Trigonometry(Addition,Double Angle & R Formulae) - Edexcel Past Exam Questions MARK SCHEME

Question 1: June 07 Q6

Question Number	Scheme	Marks
(a)	$\cos 2A = \cos^2 A - \sin^2 A$ (+ use of $\cos^2 A + \sin^2 A \equiv 1$)	M1
	$= (1 - \sin^2 A); - \sin^2 A = 1 - 2\sin^2 A$ (*)	A1 (2)
(b)	$2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv 4\sin\theta\cos\theta; -3(1 - 2\sin^2\theta) - 3\sin\theta + 3$	B1; M1
	$\equiv 4\sin\theta\cos\theta + 6\sin^2\theta - 3\sin\theta$	M1
	$\equiv \sin\theta(4\cos\theta + 6\sin\theta - 3)$ (*)	A1 (4)
(c)	$4\cos\theta + 6\sin\theta \equiv R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$	
	Complete method for R (may be implied by correct answer)	
	$[\ R^2 = 4^2 + 6^2,\ R\sin\alpha = 4,\ R\cos\alpha = 6\ ]$	M1
	$R = \sqrt{52}$ or 7.21	A1
	Complete method for α; $\alpha = 0.588$ (allow 33.7°)	M1 A1
		(4)
(d)	$\sin\theta\ (4\cos\theta + 6\sin\theta - 3) = 0$	M1
	$\theta = 0$	B1
	$\sin(\theta + 0.588) = \dfrac{3}{\sqrt{52}} = 0.4160..$ (24.6°)	M1
	$\theta + 0.588 = (0.4291),\ 2.7125$ [or $\theta + 33.7° = (24.6°),\ 155.4°$]	dM1
	$\theta = 2.12$ cao	A1
		(5)

Question 2: Jan 06 Q6

(a)	$R\cos\alpha = 12, \quad R\sin\alpha = 4$	
	$R = \sqrt{(12^2+4^2)} = \sqrt{160}$ Accept if just written down, awrt 12.6	M1 A1
	$\tan\alpha = \frac{4}{12}, \quad \Rightarrow \quad \alpha \approx 18.43°$ awrt 18.4°	M1, A1 **(4)**
(b)	$\cos(x + \text{ their } \alpha) = \frac{7}{\text{their } R} \quad (\approx 0.5534)$	M1
	$x + \text{their } \alpha = 56.4°$ awrt 56°	A1
	$= \ldots, 303.6°$ 360° – their principal value	M1
	$x = 38.0°, 285.2°$ Ignore solutions out of range	A1, A1 **(5)**
	If answers given to more than 1 dp, penalise first time then accept awrt above.	
(c)(i)	minimum value is $-\sqrt{160}$ ft their R	B1ft
(ii)	$\cos(x + \text{ their } \alpha) = -1$	M1
	$x \approx 161.57°$ cao	A1 **(3)**
		[12]

Question 3: Jan 06 Q7

Question Number	Scheme	Marks
	(a) (i) Use of $\cos 2x = \cos^2 x - \sin^2 x$ in an attempt to prove the identity.	M1
	$\frac{\cos 2x}{\cos x + \sin x} = \frac{\cos^2 x - \sin^2 x}{\cos x + \sin x} = \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x + \sin x} = \cos x - \sin x$ * cso	A1 **(2)**
	(ii) Use of $\cos 2x = 2\cos^2 x - 1$ in an attempt to prove the identity.	M1
	Use of $\sin 2x = 2\sin x\cos x$ in an attempt to prove the identity.	M1
	$\frac{1}{2}(\cos 2x - \sin 2x) = \frac{1}{2}(2\cos^2 x - 1 - 2\sin x\cos x) = \cos^2 x - \cos x\sin x - \frac{1}{2}$ * cso	A1 **(3)**
	(b) $\cos\theta(\cos\theta - \sin\theta) = \frac{1}{2}$ Using (a)(i)	M1
	$\cos^2\theta - \cos\theta\sin\theta - \frac{1}{2} = 0$	
	$\frac{1}{2}(\cos 2\theta - \sin 2\theta) = 0$ Using (a)(ii)	M1
	$\cos 2\theta = \sin 2\theta$ *	A1 **(3)**
	(c) $\tan 2\theta = 1$	M1
	$2\theta = \frac{\pi}{4}, \left(\frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}\right)$ any one correct value of 2θ	A1
	$\theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$ Obtaining at least 2 solutions in range	M1
	The 4 correct solutions	A1 **(4)**
	If decimals $(0.393, 1.963, 3.534, 5.105)$ or degrees $(22.5°, 112.5°, 202.5°, 292.5°)$ are given, but all 4 solutions are found, penalise one A mark only. Ignore solutions out of range.	**[12]**

