1. A drugs company claims that 75% of patients suffering from depression recover when treated with a new drug.

A random sample of 10 patients with depression is taken from a doctor’s records.

(a) Write down a suitable distribution to model the number of patients in this sample who recover when treated with the new drug. \(2\)

Given that the claim is correct,

(b) find the probability that the treatment will be successful for exactly 6 patients. \(2\)

The doctor believes that the claim is incorrect and the percentage who will recover is lower. From her records she took a random sample of 20 patients who had been treated with the new drug. She found that 13 had recovered.

(c) Stating your hypotheses clearly, test, at the 5% level of significance, the doctor’s belief. \(6\)

(d) From a sample of size 20, find the greatest number of patients who need to recover from the test in part (c) to be significant at the 1% level. \(4\)

2. A teacher thinks that 20% of the pupils in a school read the Deano comic regularly.

He chooses 20 pupils at random and finds 9 of them read the Deano.

(a) (i) Test, at the 5% level of significance, whether or not there is evidence that the percentage of pupils that read the Deano is different from 20%. State your hypotheses clearly.

(ii) State all the possible numbers of pupils that read the Deano from a sample of size 20 that will make the test in part (a)(i) significant at the 5% level. \(9\)
3. It is known from past records that 1 in 5 bowls produced in a pottery have minor defects. To monitor production a random sample of 25 bowls was taken and the number of such bowls with defects was recorded.

(a) Using a 5% level of significance, find critical regions for a two-tailed test of the hypothesis that 1 in 5 bowls have defects. The probability of rejecting, in either tail, should be as close to 2.5% as possible.

(b) State the actual significance level of the above test.

At a later date, a random sample of 20 bowls was taken and 2 of them were found to have defects.

(c) Test, at the 10% level of significance, whether or not there is evidence that the proportion of bowls with defects has decreased. State your hypotheses clearly.

4. Past records from a large supermarket show that 20% of people who buy chocolate bars buy the family size bar. On one particular day a random sample of 30 people was taken from those that had bought chocolate bars and 2 of them were found to have bought a family size bar.

(a) Test, at the 5% significance level, whether or not the proportion \( p \) of people who bought a family size bar of chocolate that day had decreased. State your hypotheses clearly.

5. Linda regularly takes a taxi to work five times a week. Over a long period of time she finds the taxi is late once a week. The taxi firm changes her driver and Linda thinks the taxi is late more often. In the first week, with the new driver, the taxi is late 3 times. You may assume that the number of times a taxi is late in a week has a Binomial distribution.

Test, at the 5% level of significance, whether or not there is evidence of an increase in the proportion of times the taxi is late. State your hypotheses clearly.

6. Dhriti grows tomatoes. Over a period of time, she has found that there is a probability 0.3 of a ripe tomato having a diameter greater than 4 cm. She decides to try a new fertiliser. In a random sample of 40 ripe tomatoes, 18 have a diameter greater than 4 cm. Dhriti claims that the new fertiliser has increased the probability of a ripe tomato being greater than 4 cm in diameter.

Test Dhriti’s claim at the 5% level of significance. State your hypotheses clearly.
7. Sue throws a fair coin 15 times and records the number of times it shows a head.

(a) State the distribution to model the number of times the coin shows a head. 

Find the probability that Sue records

(b) exactly 8 heads, 

(c) at least 4 heads.

Sue has a different coin which she believes is biased in favour of heads. She throws the coin 15 times and obtains 13 heads.

(d) Test Sue’s belief at the 1% level of significance. State your hypotheses clearly.

8. A single observation $x$ is to be taken from a Binomial distribution $B(20, p)$.

This observation is used to test $H_0: p = 0.3$ against $H_1: p \neq 0.3$.

(a) Using a 5% level of significance, find the critical region for this test. The probability of rejecting either tail should be as close as possible to 2.5%.

(b) State the actual significance level of this test.

The actual value of $x$ obtained is 3.

(c) State a conclusion that can be drawn based on this value, giving a reason for your answer.

9. Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager questions whether or not there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

(a) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager’s question. You should state the probability of rejection in each tail which should be less than 0.05.

(b) Write down the actual significance level of a test based on your critical region from part (a).

The manager found that 11 customers from the sample of 20 had bought baked beans in single tins.

(c) Comment on this finding in the light of your critical region found in part (a).
10. (a) Define the critical region of a test statistic. (2)

A discrete random variable $X$ has a Binomial distribution $B(30, p)$. A single observation is used to test $H_0 : p = 0.3$ against $H_1 : p \neq 0.3$

(b) Using a 1% level of significance find the critical region of this test. You should state the probability of rejection in each tail which should be as close as possible to 0.005. (5)

(c) Write down the actual significance level of the test. (1)

The value of the observation was found to be 15.

(d) Comment on this finding in light of your critical region. (2)

11. A company claims that a quarter of the bolts sent to them are faulty. To test this claim the number of faulty bolts in a random sample of 50 is recorded.

(a) Give two reasons why a binomial distribution may be a suitable model for the number of faulty bolts in the sample. (2)

(b) Using a 5% significance level, find the critical region for a two-tailed test of the hypothesis that the probability of a bolt being faulty is $\frac{1}{4}$. The probability of rejection in either tail should be as close as possible to 0.025. (3)

(c) Find the actual significance level of this test. (2)

In the sample of 50 the actual number of faulty bolts was 8.

(d) Comment on the company’s claim in the light of this value. Justify your answer. (2)

The machine making the bolts was reset and another sample of 50 bolts was taken. Only 5 were found to be faulty.

(e) Test at the 1% level of significance whether or not the probability of a faulty bolt has decreased. State your hypotheses clearly. (6)

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12. A student takes a multiple choice test. The test is made up of 10 questions each with 5 possible answers. The student gets 4 questions correct. Her teacher claims she was guessing the answers. Using a one-tailed test, at the 5% level of significance, test whether or not there is evidence to reject the teacher’s claim.

State your hypotheses clearly. (6)

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13. A shopkeeper knows, from past records, that 15% of customers buy an item from the display next to the till. After a refurbishment of the shop, he takes a random sample of 30 customers and finds that only 1 customer has bought an item from the display next to the till.

(a) Stating your hypotheses clearly, and using a 5% level of significance, test whether or not there has been a change in the proportion of customers buying an item from the display next to the till. (6)

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