**Algebraic Fractions - Edexcel Past Exam Questions MARK SCHEME**

## Question 1

| Question Number | Scheme | Marks |
|---|---|---|
| | $$\frac{5x+1}{(x+2)(x-1)} - \frac{3}{x+2}$$ | B1 |
| | $$= \frac{5x+1-3(x-1)}{(x+2)(x-1)}$$ <br> M1 for combining fractions even if the denominator is not lowest common | M1 |
| | $$= \frac{2x+4}{(x+2)(x-1)} = \frac{2(x+2)}{(x+2)(x-1)} = \frac{2}{x-1} \quad *$$ <br> M1 must have linear numerator | M1 A1 cso <br> (4) |

**Question 2**

| Question Number | Scheme | Marks |
|---|---|---|
| | $$x^2 - x - 2 = (x-2)(x+1) \qquad \text{At any stage}$$ | B1 |
| | $$\frac{2x^2 + 3x}{(2x+3)(x-2)} = \frac{x(2x+3)}{(2x+3)(x-2)} = \frac{x}{x-2}$$ | B1 |
| | $$\frac{2x^2+3x}{(2x+3)(x-2)} - \frac{6}{x^2-x-2} = \frac{x(x+1)-6}{(x-2)(x+1)}$$ | M1 |
| | $$= \frac{x^2+x-6}{(x-2)(x+1)}$$ | A1 |
| | $$= \frac{(x+3)(x-2)}{(x-2)(x+1)}$$ | M1 A1 |
| | $$= \frac{x+3}{x+1}$$ | A1 (7) |
| | | [7] |
| | **Alternative method** | |
| | $$x^2 - x - 2 = (x-2)(x+1) \qquad \text{At any stage}$$ | B1 |
| | $(2x+3)$ appearing as a factor of the numerator at any stage | B1 |
| | $$\frac{2x^2+3x}{(2x+3)(x-2)} - \frac{6}{(x-2)(x+1)} = \frac{(2x^2+3x)(x+1)-6(2x+3)}{(2x+3)(x-2)(x+1)}$$ | M1 |
| | $$= \frac{2x^3+5x^2-9x-18}{(2x+3)(x-2)(x+1)} \qquad \text{can be implied}$$ | A1 |
| | $$= \frac{(x-2)(2x^2+9x+9)}{(2x+3)(x-2)(x+1)} \quad \text{or} \quad \frac{(2x+3)(x^2+x-6)}{(2x+3)(x-2)(x+1)} \quad \text{or} \quad \frac{(x+3)(2x^2-x-6)}{(2x+3)(x-2)(x+1)}$$ | M1 |
| | Any one linear factor × quadratic | |
| | $$= \frac{(2x+3)(x-2)(x+3)}{(2x+3)(x-2)(x+1)} \qquad \text{Complete factors}$$ | A1 |
| | $$= \frac{x+3}{x+1}$$ | A1 (7) |

**Question 3**

| Question Number | Scheme | Marks |
|---|---|---|
| (a) | $\dfrac{(3x+2)(x-1)}{(x+1)(x-1)}, \quad = \quad \dfrac{3x+2}{x+1}$ <br><br> **Notes** <br> M1 attempt to factorise numerator, *usual rules* <br> B1 factorising denominator seen anywhere in (a), <br> A1 given answer <br> If factorisation of denom. not seen, correct answer implies B1 | M1B1, A1 (3) |
| (b) | Expressing over common denominator <br><br> $\dfrac{3x+2}{x+1} - \dfrac{1}{x(x+1)} = \dfrac{x(3x+2)-1}{x(x+1)}$ <br><br> [Or "Otherwise" : $\dfrac{(3x^2 - x - 2)x - (x-1)}{x(x^2-1)}$ ] <br><br> Multiplying out numerator and attempt to factorise <br><br> $\left[3x^2 + 2x - 1 \equiv (3x-1)(x+1)\right]$ <br><br> Answer: $\dfrac{3x-1}{x}$ | M1 <br><br><br><br> M1 <br><br><br> A1      (3) <br><br> **(6 marks)** |

