



## Trigonometric Equations and Identities - Edexcel Past Exam Questions **MARK SCHEME**

### Question 1 : Jan 05 Q4

Question Number	Scheme	Marks
4.	<p>(a) <math>5(1 - \sin^2 x) = 3(1 + \sin x)</math>  <math>5 - 5\sin^2 x = 3 + 3\sin x</math>  <math>0 = 5\sin^2 x + 3\sin x - 2</math> *</p> <p>(b) <math>0 = (5\sin x - 2)(\sin x + 1)</math>  <math>\sin x = \frac{2}{5}, -1</math> (both)  <math>\sin x = \frac{2}{5} \Rightarrow x = \underline{23.6}</math> (<math>\alpha = 23.6</math> or <math>156.4</math>)  <math>\phantom{\sin x = \frac{2}{5} \Rightarrow x = } , \underline{156.4}</math> (<math>180 - \alpha</math>)  <math>\sin x = -1 \Rightarrow x = \underline{270}</math></p>	<p>M1</p> <p>A1 cso (2)</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>B1 (5)</p> <p>(7)</p>
	<p>(a) M1 for use of <math>\cos^2 x = 1 - \sin^2 x</math>. Condone missing ( )</p> <p>(b) 1<sup>st</sup> M1 for attempt to solve <math>\rightarrow \sin x =</math>  1<sup>st</sup> B1 for correct solution, <math>\alpha</math> to <math>\sin x = \frac{2}{5}</math>. Must be 1 d.p.  2<sup>nd</sup> M1 for <math>180 - \alpha</math>, accept nearest degree or awrt.</p> <p>Answer only in (b) scores M0A0 but then could score B1M1B1</p> <p>Incorrect factorisation probably only gets <math>\frac{2}{5}</math>.</p>	



## Question 2 : June 05 Q5

Question number	Scheme	Marks
5.	<p>(a) <math>(x + 10 =) \quad 60 \quad \alpha</math>  <math>120</math> (M: <math>180 - \alpha</math> or <math>\pi - \alpha</math>)  <math>x = 50 \quad x = 110</math> (or 50.0 and 110.0) (M: Subtract 10)</p> <p>(b) <math>(2x =) \quad 154.2 \quad \beta</math> Allow a.w.r.t. 154 or a.w.r.t. 2.69 (radians)  <math>205.8</math> (M: <math>360 - \beta</math> or <math>2\pi - \beta</math>)  <math>x = 77.1 \quad x = 102.9</math> (M: Divide by 2)</p>	<p>B1  M1  M1 A1 (4)</p> <p>B1  M1  M1 A1 (4)</p> <p><b>8</b></p>
	<p>(a) First M: Must be subtracting from 180 <u>before</u> subtracting 10.  (b) First M: Must be subtracting from 360 <u>before</u> dividing by 2, <u>or</u> dividing by 2 then subtracting from 180.</p> <p>In each part:  Extra solutions outside 0 to 180 : Ignore.  Extra solutions between 0 and 180 : A0.</p> <p><u>Alternative for (b): (double angle formula)</u></p> <p><math>1 - 2\sin^2 x = -0.9</math>      <math>2\sin^2 x = 1.9</math>      B1  <math>\sin x = \sqrt{0.95}</math>      M1  <math>x = 77.1</math>  <math>x = 180 - 77.1 = 102.9</math>      M1 A1</p>	



### Question 3 : Jan 06 Q8

Question number	Scheme	Marks
8.	<p>(a) <math>\sin(\theta + 30) = \frac{3}{5}</math> <span style="float: right;">(<math>\frac{3}{5}</math> on RHS)</span></p> <p style="text-align: center;"><math>\theta + 30 = 36.9</math> <span style="float: right;">(<math>\alpha = \text{AWRT } 37</math>)</span></p> <p>or <math>\theta = 143.1</math> <span style="float: right;">(<math>180 - \alpha</math>)</span></p> <p style="text-align: center;"><u><math>\theta = 6.9, 113.1</math></u></p> <p>(b) <math>\tan \theta = \pm 2</math> or <math>\sin \theta = \pm \frac{2}{\sqrt{5}}</math> or <math>\cos \theta = \pm \frac{1}{\sqrt{5}}</math></p> <p>(<math>\tan \theta = 2 \Rightarrow</math>) <math>\theta = \underline{63.4}</math> <span style="float: right;">(<math>\beta = \text{AWRT } 63.4</math>)</span></p> <p>or <math>\underline{243.4}</math> <span style="float: right;">(<math>180 + \beta</math>)</span></p> <p>(<math>\tan \theta = -2 \Rightarrow</math>) <math>\theta = \underline{116.6}</math> <span style="float: right;">(<math>180 - \beta</math>)</span></p> <p>or <math>\underline{296.6}</math> <span style="float: right;">(<math>180 + \text{their } 116.6</math>)</span></p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1cao (4)</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>M1 (5)</p> <p><b>9</b></p>
	<p>(a) M1 for 180 – their first solution. Must be at the correct stage i.e. for <math>\theta + 30</math></p> <p>(b) ALL M marks in (b) must be for <math>\theta = \dots</math></p> <p>1<sup>st</sup> M1 for 180 + their first solution</p> <p>2<sup>nd</sup> M1 for 180 – their first solution</p> <p>3<sup>rd</sup> M1 for 180+ their 116.6 or 360 – their first solution</p> <p><u>Answers Only</u> can score full marks in both parts</p> <p><u>Not 1 d.p.:</u> loses A1 in part (a). In (b) all answers are AWRT.</p> <p>Ignore extra solutions outside range</p> <p><u>Radians</u> Allow M marks for consistent work with radians only, but all A and B marks for angles must be in degrees. Mixing degrees and radians is M0.</p>	



