Differentiating from First Principles - Edexcel Past Exam Questions SOLUTIONS

1. (a) Given that $y = 2x^2 - 5x + 3$, find $\frac{dy}{dx}$ from first principles. **[5]**

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\left[\begin{array}{l} f(x) = 2x^2 - 5x + 3 \\ f(x+h) = 2(x+h)^2 - 5(x+h) + 3 \end{array} \right]$$

Replace all 'x' by 'x+h'

$$= \lim_{h \to 0} \frac{2(x+h)^2 - 5(x+h) + 3 - [2x^2 - 5x + 3]}{h}$$

$$= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) - 5x - 5h + 3 - 2x^2 + 5x - 3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{- 5x} - 5h \cancel{+ 3} \cancel{- 2x^2} \cancel{+ 5x} \cancel{- 3}}{h}$$

$$= \lim_{h \to 0} \frac{4x\cancel{h} + 2h^{\cancel{2}}}{\cancel{h}}$$

$$= \lim_{h \to 0} 4x + 2h$$

As $h \to 0$ $2h \to 0$ $\therefore \frac{dy}{dx} = 4x$

(b) Given that $y = \frac{a}{x} + 2x^{\frac{3}{2}}$ and $\frac{dy}{dx} = 7$ when $x = 4$, find the value of the constant a. **[4]**

$$\left[\begin{array}{l} y = \frac{a}{x} + 2x^{\frac{3}{2}} \\ \quad = ax^{-1} + 2x^{\frac{3}{2}} \end{array} \right]$$

$$\frac{dy}{dx} = -ax^{-2} + 3x^{\frac{1}{2}}$$

$$\left.\frac{dy}{dx}\right|_{x=4} = -a(4)^{-2} + 3(4)^{\frac{1}{2}}$$

$$= -\frac{a}{4^2} + 3\sqrt{4}$$

$$= -\frac{a}{16} + (3 \times 2)$$

$$= -\frac{a}{16} + 6$$

$$\frac{dy}{dx} = 7 \Rightarrow -\frac{a}{16} + 6 = 7$$

$$-a + 96 = 112$$

$$a = 112 - 96 = 16$$

2. (a) Given that $y = x^2 - 3x + 4$, show from first principles that **[5]**

$$\frac{dy}{dx} = 2x - 3$$

$$\left[\begin{array}{l} f(x) = x^2 - 3x + 4 \\ f(x+h) = (x+h)^2 - 3(x+h) + 4 \end{array} \right]$$

Replace all 'x' by 'x+h'

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) + 4 - [x^2 - 3x + 4]}{h}$$

$$= \lim_{h \to 0} \frac{(\cancel{x^2} + 2xh + h^2 - \cancel{3x} - 3h \cancel{+4} \cancel{-x^2} \cancel{+3x} \cancel{-4})}{h}$$

$$= \lim_{h \to 0} \frac{2x\cancel{h} + h^{\cancel{2}} - 3\cancel{h}}{\cancel{h}}$$

$$= \lim_{h \to 0} 2x + h - 3$$

As $h \to 0$, $h \to 0$ $\quad \therefore \frac{dy}{dx} = 2x - 3$

(b) Differentiate $y = \frac{2}{x^2} + 7\sqrt{x}$ with respect to x. **[2]**

$$y = 2x^{-2} + 7x^{1/2}$$

$$\frac{dy}{dx} = (-2)(2)x^{-2-1} + (7)\left(\tfrac{1}{2}\right)x^{\frac{1}{2}-1}$$

$$= -4x^{-3} + \frac{7}{2}x^{-\frac{1}{2}}$$ ← You can leave your answer like this

$$= \frac{-4}{x^3} + \frac{7}{2\sqrt{x}}$$

3. Given that $y = x^2 - 7x + 2$, find $\frac{dy}{dx}$ from first principles. **[5]**

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\left[\begin{array}{l} f(x) = x^2 - 7x + 2 \\ f(x+h) = (x+h)^2 - 7(x+h) + 2 \end{array} \right]$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 7(x+h) + 2 - \left[x^2 - 7x + 2\right]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{7x} - 7h + \cancel{2} \cancel{- x^2} + \cancel{7x} \cancel{- 2}}{h}$$