**Question 4**

| | | |
|---|---|---|
| (a) $f(x) = \dfrac{(x+2)^2, -3(x+2)+3}{(x+2)^2}$ | M1 A1, A1 | |
| $= \dfrac{x^2+4x+4-3x-6+3}{(x+2)^2} = \dfrac{x^2+x+1}{(x+2)^2}$ **\***   cso | A1 | (4) |
| (b) $x^2+x+1 = \left(x+\dfrac{1}{2}\right)^2 + \dfrac{3}{4}, \quad > 0$ for all values of $x$. | M1 A1, A1 | **(3)** |
| (c) $f(x) = \dfrac{\left(x+\dfrac{1}{2}\right)^2 + \dfrac{3}{4}}{(x+2)^2}$ <br> Numerator is positive from (b) <br> $x \neq -2 \Rightarrow (x+2)^2 > 0$ (Denominator is positive) <br> Hence $f(x) > 0$ | B1 | (1) |
| | | [8] |
| *Alternative to* (b) <br> $\dfrac{d}{dx}(x^2+x+1) = 2x+1 = 0 \Rightarrow x = -\dfrac{1}{2} \Rightarrow x^2+x+1 = \dfrac{3}{4}$ | M1 A1 | |
| A parabola with positive coefficient of $x^2$ has a minimum $\Rightarrow x^2+x+1 > 0$ <br> Accept equivalent arguments | A1 | (3) |

**Question 5**

| Question Number | Scheme | Marks |
|---|---|---|
| (a) | $2x^2 + 3x - 2 = (2x-1)(x+2)$      at any stage | B1 |
| | $f(x) = \dfrac{(2x+3)(2x-1)-(9+2x)}{(2x-1)(x+2)}$    f.t. on error in denominator factors | M1, A1√ |
| | (need not be single fraction) | |
| | Simplifying numerator to quadratic form | M1 |
| | Correct **numerator**          $= \dfrac{4x^2+2x-12}{[(2x-1)(x+2)]}$ | A1 |
| | Factorising numerator, **with** a denominator    $= \dfrac{2(2x-3)(x+2)}{(2x-1)(x+2)}$   o.e. | M1 |
| | $= \dfrac{4x-6}{2x-1}$     (✱) | A1 cso   (7) |
| Alt.(a) | $2x^2 + 3x - 2 = (2x-1)(x+2)$      at any stage | B1 |
| | $f(x) = \dfrac{(2x+3)(2x^2+3x-2)-(9+2x)(x+2)}{(x+2)(2x^2+3x-2)}$ | M1A1 f.t. |
| | $= \dfrac{4x^3+10x^2-8x-24}{(x+2)(2x^2+3x-2)}$ | |
| | $= \dfrac{2(x+2)(2x^2+x-6)}{(x+2)(2x^2+3x-2)}$   or   $\dfrac{2(2x-3)(x^2+4x+4)}{(x+2)(2x^2+3x+2)}$   o.e. | |
| | Any one linear factor × quadratic factor in **numerator** | M1, A1 |
| | $= \dfrac{2(x+2)(x+2)(2x-3)}{(x+2)(2x^2+3x-2)}$   o.e. | M1 |
| | $= \dfrac{2(2x-3)}{2x-1}$   $\dfrac{4x-6}{2x-1}$   (✱) | A1 |

Notes:    (a)   $1^{st}$ M1 in either version is for correct method

         $1^{st}$ A1   Allow $\dfrac{2x+3(2x-1)-(9+2x)}{(2x-1)(x+2)}$ or $\dfrac{(2x+3)(2x-1)-9+2x}{(2x-1)(x+2)}$ or $\dfrac{2x+3(2x-1)-9+2x}{(2x-1)(x+2)}$ ( fractions)