#### Question 4 : June 06 Q6

Question number	Scheme	Marks
6.	<p>(a) <math>\tan \theta = 5</math></p> <p>(b) <math>\tan \theta = k</math>      <math>(\theta = \tan^{-1} k)</math></p> <p><math>\theta = 78.7, \quad 258.7</math>      (Accept awrt)</p>	<p>B1      (1)</p> <p>M1</p> <p>A1, A1ft      (3)</p> <p><b>4</b></p>
	<p>(a) Must be seen explicitly, e.g. <math>\tan \theta = \tan^{-1} 5 = 78.7</math> or equiv. is B0, unless <math>\tan \theta = 5</math> is also seen.</p> <p>(b) The M mark may be implied by working in (a).</p> <p>A1ft for <math>180 + \alpha</math>. (<math>\alpha \neq k</math>).</p> <p>Answers in radians would lose both the A marks.</p> <p>Extra answers between 0 and 360: Deduct the final mark.</p> <p><u>Alternative:</u></p> <p>Using <math>\cos^2 \theta = 1 - \sin^2 \theta</math> (or equiv.) and proceeding to <math>\sin \theta = k</math> (or equiv.): M1 then A marks as in main scheme.</p>	



# Question 5 : Jan 08 Q4

Question Number	Scheme	Marks
(a)	$3 \sin^2 \theta - 2 \cos^2 \theta = 1$ $3 \sin^2 \theta - 2(1 - \sin^2 \theta) = 1$ (M1: Use of $\sin^2 \theta + \cos^2 \theta = 1$ ) $3 \sin^2 \theta - 2 + 2 \sin^2 \theta = 1$ $5 \sin^2 \theta = 3$ cso AG	M1 A1 (2)
(b)	$\sin^2 \theta = \frac{3}{5}$ , so $\sin \theta = (\pm)\sqrt{0.6}$ Attempt to solve both $\sin \theta = +..$ and $\sin \theta = - ...$ (may be implied by later work) $\theta = 50.7685^\circ$ awrt $\theta = 50.8^\circ$ (dependent on first M1 only) $\theta (= 180^\circ - 50.7685^\circ)$ ; $= 129.23...^\circ$ awrt $129.2^\circ$ [f.t. dependent on first M and 3rd M] $\sin \theta = -\sqrt{0.6}$ $\theta = 230.785^\circ$ and $309.23152^\circ$ awrt $230.8^\circ, 309.2^\circ$ (both)	M1 M1 A1 M1; A1 ✓ M1A1 (7) [9]
Notes:	(a) N.B: AG; need to see at least one line of working after substituting $\cos^2 \theta$ (b) First M1: Using $5 \sin^2 \theta = 3$ to find value for $\sin \theta$ or $\theta$ $\left[ \text{Allow such results as } \sin \theta = \frac{3}{5}, \sin \theta = \frac{\sqrt{3}}{5} \dots \text{for M1} \right]$ Second M1: Considering the $-$ value for $\sin \theta$ . (usually later) First A1: Given for awrt $50.8^\circ$ . <b>Not</b> dependent on second M. Third M1: For $(180 - \text{candidate}'s 50.8)^\circ$ , need not see written down Final M1: <b>Dependent</b> on second M (but <b>may be implied by answers</b> ) For $(180 + \text{candidate}'s 50.8)^\circ$ or $(360 - \text{candidate}'s 50.8)^\circ$ or equiv. Final A1: Requires both values. ( <b>no follow through</b> ) [ Finds $\cos^2 \theta = k$ ( $k = 2/5$ ) and so $\cos \theta = (\pm)...$ M1, then mark equivalently] <b>NB</b> Candidates who <b>only</b> consider positive value for $\sin \theta$ can score max of 4 marks: M1M0A1M1A1M0A0 – Very common. Candidates who score first M1 but have wrong $\sin \theta$ can score maximum $M1M1A0M1A\checkmark M1A0$ <b>SC</b> Candidates who obtain one value from each set, e.g $50.8$ and $309.2$ $M1M1(\text{bod})A1M0A0M1(\text{bod})A0$ Extra values out of range – no penalty <b>Any very tricky or " outside scheme methods" , send to TL</b>	