Question 4: June 06 Q8

Question Number	Scheme	Marks
(a)	Method for finding $\sin A$	M1
	$\sin A = -\frac{\sqrt{7}}{4}$	A1 A1
	Note: First A1 for $\frac{\sqrt{7}}{4}$, exact. Second A1 for sign (even if dec. answer given)	
	Use of $\sin 2A \equiv 2\sin A\cos A$	M1
	$\sin 2A = -\frac{3\sqrt{7}}{8}$ or equivalent exact	A1√ (5)
	Note: ± f.t. Requires exact value, dependent on 2nd M	
(b)(i)	$\cos\left(2x+\frac{\pi}{3}\right)+\cos\left(2x-\frac{\pi}{3}\right) \equiv \cos 2x\cos\frac{\pi}{3} - \sin 2x\sin\frac{\pi}{3} + \cos 2x\cos\frac{\pi}{3} + \sin 2x\sin\frac{\pi}{3}$	M1
	$\equiv 2\cos 2x\cos\frac{\pi}{3}$	A1
	[This can be just written down (using factor formulae) for M1A1]	
	$\equiv \cos 2x$ AG	A1* (3)
	Note:	
	M1A1 earned, if $\equiv 2\cos 2x\cos\frac{\pi}{3}$ just written down, using factor theorem	
	Final A1* requires some working after first result.	

Question 5: Jan 07 Q1

Question Number	Scheme	Marks
	(a) $\sin 3\theta = \sin(2\theta+\theta) = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$	B1
	$= 2\sin\theta\cos^2\theta + (1-2\sin^2\theta)\sin\theta$	B1 B1
	$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$	M1
	$= 3\sin\theta - 4\sin^3\theta$ ✱ cso	A1 **(5)**
	(b) $\sin 3\theta = 3\times\frac{\sqrt{3}}{4} - 4\left(\frac{\sqrt{3}}{4}\right)^3 = \frac{3\sqrt{3}}{4} - \frac{3\sqrt{3}}{16} = \frac{9\sqrt{3}}{16}$ or exact equivalent	M1 A1 **(2)**
		[7]

Question 6: Jan 07 Q5

Question Number	Scheme	Marks
	(a) $R^2 = \left(\sqrt{3}\right)^2 + 1^2 \Rightarrow R = 2$	M1 A1
	$\tan\alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$ accept awrt 1.05	M1 A1 **(4)**
	(b) $\sin\left(x + \text{ their } \alpha\right) = \frac{1}{2}$	M1
	$x + \text{ their } \alpha = \frac{\pi}{6}\ \left(\frac{5\pi}{6}, \frac{13\pi}{6}\right)$	A1
	$x = \frac{\pi}{2}, \frac{11\pi}{6}$ accept awrt 1.57, 5.76	M1 A1 **(4)**
		[8]
	The use of degrees loses only one mark in this question. Penalise the first time it occurs in an answer and then ignore.	