         $2^{nd}$ M1 in (main a) is for forming 3 term quadratic in **numerator**

         $3^{rd}$ M1 is for factorising their quadratic (usual rules) ; factor of 2 need not be extracted

         (✱) A1 is given answer so is cso

     Alt :(a)   $3^{rd}$ M1 is for factorising resulting quadratic

**Question 6**

| Question Number | Scheme | Marks |
|---|---|---|
| | $$x^2-1 \overline{\left)\begin{array}{l} 2x^2 \quad\quad -1 \\ \overline{2x^4 -3x^2 +x+1} \\ 2x^4 -2x^2 \\ \overline{\phantom{2x^4} -x^2 +x+1} \\ \phantom{2x^4} -x^2 \quad\quad +1 \\ \overline{\phantom{2x^4 -x^2 +x} x} \end{array}\right.}$$ $a = 2$ stated or implied<br>$c = -1$ stated or implied $$2x^2 -1 + \frac{x}{x^2-1}$$ $a=2, b=0, c=-1, d=1, e=0$ $d=1$ and $b=0$, $e=0$ stated or implied | M1<br>A1<br>A1<br><br><br><br><br><br>A1<br>[4] |

**Question 7**

| Question Number | Scheme | Marks |
|---|---|---|
| (a) | $x^2 -2x-3 = (x-3)(x+1)$ | B1 |
| | $f(x) = \dfrac{2(x-1)-(x+1)}{(x-3)(x+1)}$ $\left( or\ \dfrac{2(x-1)}{(x-3)(x+1)} - \dfrac{x+1}{(x-3)(x+1)} \right)$ | M1 A1 |
| | $= \dfrac{x-3}{(x-3)(x+1)} = \dfrac{1}{x+1}$ ✱ | A1 cso (4) |

**Question 8**

| Question Number | Scheme | Marks |
|---|---|---|
| (a) | $$\dfrac{2x+2}{x^2-2x-3}-\dfrac{x+1}{x-3}=\dfrac{2x+2}{(x-3)(x+1)}-\dfrac{x+1}{x-3}$$ $$=\dfrac{2x+2-(x+1)(x+1)}{(x-3)(x+1)}$$ $$=\dfrac{(x+1)(1-x)}{(x-3)(x+1)}$$ $$=\dfrac{1-x}{x-3}\qquad\text{Accept }-\dfrac{x-1}{x-3},\ \dfrac{x-1}{3-x}$$ | M1 A1  M1  A1  (4) |
| | *Alternative to* (a) $$\dfrac{2x+2}{x^2-2x-3}=\dfrac{2(x+1)}{(x-3)(x+1)}=\dfrac{2}{x-3}$$ $$\dfrac{2}{x-3}-\dfrac{x+1}{x-3}=\dfrac{2-(x+1)}{x-3}$$ $$=\dfrac{1-x}{x-3}$$ | M1 A1  M1  A1  (4) |

**Question 9**

| Question Number | Scheme | Marks |
|---|---|---|
| Q | $f(x) = 1 - \dfrac{2}{(x+4)} + \dfrac{x-8}{(x-2)(x+4)}$ <br><br> $x \in \mathbb{R}, \ x \neq -4, \ x \neq 2.$ | |
| (a) | $f(x) = \dfrac{(x-2)(x+4) - 2(x-2) + x - 8}{(x-2)(x+4)}$ | An attempt to combine to one fraction   M1 <br><br> Correct result of combining all three fractions   A1 |
| | $= \dfrac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$ | |
| | $= \dfrac{x^2 + x - 12}{[(x+4)(x-2)]}$ | Simplifies to give the correct numerator. Ignore omission of   A1 <br> denominator |
| | $= \dfrac{(x+4)(x-3)}{[(x+4)(x-2)]}$ | An attempt to factorise the   dM1 <br> numerator. |
| | $= \dfrac{(x-3)}{(x-2)}$ | Correct result   A1 cso AG |
| | | (5) |