# Question 6 : June 08 Q9

Question Number	Scheme	Marks
(a)	45 $(\alpha)$ $180 - \alpha$ , Add 20 (for at least one angle) 65 155	B1 M1 M1 A1 (4)
(b)	120 or 240 $(\beta)$ : $360 - \beta$ , $360 + \beta$ Dividing by 3 (for at least one angle) 40 80 160 200 280 320	B1 M1 M1 M1 A1 A1 (6)
		(10 marks)



# Question 7 : Jan 09 Q8

Question Number	Scheme	Marks
(a)	$4(1 - \cos^2 x) + 9 \cos x - 6 = 0$ $4 \cos^2 x - 9 \cos x + 2 = 0$ (*)	M1 A1 (2)
(b)	$(4 \cos x - 1)(\cos x - 2) = 0$ $\cos x = \dots, \frac{1}{4}$ $x = 75.5$ ( $\alpha$ ) $360 - \alpha, 360 + \alpha$ or $720 - \alpha$ $284.5, 435.5, 644.5$	M1 A1  B1 M1, M1 A1 (6) <b>[8]</b>
(a)	<b>M1:</b> Uses $\sin^2 x = 1 - \cos^2 x$ (may omit bracket) <b>not</b> $\sin^2 x = \cos^2 x - 1$ <b>A1:</b> Obtains the printed answer without error – <b>must have</b> $= 0$	
(b)	<b>M1:</b> Solves the quadratic with usual conventions <b>A1:</b> Obtains $\frac{1}{4}$ accurately- ignore extra answer 2 but penalise e.g. -2. <b>B1:</b> allow answers which round to 75.5 <b>M1:</b> $360 - \alpha$ ft their value, <b>M1:</b> $360 + \alpha$ ft their value or $720 - \alpha$ ft <b>A1:</b> Three <b>and only three</b> correct exact answers in the range achieves the mark	
Special cases	In part (b) Error in solving quadratic $(4\cos x - 1)(\cos x + 2)$ Could yield, M1A0B1M1M1A1 losing one mark for the error  Works in radians: Complete work in radians :Obtains 1.3 B0. Then allow M1 M1 for $2\pi - \alpha, 2\pi + \alpha$ or $4\pi - \alpha$ Then gets 5.0, 7.6, 11.3 A0 so 2/4 Mixed answer 1.3, $360 - 1.3, 360 + 1.3, 720 - 1.3$ still gets B0M1M1A0	





