1. A stone is thrown vertically upwards with speed $16 \text{ m s}^{-1}$ from a point $h$ metres above the ground. The stone hits the ground 4 s later. Find

(a) the value of $h$, 

(b) the speed of the stone as it hits the ground.

\[ s = ut - \frac{1}{2}at^2 \]

\[ -h = 16(4) + \frac{1}{2}(-9.8)(4^2) \]

\[ -h = 64 - 4.9(16) \]

\[ -h = 14.4 \]

\[ h = 14.4 \text{ m} \]

\[ v = u + at \]

\[ v = 16 + (-9.8)(4) \]

\[ v = -23.2 \text{ m s}^{-1} \]

\[ \text{negative as direction is opposite to } +ve \text{ direction} \]

\[ \text{speed of stone} = 23.2 \text{ m s}^{-1} \]
2. A ball is projected vertically upwards with speed 21 m s\(^{-1}\) from a point \(A\), which is 1.5 m above the ground. After projection, the ball moves freely under gravity until it reaches the ground. Modelling the ball as a particle, find

(a) the greatest height above \(A\) reached by the ball, 

(b) the speed of the ball as it reaches the ground, 

(c) the time between the instant when the ball is projected from \(A\) and the instant when the ball reaches the ground.

\(\text{Jan 07 Q5}\)

(a) Using \(v^2 = u^2 + 2as\)

\[
0 = 21^2 + 2(-9.8)s \\
s = \frac{441}{19.6} \\
\approx 22.5 \text{ m}
\]

(b) \(s = -1.5 \text{ m}\)

\[
u = \frac{v}{t} = \frac{21}{4.4} = 4.8 \text{ m/s}
\]

(c) \(s = ut + \frac{1}{2}at^2\)

\[
-1.5 = 21t + \frac{1}{2}(-9.8)t^2 \\
-1.5 = 21t - 4.9t^2 \\
4.9t^2 - 21t + 1.5 = 0 \\
t = 21 \pm \sqrt{(21)^2 - 4(4.9)(-1.5)} \\
= 21 \pm \sqrt{441 + 34.2} \\
= 4.355... \\
= 4.4 \text{ s (2 s.f.)}
\]
3. A firework rocket starts from rest at ground level and moves vertically. In the first 3 s of its motion, the rocket rises 27 m. The rocket is modelled as a particle moving with constant acceleration \( a \) m s\(^{-2}\). Find

(a) the value of \( a \),
(2)

(b) the speed of the rocket 3 s after it has left the ground.
(2)

After 3 s, the rocket burns out. The motion of the rocket is now modelled as that of a particle moving freely under gravity.

(c) Find the height of the rocket above the ground 5 s after it has left the ground.
(4)

Jan 08 Q2

(a) \( S = ut + \frac{1}{2} at^2 \)

\[ S = 27, \quad u = 0, \quad t = 3 \]

\[ a = \frac{27}{4.5} = 6 \text{ m s}^{-2} \]

(b) \( v^2 = u^2 + 2as \)

\[ v = \sqrt{v^2}, \quad v = 18 \text{ m s}^{-1} \]

(c) \( S = ut + \frac{1}{2} at^2 \)

\[ S = 18, \quad u = 18, \quad a = -9.8 \]

\[ t = 2 \text{ s} \]

\[ \text{Height above ground} = 16.4 + 27 = 43.4 \text{ m} \]
4. At time \( t = 0 \), a particle is projected vertically upwards with speed \( u \) m s\(^{-1}\) from a point 10 m above the ground. At time \( T \) seconds, the particle hits the ground with speed 17.5 m s\(^{-1}\). Find

(a) the value of \( u \),

(b) the value of \( T \).

\[ \begin{align*}
\text{(a)} & \quad \text{the value of } u, \\
\text{(b)} & \quad \text{the value of } T.
\end{align*} \]

\[ \begin{align*}
\text{Alternative method} \\
\text{(t)} & \quad S = \frac{(u + \nu) t}{2} \\
-10 & = \frac{(10.5 + -17.5) t}{2} \\
t & = \frac{20}{7} \text{ s}
\end{align*} \]

\[ \begin{align*}
\text{(t)} & \quad V^2 = u^2 + 2as \\
(-17.5)^2 & = u^2 + 2 (-9.8) (-10) \\
306.25 & = u^2 + 196 \\
u^2 & = 110.25 \\
u & = \sqrt{110.25} = 10.5 \text{ m s}^{-1}
\end{align*} \]

\[ \begin{align*}
\text{(t)} & \quad S = ut + \frac{1}{2}at^2 \\
-10 & = 10.5 t + \frac{1}{2} (-9.8) t^2 \\
-10 & = 10.5 t - 4.9t^2 \\
4.9t^2 & = 10.5 t - 10 \\
t & = 10.5 \pm \sqrt{(-10.5)^2 - 4(4.9)(-10)} \\
t & = 2 \frac{1}{2} \text{ s or } 2.9 \text{ s}
\end{align*} \]
5. A ball is projected vertically upwards with a speed of 14.7 m s\(^{-1}\) from a point which is 49 m above horizontal ground. Modelling the ball as a particle moving freely under gravity, find

(a) the greatest height, above the ground, reached by the ball,

(b) the speed with which the ball first strikes the ground,

(c) the total time from when the ball is projected to when it first strikes the ground.

\[ s = \text{?} \]
\[ u = 14.7 \]
\[ v = \text{?} \]
\[ a = -9.8 \] (\(t\))
\[ t = \text{?} \]

\[ v^2 = u^2 + 2as \]
\[ 0 = 14.7^2 + 2(-9.8)s \]
\[ s = 11.025 \]

\[ \text{Height above the ground} \]
\[ = 49 + 11.025 \]
\[ = 60.025 \]
\[ = 60 \text{ m (2 s.f)} \]

\[ v^2 = 1176.49 \]
\[ v = \sqrt{1176.49} = 34.3 \text{ m s}^{-1} \]

\[ s = -49 \] (\(t\))
\[ u = 14.7 \]
\[ v = -34.3 \] (\(t\))
\[ a = -9.8 \] (\(t\))
\[ t = ? \]

\[ v = u + at \]
\[ -34.3 = (14.7 + (-9.8))t \]
\[ t = \frac{5s}{5s} \]

**Alternative method**

\[ s = ut + \frac{1}{2}at^2 \]
\[ -49 = 14.7t + \frac{1}{2}(-9.8)t^2 \]
\[ -49 = 14.7t - 4.9t^2 \]
\[ 4.9t^2 - 14.7t - 49 = 0 \]
\[ t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ t = \frac{14.7 \pm \sqrt{14.7^2 - 4(4.9)(-49)}}{2(4.9)} \]
\[ = 5s \]
6. A ball is thrown vertically upwards with speed \( u \) m s\(^{-1}\) from a point \( P \) at height \( h \) metres above the ground. The ball hits the ground 0.75 s later. The speed of the ball immediately before it hits the ground is 6.45 m s\(^{-1}\). The ball is modelled as a particle.

(a) Show that \( u = 0.9 \).

(b) Find the height above \( P \) to which the ball rises before it starts to fall towards the ground again.

(c) Find the value of \( h \).

\[
\begin{align*}
\text{(a)} & \quad \text{Show that } u = 0.9. \\
\text{(b)} & \quad \text{Find the height above } P \text{ to which the ball rises before it starts to fall towards the ground again.} \\
\text{(c)} & \quad \text{Find the value of } h.
\end{align*}
\]