## Question 10

| Question Number | Scheme | Marks |
|---|---|---|
| | $\dfrac{x+1}{3x^2-3}-\dfrac{1}{3x+1}$ | |
| | $=\dfrac{x+1}{3(x^2-1)}-\dfrac{1}{3x+1}$ | |
| | $=\dfrac{x+1}{3(x+1)(x-1)}-\dfrac{1}{3x+1}$    $\begin{array}{l}x^2-1\to(x+1)(x-1)\text{ or}\\ 3x^2-3\to(x+1)(3x-3)\text{ or}\\ 3x^2-3\to(3x+3)(x-1)\\ \text{seen or implied anywhere in candidate's}\\ \text{working.}\end{array}$ | Award below |
| | $=\dfrac{1}{3(x-1)}-\dfrac{1}{3x+1}$ | |
| | $=\dfrac{3x+1-3(x-1)}{3(x-1)(3x+1)}$    Attempt to combine. | M1 |
| | or $\dfrac{3x+1}{3(x-1)(3x+1)}-\dfrac{3(x-1)}{3(x-1)(3x+1)}$    Correct result. | A1 |
| |    *Decide to award M1 here!!* | M1 |
| | $=\dfrac{4}{3(x-1)(3x+1)}$    Either $\dfrac{4}{3(x-1)(3x+1)}$ <br> or $\dfrac{\frac{4}{3}}{(x-1)(3x+1)}$ or $\dfrac{4}{(3x-3)(3x+1)}$ <br> or $\dfrac{4}{9x^2-6x-3}$ | A1 **aef** |
| | | [4] |

## Question 11

| Question Number | | Scheme | Marks |
|---|---|---|---|
| | (a) | $\dfrac{(x+5)(2x-1)}{(x+5)(x-3)}=\dfrac{(2x-1)}{(x-3)}$ | M1 B1 A1 aef |
| | | | (3) |

**Question 12**

| Question Number | Scheme | Marks |
|---|---|---|
| (a) | $\dfrac{4x-1}{2(x-1)} - \dfrac{3}{2(x-1)(2x-1)}$ | |
| | $= \dfrac{(4x-1)(2x-1) - 3}{2(x-1)(2x-1)}$      An attempt to form a single fraction | M1 |
| | $= \dfrac{8x^2 - 6x - 2}{\{2(x-1)(2x-1)\}}$      Simplifies to give a correct quadratic numerator over a correct quadratic denominator | A1 aef |
| | $= \dfrac{2(x-1)(4x+1)}{\{2(x-1)(2x-1)\}}$      An attempt to factorise a 3 term quadratic numerator | M1 |
| | $= \dfrac{4x+1}{2x-1}$ | A1 |
| | | (4) |
| (b) | $f(x) = \dfrac{4x-1}{2(x-1)} - \dfrac{3}{2(x-1)(2x-1)} - 2, \quad x > 1$ | |
| | $f(x) = \dfrac{(4x+1)}{(2x-1)} - 2$ | |
| | $= \dfrac{(4x+1) - 2(2x-1)}{(2x-1)}$      An attempt to form a single fraction | M1 |
| | $= \dfrac{4x+1-4x+2}{(2x-1)}$ | |
| | $= \dfrac{3}{(2x-1)}$      Correct result | A1 * |
| | | (2) |

**Question 13**

| Question Number | Scheme | Marks |
|---|---|---|
| (a) | $$x^2 - 9 = (x+3)(x-3)$$ | B1 |
| | $$\frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{(x+3)(x-3)}$$ | |
| | $$= \frac{(4x-5)(x+3)}{(2x+1)(x-3)(x+3)} - \frac{2x(2x+1)}{(2x+1)(x+3)(x-3)}$$ | M1 |
| | $$= \frac{5x-15}{(2x+1)(x-3)(x+3)}$$ | M1A1 |
| | $$= \frac{5(x-3)}{(2x+1)(x-3)(x+3)} = \frac{5}{(2x+1)(x+3)}$$ | A1* |
| | | (5) |