# Question 8 : June 09 Q7

Question Number	Scheme	Marks
Q (i)	$\tan \theta = -1 \Rightarrow \theta = -45, 135$ $\sin \theta = \frac{2}{5} \Rightarrow \theta = 23.6, 156.4$ (AWRT: 24, 156)	B1, B1ft B1, B1ft (4)
(ii)	$4 \sin x = \frac{3 \sin x}{\cos x}$ $4 \sin x \cos x = 3 \sin x \Rightarrow \sin x(4 \cos x - 3) = 0$ Other possibilities (after squaring): $\sin^2 x(16 \sin^2 x - 7) = 0$ , $(16 \cos^2 x - 9)(\cos^2 x - 1) = 0$ $x = 0, 180$ <u>seen</u> $x = 41.4, 318.6$ (AWRT: 41, 319)	M1  M1  B1, B1 B1, B1ft (6)
<b>[10]</b>		
(i)	1 <sup>st</sup> B1 for $-45$ seen ( $\alpha$ , where $ \alpha  < 90$ ) 2 <sup>nd</sup> B1 for $135$ seen, <u>or ft</u> $(180 + \alpha)$ if $\alpha$ is negative, or $(\alpha - 180)$ if $\alpha$ is positive. If $\tan \theta = k$ is obtained from <u>wrong working</u> , 2 <sup>nd</sup> B1ft is still available. 3 <sup>rd</sup> B1 for awrt 24 ( $\beta$ , where $ \beta  < 90$ ) 4 <sup>th</sup> B1 for awrt 156, <u>or ft</u> $(180 - \beta)$ if $\beta$ is positive, or $-(180 + \beta)$ if $\beta$ is negative. If $\sin \theta = k$ is obtained from <u>wrong working</u> , 4 <sup>th</sup> B1ft is still available.	
(ii)	1 <sup>st</sup> M1 for use of $\tan x = \frac{\sin x}{\cos x}$ . Condone $\frac{3 \sin x}{3 \cos x}$ . 2 <sup>nd</sup> M1 for correct work leading to 2 factors (may be implied). 1 <sup>st</sup> B1 for 0, 2 <sup>nd</sup> B1 for 180. 3 <sup>rd</sup> B1 for awrt 41 ( $\gamma$ , where $ \gamma  < 180$ ) 4 <sup>th</sup> B1 for awrt 319, <u>or ft</u> $(360 - \gamma)$ . If $\cos \theta = k$ is obtained from <u>wrong working</u> , 4 <sup>th</sup> B1ft is still available. N.B. Losing $\sin x = 0$ usually gives a maximum of 3 marks M1M0B0B0B1B1 <u>Alternative:</u> (squaring both sides) 1 <sup>st</sup> M1 for squaring both sides and using a 'quadratic' identity. e.g. $16 \sin^2 \theta = 9(\sec^2 \theta - 1)$ 2 <sup>nd</sup> M1 for reaching a factorised form. e.g. $(16 \cos^2 \theta - 9)(\cos^2 \theta - 1) = 0$ Then marks are equivalent to the main scheme. Extra solutions, if not rejected, are penalised as in the main scheme.  <u>For both parts of the question:</u> <u>Extra solutions outside required range:</u> Ignore <u>Extra solutions inside required range:</u> For each pair of B marks, the 2 <sup>nd</sup> B mark is lost if there are two correct values and one or more extra solution(s), e.g. $\tan \theta = -1 \Rightarrow \theta = 45, -45, 135$ is B1 B0 <u>Answers in radians:</u> Loses a maximum of 2 B marks in the whole question (to be deducted at the first and second occurrence).	





# Question 9 : Jan 10 Q2

Question Number	Scheme	Marks
(a)	$5 \sin x = 1 + 2(1 - \sin^2 x)$ $2 \sin^2 x + 5 \sin x - 3 = 0 \quad (*)$	M1 A1cso (2)
(b)	$(2s-1)(s+3) = 0 \text{ giving } s =$ $[\sin x = -3 \text{ has no solution}] \text{ so } \sin x = \frac{1}{2}$ $\therefore x = 30, 150$	M1 A1 B1, B1ft (4) [6]
(a)	<p>M1 for a correct method to change <math>\cos^2 x</math> into <math>\sin^2 x</math> (must use <math>\cos^2 x = 1 - \sin^2 x</math>)</p> <p>A1 need 3 term quadratic printed in any order with =0 included</p>	
(b)	<p>M1 for attempt to solve given quadratic (usual rules for solving quadratics) (can use any variable here, <math>s, y, x</math>, or <math>\sin x</math>)</p> <p>A1 requires no incorrect work seen and is for <math>\sin x = \frac{1}{2}</math> or <math>x = \sin^{-1} \frac{1}{2}</math></p> <p><math>y = \frac{1}{2}</math> is A0 (unless followed by <math>x = 30</math>)</p> <p>B1 for 30 (<math>\alpha</math>) not dependent on method</p> <p>2<sup>nd</sup> B1 for <math>180 - \alpha</math> provided in required range (otherwise <math>540 - \alpha</math>)</p> <p><u>Extra solutions outside required range:</u> Ignore</p> <p><u>Extra solutions inside required range:</u> Lose final B1</p> <p><u>Answers in radians:</u> Lose final B1</p> <p>S.C. Merely writes down two correct answers is M0A0B1B1</p> <p>Or <math>\sin x = \frac{1}{2} \therefore x = 30, 150</math> is M1A1B1B1</p> <p>Just gives one answer : 30 only is M0A0B1B0 or 150 only is M0A0B0B1</p> <p>NB Common error is to factorise wrongly giving <math>(2 \sin x + 1)(\sin x - 3) = 0</math></p> <p><math>[\sin x = 3 \text{ gives no solution}] \sin x = -\frac{1}{2} \Rightarrow x = 210, 330</math></p> <p>This earns M1 A0 B0 B1ft</p> <p>Another common error is to factorise correctly <math>(2 \sin x - 1)(\sin x + 3) = 0</math> and follow this with <math>\sin x = \frac{1}{2}, \sin x = 3</math> then <math>x = 30^\circ, 150^\circ</math></p> <p>This would be M1 A0 B1 B1</p>	