Question 7: June 07 Q6

Question Number	Scheme	Marks
(a)	Complete method for R: e.g. $R\cos\alpha = 3$, $R\sin\alpha = 2$, $R = \sqrt{(3^2 + 2^2)}$	M1
	$R = \sqrt{13}$ or 3.61 (or more accurate)	A1
	Complete method for $\tan\alpha = \frac{2}{3}$ [Allow $\tan\alpha = \frac{3}{2}$]	M1
	$\alpha = 0.588$ (Allow 33.7°)	A1 (4)
(b)	Greatest value $= \left(\sqrt{13}\right)^4 = 169$	M1, A1 (2)
(c)	$\sin(x + 0.588) = \frac{1}{\sqrt{13}}$ $(= 0.27735\ldots)$ $\sin(x + \text{their } \alpha) = \frac{1}{\text{their } R}$	M1
	$(x + 0.588) = 0.281(03\ldots)$ or 16.1°	A1
	$(x + 0.588) = \pi - 0.28103\ldots$	M1
	Must be $\pi -$ their 0.281 or 180° − their 16.1°	
	or $(x + 0.588) = 2\pi + 0.28103\ldots$	M1
	Must be $2\pi +$ their 0.281 or 360° + their 16.1°	
	$x = 2.273$ or $x = 5.976$ (awrt) Both (radians only)	A1 (5)
	If 0.281 or 16.1° not seen, correct answers imply this A mark	**(11 marks)**

Notes: (a) 1st M1 for correct method for R
2nd M1 for correct method for $\tan\alpha$
No working at all: M1A1 for $\sqrt{13}$, M1A1 for 0.588 or 33.7°.
N.B. $R\cos\alpha = 2$, $R\sin\alpha = 3$ used, can still score M1A1 for R, but loses the A mark for α.
$\cos\alpha = 3$, $\sin\alpha = 2$: apply the same marking.

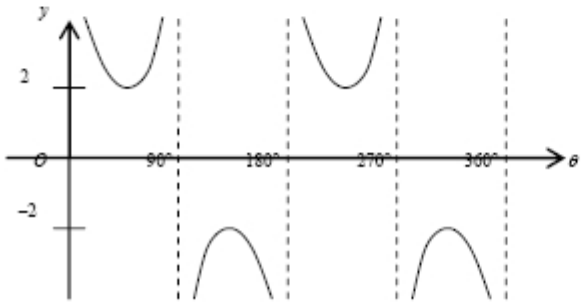
(b) M1 for realising $\sin(x+\alpha) = \pm 1$, so finding R^4.

(c) Working in mixed degrees/rads : first two marks available
Working consistently in degrees: Possible to score first 4 marks
[Degree answers, just for reference only, are 130.2° and 342.4°]
Third M1 can be gained for candidate's 0.281 – candidate's 0.588 + 2π or equiv. in degrees
One of the answers correct in radians or degrees implies the corresponding M mark.

Alt: (c) (i) Squaring to form quadratic in $\sin x$ or $\cos x$ — M1
$[13\cos^2 x - 4\cos x - 8 = 0,\quad 13\sin^2 x - 6\sin x - 3 = 0]$
Correct values for $\cos x = 0.953\ldots,\ -0.646$; or $\sin x = 0.767,\ 2.27$ awrt — A1
For any one value of $\cos x$ or $\sin x$, correct method for two values of x — M1
$x = 2.273$ or $x = 5.976$ (awrt) Both seen anywhere — A1
Checking other values (0.307, 4.011 or 0.869, 3.449) and discarding — M1

(ii) Squaring and forming equation of form $a\cos 2x + b\sin 2x = c$
$9\sin^2 x + 4\cos^2 x + 12\sin 2x = 1 \Rightarrow 12\sin 2x + 5\cos 2x = 11$
Setting up to solve using R formula e.g. $\sqrt{13}\cos(2x - 1.176) = 11$ — M1
$(2x - 1.176) = \cos^{-1}\left(\frac{11}{\sqrt{13}}\right) = 0.562(0\ldots \quad (\alpha)$ — A1
$(2x - 1.176) = 2\pi - \alpha,\ 2\pi + \alpha, \ldots\ldots$ — M1
$x = 2.273$ or $x = 5.976$ (awrt) Both seen anywhere — A1
Checking other values and discarding — M1

