Exponential and Natural Logarithms - Edexcel Past Exam Questions

1. A particular species of orchid is being studied. The population p at time t years after the study started is assumed to be

$$p = \frac{2800ae^{0.2t}}{1 + ae^{0.2t}}, \text{ where } a \text{ is a constant.}$$

Given that there were 300 orchids when the study started,

(*a*) show that $a = 0.12$, **(3)**

(*b*) use the equation with $a = 0.12$ to predict the number of years before the population of orchids reaches 1850. **(4)**

(*c*) Show that $p = \dfrac{336}{0.12 + e^{-0.2t}}$. **(1)**

(*d*) Hence show that the population cannot exceed 2800. **(2)**

June 05 Q7

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2. A heated metal ball is dropped into a liquid. As the ball cools, its temperature, T °C, t minutes after it enters the liquid, is given by

$$T = 400e^{-0.05t} + 25, \quad t \geq 0.$$

(*a*) Find the temperature of the ball as it enters the liquid. **(1)**

(*b*) Find the value of t for which $T = 300$, giving your answer to 3 significant figures. **(4)**

(*c*) Find the rate at which the temperature of the ball is decreasing at the instant when $t = 50$. Give your answer in °C per minute to 3 significant figures. **(3)**

(*d*) From the equation for temperature T in terms of t, given above, explain why the temperature of the ball can never fall to 20 °C. **(1)**

June 06 Q4

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3. Find the exact solutions to the equations

(*a*) $\ln x + \ln 3 = \ln 6$, **(2)**

(*b*) $e^x + 3e^{-x} = 4$. **(4)**

June 07 Q1

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4. The amount of a certain type of drug in the bloodstream t hours after it has been taken is given by the formula

$$x = De^{-\frac{1}{8}t},$$

where x is the amount of the drug in the bloodstream in milligrams and D is the dose given in milligrams.

A dose of 10 mg of the drug is given.

(*a*) Find the amount of the drug in the bloodstream 5 hours after the dose is given.
Give your answer in mg to 3 decimal places. **(2)**

A second dose of 10 mg is given after 5 hours.

(*b*) Show that the amount of the drug in the bloodstream 1 hour after the second dose is 13.549 mg to 3 decimal places. **(2)**

No more doses of the drug are given. At time T hours after the second dose is given, the amount of the drug in the bloodstream is 3 mg.

(*c*) Find the value of T. **(3)**

June 07 Q8

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5. The radioactive decay of a substance is given by

$$R = 1000e^{-ct}, \quad t \geq 0.$$

where R is the number of atoms at time t years and c is a positive constant.

(*a*) Find the number of atoms when the substance started to decay. **(1)**
It takes 5730 years for half of the substance to decay.

(*b*) Find the value of c to 3 significant figures. **(4)**

(*c*) Calculate the number of atoms that will be left when $t = 22\,920$. **(2)**

(*d*) Sketch the graph of R against t . **(2)**

Jan 08 Q5

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6. Rabbits were introduced onto an island. The number of rabbits, P, t years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}, \quad t \in \mathbb{R}, \quad t \geq 0.$$

(a) Write down the number of rabbits that were introduced to the island. **(1)**

(b) Find the number of years it would take for the number of rabbits to first exceed 1000. **(2)**

(c) Find $\frac{dP}{dt}$. **(2)**

(d) Find P when $\frac{dP}{dt} = 50$. **(3)**

June 09 Q3

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7. Find the exact solutions to the equations

(a) $\ln(3x - 7) = 5$, **(3)**

(b) $3^x e^{7x+2} = 15$. **(5)**

Jan 10 Q9

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8. (a) Simplify fully

$$\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}.$$

(3)

Given that

$$\ln(2x^2 + 9x - 5) = 1 + \ln(x^2 + 2x - 15), \quad x \neq -5,$$

(b) find x in terms of e. **(4)**

June 10 Q8

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9. Joan brings a cup of hot tea into a room and places the cup on a table. At time t minutes after Joan places the cup on the table, the temperature, θ °C, of the tea is modelled by the equation

$$\theta = 20 + Ae^{-kt},$$

where A and k are positive constants.

Given that the initial temperature of the tea was 90 °C,

(a) find the value of A. **(2)**

The tea takes 5 minutes to decrease in temperature from 90 °C to 55 °C.

(b) Show that $k = \frac{1}{5} \ln 2$. **(3)**

(c) Find the rate at which the temperature of the tea is decreasing at the instant when $t = 10$. Give your answer, in °C per minute, to 3 decimal places. **(3)**

Jan 11 Q4

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10. The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$$m = pe^{-kt},$$

where k and p are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(a) Write down the value of p. **(1)**

(b) Show that $k = \frac{1}{4} \ln 3$. **(4)**

(c) Find the value of t when $\frac{dm}{dt} = -0.6 \ln 3$. **(6)**

June 11 Q5