# Question 10 : June 10 Q5

Question Number	Scheme	Marks
	<p>(a) <math>\tan \theta = \frac{2}{5}</math> (or 0.4) (i.s.w. if a value of <math>\theta</math> is subsequently found)</p> <p>Requires the correct value with no incorrect working seen.</p>	B1 (1)
	<p>(b) awrt 21.8 (<math>\alpha</math>)</p> <p>(Also allow awrt 68.2, ft from <math>\tan \theta = \frac{5}{2}</math> in (a), but no other ft)</p> <p>(This value must be seen in part (b). It may be implied by a correct solution, e.g. 10.9)</p> <p>180 + <math>\alpha</math> (= 201.8), or 90 + (<math>\alpha/2</math>) (if division by 2 has already occurred) (<math>\alpha</math> found from <math>\tan 2x = \dots</math> or <math>\tan x = \dots</math> or <math>\sin 2x = \pm \dots</math> or <math>\cos 2x = \pm \dots</math>)</p> <p>360 + <math>\alpha</math> (= 381.8), or 180 + (<math>\alpha/2</math>) (<math>\alpha</math> found from <math>\tan 2x = \dots</math> or <math>\sin 2x = \dots</math> or <math>\cos 2x = \dots</math>)</p> <p>OR 540 + <math>\alpha</math> (= 561.8), or 270 + (<math>\alpha/2</math>) (<math>\alpha</math> found from <math>\tan 2x = \dots</math>)</p> <p>Dividing at least one of the angles by 2 (<math>\alpha</math> found from <math>\tan 2x = \dots</math> or <math>\sin 2x = \dots</math> or <math>\cos 2x = \dots</math>)</p> <p><math>x = 10.9, 100.9, 190.9, 280.9</math> (Allow awrt)</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1 (5)</p>

(b) Extra solution(s) in range: Loses the final A mark.

Extra solutions outside range: Ignore (whether correct or not).

Common answers:

10.9 and 100.9 would score B1 M1 M0 M1 A0 (Ensure that these M marks are awarded)

10.9 and 190.9 would score B1 M0 M1 M1 A0 (Ensure that these M marks are awarded)

Alternatives:

$$(i) 2 \cos 2x - 5 \sin 2x = 0 \quad R \cos(2x + \lambda) = 0 \quad \lambda = 68.2 \Rightarrow 2x + 68.2 = 90 \quad \text{B1}$$

$$2x + \lambda = 270 \quad \text{M1}$$

$$2x + \lambda = 450 \quad \text{or} \quad 2x + \lambda = 630 \quad \text{M1}$$

$$\text{Subtracting } \lambda \text{ and dividing by 2 (at least once)} \quad \text{M1}$$

$$(ii) 25 \sin^2 2x = 4 \cos^2 2x = 4(1 - \sin^2 2x)$$

$$29 \sin^2 2x = 4 \quad 2x = 21.8 \quad \text{B1}$$

The M marks are scored as in the main scheme, but extra solutions will be likely, losing the A mark.

Using radians:

B1: Can be given for awrt 0.38 ( $\beta$ )

M1: For  $\pi + \beta$  or  $180 + \beta$

M1: For  $2\pi + \beta$  or  $3\pi + \beta$  (Must now be consistently radians)

M1: For dividing at least one of the angles by 2

A1: For this mark, the answers must be in degrees.

(Correct) answers only (or by graphical methods):

B and M marks can be awarded by implication, e.g.

10.9 scores B1 M0 M0 M1 A0

10.9, 100.9 scores B1 M1 M0 M1 A0

10.9, 100.9, 190.9, 280.9 scores full marks.

Using 11, etc. instead of 10.9 can still score the M marks by implication.



