Solving Trigonometric Equations - Edexcel Past Exam Questions MARK SCHEME

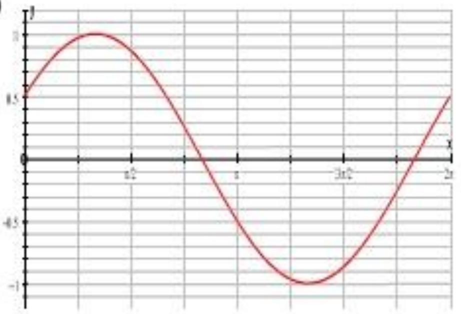
Question 1: Jan 07 Q6

Question Number	Scheme	Marks
6.	$2(1-\sin^2 x)+1=5\sin x$	M1
	$2\sin^2 x+5\sin x-3=0$ $(2\sin x-1)(\sin x+3)=0$	
	$\sin x=\frac{1}{2}$	M1, A1
	$x=\frac{\pi}{6},\frac{5\pi}{6}$	M1, M1, A1cso **(6)**

Notes

Use of $\cos^2 x=1-\sin^2 x$. Condone invisible brackets in first line if $2-2\sin^2 x$ is present (or implied) in a subsequent line. Must be using $\cos^2 x=1-\sin^2 x$. Using $\cos^2 x=1+\sin^2 x$ is M0.	M1
Attempt to solve a 2 or 3 term quadratic in $\sin x$ up to $\sin x=\ldots$ Usual rules for solving quadratics. Method may be factorising, formula or completing the square	M1
Correct factorising for correct quadratic and $\sin x=\frac{1}{2}$. So, e.g. $(\sin x+3)$ as a factor $\rightarrow$ $\sin x=3$ can be ignored.	A1
Method for finding any angle in any range consistent with (either of) their trig. equation(s) in degrees or radians (even if x not exact). [Generous M mark] Generous mark. Solving any trig. equation that comes from minimal working (however bad). So $x=\sin^{-1}/\cos^{-1}/\tan^{-1}(\text{number})\rightarrow$ answer in degrees or radians correct for their equation (in any range)	M1
Method for finding second angle consistent with (either of) their trig. equation(s) in radians. Must be in range $0\le x<2\pi$. Must involve using π (e.g. $\pi\pm\ldots, 2\pi-\ldots$) but … can be inexact. Must be using the same equation as they used to attempt the 3rd M mark. Use of π must be consistent with the trig. equation they are using (e.g. if using $\cos^{-1}$ then must be using $2\pi-\ldots$) If finding both angles in degrees: method for finding 2nd angle equivalent to method above in degrees and an attempt to change both angles to radians.	M1
$\frac{\pi}{6},\frac{5\pi}{6}$ c.s.o. Recurring decimals are okay (instead of $\frac{1}{6}$ and $\frac{5}{6}$). Correct decimal values (corrected or truncated) before the final answer of $\frac{\pi}{6},\frac{5\pi}{6}$ is acceptable.	A1 cso
Ignore extra solutions outside range; deduct final A mark for extra solutions in range.	
Special case Answer only $\frac{\pi}{6},\frac{5\pi}{6}$ M0, M0, A0, M1, M1 A1 Answer only $\frac{\pi}{6}$ M0, M0, A0, M1, M0 A0	

Question 2: June 07 Q9

Question number	Scheme	Marks
	(a)  Sine wave (anywhere) with at least 2 turning points.	M1
	Starting on positive y-axis, going up to a max., then min. below x-axis, no further turning points in range, finishing above x-axis at $x = 2\pi$ or 360°. There must be <u>some</u> indication of scale on the y-axis… (e.g. 1, −1 or 0.5) Ignore parts of graph outside 0 to 2π. n.b. Give credit if necessary for what is seen on an initial sketch (before any transformation has been performed).	A1 (2)
	(b) $\left(0, \frac{1}{2}\right)$, $\left(\frac{5\pi}{6}, 0\right)$, $\left(\frac{11\pi}{6}, 0\right)$ (Ignore any extra solutions) (<u>Not</u> 150°, 330°) $\left(\pi - \frac{\pi}{6}\right)$ and $\left(2\pi - \frac{\pi}{6}\right)$ are insufficient, but if <u>both</u> are seen allow B1 B0.	B1, B1, B1 (3)
	(c) awrt 0.71 radians (0.70758…), or awrt 40.5° (40.5416…) (α)	B1
	$(\pi - \alpha)$ (2.43…) or $(180 - \alpha)$ <u>if α is in degrees</u>. $\left[\underline{\text{NOT}}\ \pi - \left(\alpha - \frac{\pi}{6}\right)\right]$	M1
	Subtract $\frac{\pi}{6}$ from α (or from $(\pi - \alpha)$)… or subtract 30 <u>if α is in degrees</u>	M1
	0.18 (or 0.06π), 1.91 (or 0.61π) Allow awrt (The 1st A mark is dependent on just the 2nd M mark)	A1, A1 (5)
		10
	(b) The zeros are not required, i.e. allow 0.5, etc. (and also allow coordinates the wrong way round). These marks are also awarded if the exact intercept values are seen in part (a), but if values in (b) and (a) are contradictory, (b) takes precedence. (c) B1: If the required value of α is <u>not seen</u>, this mark can be given by implication if a final answer rounding to 0.18 or 0.19 (or a final answer rounding to 1.91 or 1.90) is achieved. (Also see premature approx. note*). <u>Special case:</u> $\sin\left(x + \frac{\pi}{6}\right) = 0.65 \Rightarrow \sin x + \sin\frac{\pi}{6} = 0.65 \Rightarrow \sin x = 0.15$ $x = \arcsin 0.15 = 0.15056\ldots$ and $x = \pi - 0.15056 = 2.99$ (B0 M1 M0 A0 A0) (This special case mark is also available for degrees… 180 − 8.62…) Extra solutions outside 0 to 2π : Ignore. Extra solutions between 0 and 2π : Loses the final A mark. *<u>Premature approximation</u> in part (c): e.g. $\alpha = 41°$, $180 - 41 = 139$, $41 - 30 = 11$ and $139 - 30 = 109$ Changing to radians: 0.19 and 1.90 This would score B1 (required value of α not seen, but there is a final answer 0.19 (or 1.90)), M1 M1 A0 A0.	