Question 8: June 07 Q7

Question Number	Scheme	Marks
(a)	$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$	M1
	M1 Use of common denominator to obtain single fraction	
	$=\frac{1}{\cos\theta\sin\theta}$	M1
	M1 Use of appropriate trig identity (in this case $\sin^2\theta+\cos^2\theta=1$)	
	$=\frac{1}{\frac{1}{2}\sin 2\theta}$ Use of $\sin 2\theta=2\sin\theta\cos\theta$	M1
	$=2\text{cosec}2\theta$ (✱)	A1 cso (4)
Alt.(a)	$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\tan\theta+\frac{1}{\tan\theta}=\frac{\tan^2\theta+1}{\tan\theta}$ M1	
	$=\frac{\sec^2\theta}{\tan\theta}$ M1	
	$=\frac{1}{\cos\theta\sin\theta}=\frac{1}{\frac{1}{2}\sin 2\theta}$ M1	
	$=2\text{cosec}2\theta$ (✱) (cso) A1	
	If show two expressions are equal, need conclusion such as QED, tick, true.	
(b)	 Shape (May be translated but need to see 4"sections")	B1
	T.P.s at $y=\pm 2$, asymptotic at correct x-values (dotted lines not required)	B1 dep. (2)
(c)	$2\text{cosec}2\theta=3$	
	$\sin 2\theta=\frac{2}{3}$ Allow $\frac{2}{\sin 2\theta}=3$ [M1 for equation in $\sin 2\theta$]	M1, A1
	(2θ) = [41.810…°, 138.189…° ; 401.810…°, 498.189…°]	
	1st M1 for α, $180-\alpha$; 2nd M1 adding 360° to at least one of values	M1; M1
	$\theta=20.9°, 69.1°, 200.9°, 249.1°$ (1 d.p.) awrt	
Note	1st A1 for any two correct, 2nd A1 for other two Extra solutions in range lose final A1 only SC: Final 4 marks: $\theta=20.9°$, after M0M0 is B1; record as M0M0A1A0	A1,A1 (6)
Alt.(c)	$\tan\theta+\frac{1}{\tan\theta}=3$ and form quadratic , $\tan^2\theta-3\tan\theta+1=0$ M1, A1	
	(M1 for attempt to multiply through by tanθ, A1 for correct equation above)	
	Solving quadratic [$\tan\theta=\frac{3\pm\sqrt{5}}{2}$ = 2.618… or = 0.3819…] M1	
	θ = 69.1°, 249.1° $\theta=20.9°, 200.9°$ (1 d.p.) M1, A1, A1	
	(M1 is for one use of 180° + α°, A1A1 as for main scheme)	**(12 marks)**

Question 9: Jan 08 Q6

Question Number	Scheme	Marks
	(a) $\cos(2x+x) = \cos 2x \cos x - \sin 2x \sin x$	M1
	$= (2\cos^2 x - 1)\cos x - (2\sin x \cos x)\sin x$	M1
	$= (2\cos^2 x - 1)\cos x - 2(1-\cos^2 x)\cos x$ any correct expression	A1
	$= 4\cos^3 x - 3\cos x$	A1 (4)
	(b)(i) $\dfrac{\cos x}{1+\sin x} + \dfrac{1+\sin x}{\cos x} = \dfrac{\cos^2 x + (1+\sin x)^2}{(1+\sin x)\cos x}$	M1
	$= \dfrac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{(1+\sin x)\cos x}$	A1
	$= \dfrac{2(1+\sin x)}{(1+\sin x)\cos x}$	M1
	$= \dfrac{2}{\cos x} = 2\sec x$ * cso	A1 (4)
	(c) $\sec x = 2$ *or* $\cos x = \dfrac{1}{2}$	M1
	$x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$ accept awrt 1.05, 5.24	A1, A1 (3)
		[11]

Question 10: June 08 Q2

Question Number	Scheme		Marks
(a)	$R^2 = 5^2 + 12^2$		M1
	$R = 13$		A1
	$\tan\alpha = \frac{12}{5}$		M1
	$\alpha \approx 1.176$		A1 cao (4)
(b)	$\cos(x-\alpha) = \frac{6}{13}$		M1
	$x - \alpha = \arccos\frac{6}{13} = 1.091\ \ldots$		A1
	$x = 1.091\ \ldots + 1.176\ \ldots \approx 2.267\ldots$	awrt 2.3	A1
	$x - \alpha = -1.091\ \ldots$	accept … = 5.19 … for M	M1
	$x = -1.091\ \ldots + 1.176\ \ldots \approx 0.0849\ldots$	awrt 0.084 or 0.085	A1 (5)
(c)(i)	$R_{max} = 13$	ft their R	B1 ft
(ii)	At the maximum, $\cos(x-\alpha) = 1$ *or* $x - \alpha = 0$		M1
	$x = \alpha = 1.176\ \ldots$	awrt 1.2, ft their α	A1ft (3)
			(12 marks)