# Question 11 : Jan11 Q7

Question Number	Scheme	Marks
(a)	$3\sin^2 x + 7\sin x = \cos^2 x - 4; \quad 0 \leq x < 360^\circ$ $3\sin^2 x + 7\sin x = (1 - \sin^2 x) - 4$ $4\sin^2 x + 7\sin x + 3 = 0 \quad \text{AG}$	M1 A1 * cso (2)
(b)	$(4\sin x + 3)(\sin x + 1) \{= 0\}$ $\sin x = -\frac{3}{4}, \quad \sin x = -1$ $( \alpha  = 48.59\dots)$ $x = 180 + 48.59 \quad \text{or} \quad x = 360 - 48.59$ $x = 228.59\dots, \quad x = 311.41\dots$ $\{\sin x = -1\} \Rightarrow x = 270$	Valid attempt at factorisation and $\sin x = \dots$ Both $\sin x = -\frac{3}{4}$ and $\sin x = -1$ . Either $(180 +  \alpha )$ or $(360 -  \alpha )$ Both awrt 228.6 and awrt 311.4 270 M1 A1 dM1 A1 B1 (5) [7]
<b>Notes</b>		
(a)	M1 for a correct method to change $\cos^2 x$ into $\sin^2 x$ (must use $\cos^2 x = 1 - \sin^2 x$ ). Note that applying $\cos^2 x = \sin^2 x - 1$ , scores M0. A1 for obtaining the printed answer without error (except for implied use of zero.), although the equation at the end of the proof must be $= 0$ . Solution just written only as above would score M1A1.	
(b)	1 <sup>st</sup> M1 for a valid attempt at factorisation, can use any variable here, $s$ , $y$ , $x$ or $\sin x$ , and an attempt to find at least one of the solutions. Alternatively, using a correct formula for solving the quadratic. Either the formula must be stated correctly or the correct form must be implied by the substitution. 1 <sup>st</sup> A1 for the two correct values of $\sin x$ . If they have used a substitution, a correct value of their $s$ or their $y$ or their $x$ . 2 <sup>nd</sup> M1 for solving $\sin x = -k$ , $0 < k < 1$ and realising a solution is either of the form $(180 +  \alpha )$ or $(360 -  \alpha )$ where $\alpha = \sin^{-1}(k)$ . Note that you <b>cannot</b> access this mark from $\sin x = -1 \Rightarrow x = 270$ . Note that this mark is dependent upon the 1 <sup>st</sup> M1 mark awarded. 2 <sup>nd</sup> A1 for both awrt 228.6 and awrt 311.4 B1 for 270. If there are any EXTRA solutions inside the range $0 \leq x < 360^\circ$ and the candidate would otherwise score FULL MARKS then withhold the final ba2 mark (the fourth mark in this part of the question). Also ignore EXTRA solutions outside the range $0 \leq x < 360^\circ$ . <b>Working in Radians:</b> Note the answers in radians are $x = 3.9896\dots, 5.4351\dots, 4.7123\dots$ If a candidate works in radians then mark part (b) as above awarding the 2 <sup>nd</sup> A1 for both awrt 4.0 and awrt 5.4 and the B1 for awrt 4.7 or $\frac{3\pi}{2}$ . If the candidate would then score FULL MARKS then withhold the final ba2 mark (the fourth mark in this part of the question). <b>No working:</b> Award B1 for 270 seen without any working. Award M0A0M1A1 for awrt 228.6 and awrt 311.4 seen without any working. Award M0A0M1A0 for any one of awrt 228.6 or awrt 311.4 seen without any working.	





