s^{-2}. Find

$\rightarrow u = 0$

(*a*) the value of a, **(2)**

(*b*) the speed of the rocket 3 s after it has left the ground. **(2)**

After 3 s, the rocket burns out. The motion of the rocket is now modelled as that of a particle moving freely under gravity.

(*c*) Find the height of the rocket above the ground 5 s after it has left the ground. **(4)**

Jan 08 Q2

a) +ve ↑
$s = 27$
$u = 0$
v
$a = ?$
$t = 3$

$(\uparrow)\ s = ut + \frac{1}{2}at^2$

$27 = 0 + \frac{1}{2}a(3)^2$

$a = \frac{27}{4.5} = 6 \text{ ms}^{-2}$

b) +ve ↑
$s = 27$
$u = 0$
$v = ?$
$a = 6$
$t = 3$

$(\uparrow)\ v^2 = u^2 + 2as$

$v^2 = 0 + 2(6)(27)$

$v^2 = 324$

$v = \sqrt{324}$

$= 18 \text{ ms}^{-1}$

c) from 3s to 5s,

height reached $\Rightarrow s = ut + \frac{1}{2}at^2$

+ve ↑
$s = ?$
$u = 18$ ← velocity after 3 s
v
$a = -9.8\ (\downarrow)$
$t = 2$

$= 18(2) + \frac{1}{2}(-9.8)(2^2)$

$= 16.4 \text{ m}$

$\therefore$ Height above ground $= 16.4 + 27$

$= 43.4 \text{ m}$

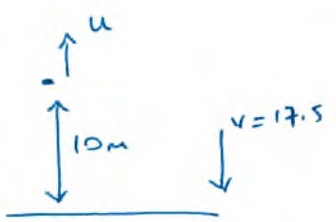
4. At time $t = 0$, a particle is projected vertically upwards with speed u m s^{-1} from a point 10 m above the ground. At time T seconds, the particle hits the ground with speed 17.5 m s^{-1}. Find

(*a*) the value of u, **(3)**

(*b*) the value of T **(4)**

June 08 Q2

a)



+ve ↑

$s = -10$ (↓)
$u = u$
$v = -17.5$ (↓)
$a = -9.8$ (↓)
$t = T$

(↑) $v^2 = u^2 + 2as$

$(-17.5)^2 = u^2 + 2(-9.8)(-10)$

$306.25 = u^2 + 196$

$u^2 = 110.25$

$u = \sqrt{110.25} = 10.5 \text{ ms}^{-1}$

b) +ve ↑

$s = -10$ (↓)
$u = 10.5$
$v = -17.5$ (↓)
$a = -9.8$ (↓)
$t = T$

(↑) $s = ut + \frac{1}{2}at^2$

$-10 = 10.5t + \frac{1}{2}(-9.8)t^2$

$-10 = 10.5t - 4.9t^2$

$4.9t^2 - 10.5t - 10 = 0$

$$t = \frac{10.5 \pm \sqrt{(-10.5)^2 - 4(4.9)(-10)}}{2(4.9)}$$

$t = \frac{20}{7}$ s or $2\frac{6}{7}$ s or 2.9 s

Alternative method

+ve

(↑) $s = \left(\frac{u+v}{2}\right)t$

$-10 = \left(\frac{10.5 + {}^{-}17.5}{2}\right)t$

$t = \frac{20}{7}$ s

5. A ball is projected vertically upwards with a speed of 14.7 m s^{-1} from a point which is 49 m above horizontal ground. Modelling the ball as a particle moving freely under gravity, find

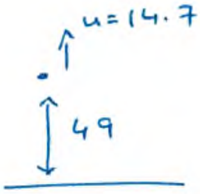
(*a*) the greatest height, above the ground, reached by the ball, **(4)**

(*b*) the speed with which the ball first strikes the ground, **(3)**

(*c*) the total time from when the ball is projected to when it first strikes the ground. **(3)**

June 10 Q6

a)



greatest height $\Rightarrow v = 0$

(+ve ↑)
$s = ?$
$u = 14.7$
$v = 0$
$a = -9.8$ (↓)
t

(↑) $v^2 = u^2 + 2as$

$0 = 14.7^2 + 2(-9.8)s$

$s = 11.025$

∴ Height above the ground

$= 49 + 11.025$

$= 60.025$

$= 60$ m (2 s.f)

(b) (+ve ↑)
$s = -49$ (↓)
$u = 14.7$
$v = ?$
$a = -9.8$ (↓)
t

(↑) $v^2 = u^2 + 2as$

$= 14.7^2 + 2(-9.8)(-49)$

$v^2 = 1176.49$

$v = \sqrt{1176.49} = 34.3$ ms^{-1}

(c) (+ve ↑)
$s = -49$ (↓)
$u = 14.7$
$v = -34.3$ (↓)
$a = -9.8$ (↓)
$t = ?$

(+ve ↑) $v = u + at$

$-34.3 = 14.7 + (-9.8)t$

$t = 5$s

Alternative method

(↑) $s = ut + \frac{1}{2}at^2$

$-49 = 14.7t + \frac{1}{2}(-9.8)($

$-49 = 14.7t - 4.9t^2$

$4.9t^2 - 14.7t - 49 = 0$

$t = \frac{+14.7 \pm \sqrt{14.7^2 - 4(4.9)(-}}{2(4.9)}$

$= 5$s

6. A ball is thrown vertically upwards with speed u m s^{-1} from a point P at height h metres above the ground. The ball hits the ground 0.75 s later. The speed of the ball immediately before it hits the ground is 6.45 m s^{-1}. The ball is modelled as a particle.

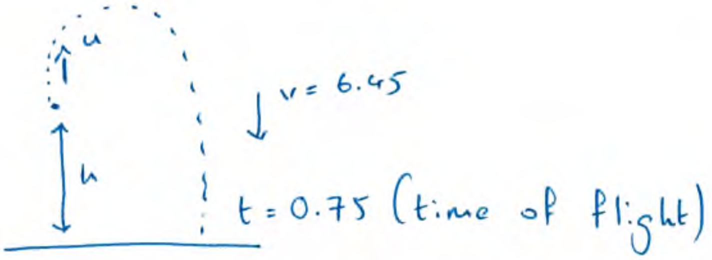
(*a*) Show that $u = 0.9$. **(3)**

(*b*) Find the height above P to which the ball rises before it starts to fall towards the ground again. **(2)**

(*c*) Find the value of h. **(3)**

Jan 11 Q2

a)



+ve ↑
$s = -h$ (↓)
$u = u$
$v = -6.45$ (↓)
$a = -9.8$ (↓)
$t = 0.75$

(↑) $v = u + at$

$-6.45 = u + (-9.8)(0.75)$

$u = -6.45 + (9.8 \times 0.75)$

$u = 0.9$ m s^{-1}

(b) Max height ⇒ $v = 0$

+ve (↑)
$s =$
$u = 0.9$
$v = 0$
$a = -9.8$ (↓)
$t =$

$v^2 = u^2 + 2as$

$0 = 0.9^2 + 2(-9.8)s$

$s = 0.04132\ldots$

$s = 0.041$ m (2 S.F)

(c) +ve ↑
$s = -h$ (↓)
$u = 0.9$
$v = -6.45$ (↓)
$a = -9.8$ (↓)
$t = 0.75$

(↑) $s = ut + \frac{1}{2}at^2$

$-h = 0.9(0.75) + \frac{1}{2}(-9.8)(0.75)^2$

$-h = 0.675 - 2.75625$

$-h = -2.08125$

$h = 2.08125$

$\therefore h = 2.1$ (2 S.F)