Question 11: Jan 09 Q6

Question Number	Scheme	Marks
(a)(i)	$\sin 3\theta = \sin(2\theta+\theta)$	
	$= \sin 2\theta\cos\theta + \cos 2\theta \sin\theta$	
	$= 2\sin\theta\cos\theta.\cos\theta + (1-2\sin^2\theta)\sin\theta$	M1 A1
	$= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$	M1
	$= 3\sin\theta - 4\sin^3\theta$ ✱ cso	A1 (4)
(ii)	$8\sin^3\theta - 6\sin\theta + 1 = 0$	
	$-2\sin 3\theta + 1 = 0$	M1 A1
	$\sin 3\theta = \frac{1}{2}$	M1
	$3\theta = \frac{\pi}{6}, \frac{5\pi}{6}$	
	$\theta = \frac{\pi}{18}, \frac{5\pi}{18}$	A1 A1 (5)
(b)	$\sin 15° = \sin(60°-45°) = \sin 60°\cos 45° - \cos 60°\sin 45°$	M1
	$= \frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}} - \frac{1}{2}\times\frac{1}{\sqrt{2}}$	M1 A1
	$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6}-\sqrt{2})$ ✱ cso	A1 (4)
		[13]
	Alternatives to (b)	
	① $\sin 15° = \sin(45°-30°) = \sin 45°\cos 30° - \cos 45°\sin 30°$	M1
	$= \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\times\frac{1}{2}$	M1 A1
	$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6}-\sqrt{2})$ ✱ cso	A1 (4)
	② Using $\cos 2\theta = 1-2\sin^2\theta$, $\cos 30° = 1-2\sin^2 15°$	
	$2\sin^2 15° = 1-\cos 30° = 1-\frac{\sqrt{3}}{2}$	
	$\sin^2 15° = \frac{2-\sqrt{3}}{4}$	M1 A1
	$\left(\frac{1}{4}(\sqrt{6}-\sqrt{2})\right)^2 = \frac{1}{16}(6+2-2\sqrt{12}) = \frac{2-\sqrt{3}}{4}$	M1
	Hence $\sin 15° = \frac{1}{4}(\sqrt{6}-\sqrt{2})$ ✱ cso	A1 (4)

Question 12: June 09 Q6

Question Number	Scheme		Marks
Q (a)	$A=B \Rightarrow \cos(A+A) = \cos 2A = \underline{\cos A\cos A - \sin A\sin A}$	Applies $A=B$ to $\cos(A+B)$ to give the underlined equation or $\cos 2A = \underline{\cos^2 A - \sin^2 A}$	M1
	$\cos 2A = \cos^2 A - \sin^2 A$ and $\cos^2 A + \sin^2 A = 1$ gives		
	$\underline{\cos 2A} = 1-\sin^2 A - \sin^2 A = \underline{1-2\sin^2 A}$ (as required)	Complete proof, with a link between LHS and RHS. No errors seen.	A1 AG
			(2)
(b)	$C_1 = C_2 \Rightarrow 3\sin 2x = 4\sin^2 x - 2\cos 2x$	Eliminating y correctly.	M1
	$3\sin 2x = 4\left(\frac{1-\cos 2x}{2}\right) - 2\cos 2x$	Using result in part (a) to substitute for $\sin^2 x$ as $\frac{\pm 1 \pm \cos 2x}{2}$ or $k\sin^2 x$ as $k\left(\frac{\pm 1 \pm \cos 2x}{2}\right)$ to produce an equation in only double angles.	M1
	$3\sin 2x = 2(1-\cos 2x) - 2\cos 2x$		
	$3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$		
	$3\sin 2x + 4\cos 2x = 2$	Rearranges to give correct result	A1 AG
			(3)
(c)	$3\sin 2x + 4\cos 2x = R\cos(2x-\alpha)$		
	$3\sin 2x + 4\cos 2x = R\cos 2x\cos\alpha + R\sin 2x\sin\alpha$		
	Equate $\sin 2x$: $3 = R\sin\alpha$ Equate $\cos 2x$: $4 = R\cos\alpha$		
	$R = \sqrt{3^2+4^2} = \sqrt{25} = 5$	$R = 5$	B1
	$\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.86989765...^\circ$	$\tan\alpha = \pm\frac{3}{4}$ or $\tan\alpha = \pm\frac{4}{3}$ or $\sin\alpha = \pm\frac{3}{\text{their } R}$ or $\cos\alpha = \pm\frac{4}{\text{their } R}$	M1
		awrt 36.87	A1
	Hence, $3\sin 2x + 4\cos 2x = 5\cos(2x-36.87)$		
			(3)