Question 3: June 11 Q7

Question Number	Scheme		Marks
	<u>**Note:**</u> A similar scheme would apply for T&I for candidates using their a and their r. So,... 1[st] M1: For attempting to find one of the correct S_n's either side (but next to) 1000. 2[nd] M1: For one of these S_n's correct for their a and their r. (You may need to get your calculators out!) 3[rd] M1: For attempting to find both of the correct S_n's either side (but next to) 1000. A1: Cannot be gained for wrong a and/or r. <u>**Trial & Improvement Cumulative Approach:**</u> A similar scheme to T&I will be applied here: 1[st] M1: For getting as far as the cumulative sum of 13 terms. 2[nd] M1: (1) S_{13} = awrt 999.7 or truncated 999. 3[rd] M1: For getting as far as the cumulative sum to 14 terms. Also at this stage $S_{13} < 1000$ and $S_{14} > 1000$. A1: BOTH (1) S_{13} = awrt 999.7 or truncated 999 AND (2) S_{14} = awrt 1005.8 or truncated 1005 AND $n = 14$. <u>**Trial & Improvement Method:**</u> for $(0.75)^n < \frac{6}{256} = 0.0234375$ 3[rd] M1: For evidence of examining both $n = 13$ and $n = 14$. Eg: $(0.75)^{13}\{=0.023757...\}$ and $(0.75)^{14}\{=0.0178179...\}$ A1: $n = 14$ <u>**Any misreads,**</u> $S_n > 10000$ etc, please escalate up to your Team Leader.		
	(a) $3\sin(x+45^\circ)=2;\ 0 \le x < 360^\circ$ (b) $2\sin^2 x + 2 = 7\cos x;\ 0 \le x < 2\pi$		
(a)	$\sin(x+45^\circ)=\frac{2}{3}$, so $(x+45^\circ)=41.8103...$ $(\alpha = 41.8103...)$	$\sin^{-1}\left(\frac{2}{3}\right)$ or awrt 41.8 or awrt 0.73^c	M1
	So, $x+45^\circ=\{138.1897...,\ 401.8103...\}$	$x+45^\circ$ = either "$180 -$ their α" or "$360^\circ +$ their α" (α could be in radians).	M1
	and $x=\{93.1897...,\ 356.8103...\}$	Either awrt 93.2° or awrt 356.8°	A1
		Both awrt 93.2° and awrt 356.8°	A1
			[4]
(b)	$2(1-\cos^2 x)+2=7\cos x$	Applies $\sin^2 x = 1-\cos^2 x$	M1
	$2\cos^2 x + 7\cos x - 4 = 0$	Correct 3 term, $2\cos^2 x + 7\cos x - 4\ \{=0\}$	A1 oe
	$(2\cos x - 1)(\cos x + 4)\ \{=0\}$, $\cos x = ...$	Valid attempt at solving and $\cos x = ...$	M1
	$\cos x = \frac{1}{2}$, $\{\cos x = -4\}$	$\cos x = \frac{1}{2}$ (See notes.)	A1 cso
	$\left(\beta = \frac{\pi}{3}\right)$		
	$x=\frac{\pi}{3}$ or $1.04719...^c$	Either $\frac{\pi}{3}$ or awrt 1.05^c	B1
	$x=\frac{5\pi}{3}$ or $5.23598...^c$	Either $\frac{5\pi}{3}$ or awrt 5.24^c or $2\pi -$ their β (See notes.)	B1 ft
			[6] **10**