$$= \lim_{h \to 0} \frac{2x\cancel{h} + h^{\cancel{2}} - 7\cancel{h}}{\cancel{h}}$$

$$= \lim_{h \to 0} 2x + h - 7$$

As $h \to 0$, $h \to 0$

$$\therefore \frac{dy}{dx} = 2x - 7$$

4. (a) Differentiate $y = x^2 - 6x + 2$ from first principles. **[5]**

$f(x) = x^2 - 6x + 2$

$f(x+h) = (x+h)^2 - 6(x+h) + 2$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 6(x+h) + 2 - [x^2 - 6x + 2]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{-6x} - 6h \cancel{+2} \cancel{-x^2} \cancel{+6x} \cancel{-2}}{h}$$

$$= \lim_{h \to 0} \frac{2x\cancel{h} + h^{\cancel{2}} - 6\cancel{h}}{\cancel{h}}$$

$$= \lim_{h \to 0} 2x + h - 6$$

As $h \to 0$, $h \to 0$ $\quad \therefore \dfrac{dy}{dx} = 2x - 6$

(b) Differentiate $\dfrac{3}{x^2} + x^{\frac{5}{2}}$ with respect to x. **[2]**

$$y = 3x^{-2} + x^{\frac{5}{2}}$$

$$\frac{dy}{dx} = (-2)(3)x^{-2-1} + x^{\frac{5}{2}-1}$$

$$= -6x^{-3} + x^{\frac{3}{2}}$$

$$= \frac{-6}{x^3} + x^{\frac{3}{2}}$$

5. (a) Given that $y = x^2 + 5x - 2$, find $\frac{dy}{dx}$ from first principles. **[5]**

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 5(x+h) - 2 - [x^2 + 5x - 2]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{5x} + 5h \cancel{-2} \cancel{-x^2} \cancel{-5x} \cancel{+2}}{h}$$

$$= \lim_{h \to 0} \frac{2x\cancel{h} + h^{\cancel{2}} + 5\cancel{h}}{\cancel{h}}$$

$$= \lim_{h \to 0} 2x + h + 5$$

As $h \to 0$, $h \to 0$ $\therefore \frac{dy}{dx} = 2x + 5$

(b) Differentiate $\frac{3}{x} - 2x^{\frac{5}{2}}$ with respect to x. **[4]**

$$y = 3x^{-1} - 2x^{\frac{5}{2}}$$

$$\frac{dy}{dx} = (-1)(3)x^{-1-1} - \left(\frac{5}{2}\right)(2)x^{\frac{5}{2}-1}$$

$$= -3x^{-2} - 5x^{\frac{3}{2}}$$

You can leave your answer like this

$$= \frac{-3}{x^2} - 5x^{\frac{3}{2}}$$

6. (a) Given that $y = 2x^2 + x + 3$, find $\frac{dy}{dx}$ from first principles. **[5]**

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x+h)^2 + (x+h) + 3 - [2x^2 + x + 3]}{h}$$

$$= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) + x + h + 3 - 2x^2 - x - 3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{+x} + h \cancel{+3} \cancel{-2x^2} \cancel{-x} \cancel{-3}}{h}$$

$$= \lim_{h \to 0} \frac{4xh + 2h^2 + h}{h} \qquad \left[\frac{h}{h} = 1 \right]$$

$$= \lim_{h \to 0} 4x + 2h + 1$$

As $h \to 0$, $2h \to 0$ $\quad \therefore \frac{dy}{dx} = 4x + 1$

(b) Given that **[5]**

$$y = \sqrt{x} + \frac{k}{x}$$

and that $\frac{dy}{dx} = 2$ when $x = 4$, find the value of the constant k. **[4]**

$$y = x^{\frac{1}{2}} + kx^{-1}$$

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - kx^{-2}$$

$$\left.\frac{dy}{dx}\right|_{x=4} = \frac{1}{2}(4)^{-\frac{1}{2}} - k(4^{-2})$$

$$= \frac{1}{2\sqrt{4}} - \frac{k}{4^2}$$

$$= \frac{1}{4} - \frac{k}{16}$$

$$\frac{dy}{dx} = 2 \Rightarrow \frac{1}{4} - \frac{k}{16} = 2$$

$$(\times 16) \quad 4 - k = 32$$

$$k = 4 - 32 = -28$$