# Question 12 : June 11 Q7

Question Number	Scheme	Marks
	<p><b>Note:</b> A similar scheme would apply for T&amp;I for candidates using their <math>a</math> and their <math>r</math>. So,...</p> <p>1<sup>st</sup> M1: For attempting to find one of the correct <math>S_n</math>'s either side (but next to) 1000.</p> <p>2<sup>nd</sup> M1: For one of these <math>S_n</math>'s correct for their <math>a</math> and their <math>r</math>. (You may need to get your calculators out!)</p> <p>3<sup>rd</sup> M1: For attempting to find both of the correct <math>S_n</math>'s either side (but next to) 1000.</p> <p>A1: Cannot be gained for wrong <math>a</math> and/or <math>r</math>.</p> <p><b><u>Trial &amp; Improvement Cumulative Approach:</u></b></p> <p>A similar scheme to T&amp;I will be applied here:</p> <p>1<sup>st</sup> M1: For getting as far as the cumulative sum of 13 terms. 2<sup>nd</sup> M1: (1)<math>S_{13}</math> = awrt 999.7 or truncated 999. 3<sup>rd</sup> M1: For getting as far as the cumulative sum to 14 terms. Also at this stage <math>S_{13} &lt; 1000</math> and <math>S_{14} &gt; 1000</math>. A1: BOTH (1)<math>S_{13}</math> = awrt 999.7 or truncated 999 AND (2) <math>S_{14}</math> = awrt 1005.8 or truncated 1005 AND <math>n = 14</math>.</p> <p><b><u>Trial &amp; Improvement Method:</u></b> for <math>(0.75)^n &lt; \frac{6}{258} = 0.0234375</math></p> <p>3<sup>rd</sup> M1: For evidence of examining both <math>n = 13</math> and <math>n = 14</math>.</p> <p>Eg: <math>(0.75)^{13} \{ = 0.023757... \}</math> and <math>(0.75)^{14} \{ = 0.0178179... \}</math></p> <p>A1: <math>n = 14</math></p> <p><b><u>Any misreads.</u></b> <math>S_n &gt; 10000</math> etc, please escalate up to your Team Leader.</p>	
(a)	<p>(a) <math>3\sin(x + 45^\circ) = 2</math>; <math>0 \leq x &lt; 360^\circ</math> (b) <math>2\sin^2 x + 2 = 7\cos x</math>; <math>0 \leq x &lt; 2\pi</math></p> <p><math>\sin(x + 45^\circ) = \frac{2}{3}</math>, so <math>(x + 45^\circ) = 41.8103... \quad (\alpha = 41.8103...)</math> <math>\sin^{-1}\left(\frac{2}{3}\right)</math> or awrt 41.8</p> <p>or awrt <math>0.73^\circ</math></p> <p>So, <math>x + 45^\circ = \{138.1897..., 401.8103...\}</math> <math>x + 45^\circ = \text{either "180 - their } \alpha" \text{ or "360}^\circ + \text{their } \alpha"</math> (<math>\alpha</math> could be in radians).</p> <p>and <math>x = \{93.1897..., 356.8103...\}</math> Either awrt <math>93.2^\circ</math> or awrt <math>356.8^\circ</math></p> <p>Both awrt <math>93.2^\circ</math> and awrt <math>356.8^\circ</math></p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[4]</p>
(b)	<p><math>2(1 - \cos^2 x) + 2 = 7\cos x</math> Applies <math>\sin^2 x = 1 - \cos^2 x</math></p> <p><math>2\cos^2 x + 7\cos x - 4 = 0</math> Correct 3 term, <math>2\cos^2 x + 7\cos x - 4 \{ = 0 \}</math></p> <p><math>(2\cos x - 1)(\cos x + 4) \{ = 0 \}</math>, <math>\cos x = ...</math> Valid attempt at solving and <math>\cos x = ...</math></p> <p><math>\cos x = \frac{1}{2}</math>, <math>\{ \cos x = -4 \}</math> <math>\cos x = \frac{1}{2}</math> (See notes.)</p> <p><math>\left( \beta = \frac{\pi}{3} \right)</math></p> <p><math>x = \frac{\pi}{3}</math> or <math>1.04719...^\circ</math> Either <math>\frac{\pi}{3}</math> or awrt <math>1.05^\circ</math></p> <p><math>x = \frac{5\pi}{3}</math> or <math>5.23598...^\circ</math> Either <math>\frac{5\pi}{3}</math> or awrt <math>5.24^\circ</math> or <math>2\pi - \text{their } \beta</math> (See notes.)</p>	<p>M1</p> <p>A1 oe</p> <p>M1</p> <p>A1 cso</p> <p>B1</p> <p>B1 ft</p> <p>[6]</p> <p>10</p>



Question Number	Scheme	Marks
(a)	<p>1<sup>st</sup> M1: can also be implied for <math>x = \text{awrt } -3.2</math></p> <p>2<sup>nd</sup> M1: for <math>x + 45^\circ =</math> either "<math>180 - \text{their } \alpha</math>" or "<math>360^\circ + \text{their } \alpha</math>". This can be implied by later working. The candidate's <math>\alpha</math> could also be in radians.</p> <p><b>Note that this mark is not</b> for <math>x =</math> either "<math>180 - \text{their } \alpha</math>" or "<math>360^\circ + \text{their } \alpha</math>".</p> <p><b>Note:</b> Imply the first two method marks or award M1M1A1 for either awrt <math>93.2^\circ</math> or awrt <math>356.8^\circ</math>.</p> <p><b>Note:</b> Candidates who apply the following incorrect working of <math>3\sin(x + 45^\circ) = 2</math>  <math>\Rightarrow 3(\sin x + \sin 45) = 2</math>, etc will usually score M0M0A0A0.</p> <p>If there are any EXTRA solutions inside the range <math>0 \leq x &lt; 360</math> and the candidate would otherwise score FULL MARKS then withhold the final aA2 mark (the final mark in this part of the question). Also ignore EXTRA solutions outside the range <math>0 \leq x &lt; 360</math>.</p> <p><b>Working in Radians:</b> Note the answers in radians are <math>x = \text{awrt } 1.6</math>, awrt <math>6.2</math></p> <p>If a candidate works in radians then mark part (a) as above awarding the A marks in the same way. If the candidate would then score FULL MARKS then withhold the final aA2 mark (the final mark in this part of the question.)</p> <p><b>No working:</b> Award M1M1A1A0 for one of awrt <math>93.2^\circ</math> or awrt <math>356.8^\circ</math> seen without any working.</p> <p>Award M1M1A1A1 for both awrt <math>93.2^\circ</math> and awrt <math>356.8^\circ</math> seen without any working.</p> <p>Allow benefit of the doubt (FULL MARKS) for final answer of <math>\sin x</math> {and not <math>x</math>} = {awrt <math>93.2</math>, awrt <math>356.8</math>}</p>	