Question Number	Scheme	Marks
(a)	1st M1: can also be implied for $x =$ awrt -3.2 2nd M1: for $x + 45^\circ =$ either "$180 -$ their α" or "$360^\circ +$ their α". This can be implied by later working. The candidate's α could also be in radians. **Note that this mark is not** for $x =$ either "$180 -$ their α" or "$360^\circ +$ their α". Note: Imply the first two method marks or award M1M1A1 for either awrt 93.2° or awrt 356.8°. Note: Candidates who apply the following incorrect working of $3\sin(x + 45^\circ) = 2$ $\Rightarrow 3(\sin x + \sin 45) = 2$, etc will usually score M0M0A0A0. If there are any EXTRA solutions inside the range $0 \le x < 360$ and the candidate would otherwise score FULL MARKS then withhold the final aA2 mark (the final mark in this part of the question). Also ignore EXTRA solutions outside the range $0 \le x < 360$. **Working in Radians:** Note the answers in radians are $x =$ awrt 1.6, awrt 6.2 If a candidate works in radians then mark part (a) as above awarding the A marks in the same way. If the candidate would then score FULL MARKS then withhold the final aA2 mark (the final mark in this part of the question.) **No working:** Award M1M1A1A0 for one of awrt 93.2° or awrt 356.8° seen without any working. Award M1M1A1A1 for both awrt 93.2° and awrt 356.8° seen without any working. Allow benefit of the doubt (FULL MARKS) for final answer of $\sin x\,\{\text{and not } x\} = \{\text{awrt } 93.2,\ \text{awrt } 356.8\}$	

Question Number	Scheme	Marks
(b)	1st M1: for a correct method to use $\sin^2 x = 1 - \cos^2 x$ on the given equation. Give bod if the candidate omits the bracket when substituting for $\sin^2 x$, but $2 - \cos^2 x + 2 = 7\cos x$, without supporting working, (eg. seeing "$\sin^2 x = 1 - \cos^2 x$") would score 1st M0. Note that applying $\sin^2 x = \cos^2 x - 1$, scores M0. 1st A1: for obtaining either $2\cos^2 x + 7\cos x - 4$ or $-2\cos^2 x - 7\cos x + 4$. 1st A1: can also awarded for a correct three term equation eg. $2\cos^2 x + 7\cos x = 4$ or $2\cos^2 x = 4 - 7\cos x$ etc. 2nd M1: for a valid attempt at factorisation of a quadratic (either 2TQ or 3TQ) in cos, can use any variable here, c, y, x or $\cos x$, and an attempt to find at least one of the solutions. See introduction to the Mark Scheme. *Alternatively*, using a correct formula for solving the quadratic. Either the formula must be stated correctly or the correct form must be implied by the substitution. 2nd A1: for $\cos x = \frac{1}{2}$, BY A CORRECT SOLUTION ONLY UP TO THIS POINT. Ignore extra answer of $\cos x = -4$, but penalise if candidate states an incorrect result e.g. $\cos x = 4$. If they have used a substitution, a correct value of their c or their y or their x. **Note:** 2nd A1 for $\cos x = \frac{1}{2}$ can be implied by later working. 1st B1: for either $\frac{\pi}{3}$ or awrt 1.05^c 2nd B1: for either $\frac{5\pi}{3}$ or awrt 5.24^c or can be ft from 2π − their β or 360° − their β where $\beta = \cos^{-1}(k)$, such that $0 < k < 1$ or $-1 < k < 0$, but $k \neq 0$, $k \neq 1$ or $k \neq -1$. If there are any EXTRA solutions inside the range $0 \le x < 2\pi$ and the candidate would otherwise score FULL MARKS then withhold the final bB2 mark (the final mark in this part of the question). Also ignore EXTRA solutions outside the range $0 \le x < 2\pi$. **Working in Degrees:** Note the answers in degrees are $x = 60,\ 300$ If a candidate works in degrees then mark part (b) as above awarding the B marks in the same way. If the candidate would then score FULL MARKS then withhold the final bB2 mark (the final mark in this part of the question.) **Answers from no working:** $x = \frac{\pi}{3}$ and $x = \frac{5\pi}{3}$ scores M0A0M0A0B1B1, $x = 60$ and $x = 300$ scores M0A0M0A0B1B0, $x = \frac{\pi}{3}$ ONLY or $x = 60$ ONLY scores M0A0M0A0B1B0, $x = \frac{5\pi}{3}$ ONLY or $x = 120$ ONLY scores M0A0M0A0B0B1. **No working:** You cannot apply the ft in the B1ft if the answers are given with NO working. Eg: $x = \frac{\pi}{5}$ and $x = \frac{9\pi}{3}$ FROM NO WORKING scores M0A0M0A0B0B0. **For candidates using trial & improvement, please forward these to your Team Leader.**	