Question Number	Scheme	Marks
(b)	<p>1<sup>st</sup> M1: for a correct method to use <math>\sin^2 x = 1 - \cos^2 x</math> on the given equation.            Give bod if the candidate omits the bracket when substituting for <math>\sin^2 x</math>, but  <math>2 - \cos^2 x + 2 = 7 \cos x</math>, without supporting working, (eg. seeing "<math>\sin^2 x = 1 - \cos^2 x</math>") would score 1<sup>st</sup> M0.            Note that applying <math>\sin^2 x = \cos^2 x - 1</math>, scores M0.            1<sup>st</sup> A1: for obtaining either <math>2\cos^2 x + 7\cos x - 4</math> or <math>-2\cos^2 x - 7\cos x + 4</math>.            1<sup>st</sup> A1: can also awarded for a correct three term equation eg. <math>2\cos^2 x + 7\cos x = 4</math> or <math>2\cos^2 x = 4 - 7\cos x</math> etc.            2<sup>nd</sup> M1: for a valid attempt at factorisation of a quadratic (either 2TQ or 3TQ) in <math>\cos</math>, can use any variable here, <math>c, y, x</math> or <math>\cos x</math>, and an attempt to find at least one of the solutions. See introduction to the Mark Scheme. <i>Alternatively</i>, using a correct formula for solving the quadratic. Either the formula must be stated correctly or the correct form must be implied by the substitution.            2<sup>nd</sup> A1: for <math>\cos x = \frac{1}{2}</math>, BY A CORRECT SOLUTION ONLY UP TO THIS POINT. Ignore extra answer of <math>\cos x = -4</math>, but penalise if candidate states an incorrect result e.g. <math>\cos x = 4</math>. If they have used a substitution, a correct value of their <math>c</math> or their <math>y</math> or their <math>x</math>.  <b>Note:</b> 2<sup>nd</sup> A1 for <math>\cos x = \frac{1}{2}</math> can be implied by later working.            1<sup>st</sup> B1: for either <math>\frac{\pi}{3}</math> or awrt 1.05°            2<sup>nd</sup> B1: for either <math>\frac{5\pi}{3}</math> or awrt 5.24° or can be ft from <math>2\pi</math> – their <math>\beta</math> or <math>360^\circ</math> – their <math>\beta</math> where <math>\beta = \cos^{-1}(k)</math>, such that <math>0 &lt; k &lt; 1</math> or <math>-1 &lt; k &lt; 0</math>, but <math>k \neq 0</math>, <math>k \neq 1</math> or <math>k \neq -1</math>.            If there are any EXTRA solutions inside the range <math>0 \leq x &lt; 2\pi</math> and the candidate would otherwise score FULL MARKS then withhold the final B2 mark (the final mark in this part of the question). Also ignore EXTRA solutions outside the range <math>0 \leq x &lt; 2\pi</math>.  <b>Working in Degrees:</b> Note the answers in degrees are <math>x = 60, 300</math>            If a candidate works in degrees then mark part (b) as above awarding the B marks in the same way. If the candidate would then score FULL MARKS then withhold the final B2 mark (the final mark in this part of the question).  <b>Answers from no working:</b>  <math>x = \frac{\pi}{3}</math> and <math>x = \frac{5\pi}{3}</math> scores M0A0M0A0B1B1,  <math>x = 60</math> and <math>x = 300</math> scores M0A0M0A0B1B0,  <math>x = \frac{\pi}{3}</math> ONLY or <math>x = 60</math> ONLY scores M0A0M0A0B1B0,  <math>x = \frac{5\pi}{3}</math> ONLY or <math>x = 120</math> ONLY scores M0A0M0A0B0B1.  <b>No working:</b> You cannot apply the ft in the B1ft if the answers are given with NO working.            Eg: <math>x = \frac{\pi}{5}</math> and <math>x = \frac{9\pi}{3}</math> FROM NO WORKING scores M0A0M0A0B0B0.  <b>For candidates using trial &amp; improvement, please forward these to your Team Leader.</b></p>	