Question Number	Scheme		Marks
(d)	$3\sin 2x + 4\cos 2x = 2$		
	$5\cos(2x - 36.87) = 2$		
	$\cos(2x - 36.87) = \frac{2}{5}$	$\cos(2x \pm \text{their } \alpha) = \frac{2}{\text{their } R}$	M1
	$(2x - 36.87) = 66.42182...^{\circ}$	awrt 66	A1
	$(2x - 36.87) = 360 - 66.42182...^{\circ}$		
	Hence, $x = 51.64591...^{\circ}$, $165.22409...^{\circ}$	One of either awrt 51.6 or awrt 51.7 or awrt 165.2 or awrt 165.3	A1
		Both awrt 51.6 AND awrt 165.2	A1
			(4)
		If there are any EXTRA solutions inside the range $0 \le x < 180^{\circ}$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 \le x < 180^{\circ}$.	
			[12]

Question 13: Jan 10 Q3

Question Number	Scheme		Marks
(a)	$5\cos x - 3\sin x = R\cos(x+\alpha)$, $R>0$, $0<x<\frac{\pi}{2}$		
	$5\cos x - 3\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$		
	Equate $\cos x$: $5 = R\cos\alpha$ Equate $\sin x$: $3 = R\sin\alpha$		
	$R = \sqrt{5^2+3^2}; = \sqrt{34}$ $\{= 5.83095..\}$	$R^2 = 5^2+3^2$ $\sqrt{34}$ or awrt 5.8	M1; A1
	$\tan\alpha = \frac{3}{5} \Rightarrow \alpha = 0.5404195003...^c$	$\tan\alpha = \pm\frac{3}{5}$ or $\tan\alpha = \pm\frac{5}{3}$ or $\sin\alpha = \pm\frac{3}{\text{their } R}$ or $\cos\alpha = \pm\frac{5}{\text{their } R}$ $\alpha =$ awrt 0.54 or $\alpha =$ awrt 0.17π or $\alpha = \frac{\pi}{\text{awrt } 5.8}$	M1 A1
	Hence, $5\cos x - 3\sin x = \sqrt{34}\cos(x+0.5404)$		(4)
(b)	$5\cos x - 3\sin x = 4$		
	$\sqrt{34}\cos(x+0.5404) = 4$		
	$\cos(x+0.5404) = \frac{4}{\sqrt{34}}$ $\{= 0.68599...\}$	$\cos(x \pm \text{their } \alpha) = \frac{4}{\text{their } R}$	M1
	$(x+0.5404) = 0.814826916...^c$	For applying $\cos^{-1}\left(\frac{4}{\text{their } R}\right)$	M1
	$x = 0.2744...^c$	awrt 0.27^c	A1
	$(x+0.5404) = 2\pi - 0.814826916...^c$ $\{= 5.468358...^c\}$	$2\pi -$ their 0.8148	ddM1
	$x = 4.9279...^c$	awrt 4.93^c	A1
	Hence, $x = \{0.27, 4.93\}$		(5) [9]

Part (b): If there are any EXTRA solutions inside the range $0 \le x < 2\pi$, then withhold the final accuracy mark if the candidate would otherwise score all 5 marks. Also ignore EXTRA solutions outside the range $0 \le x < 2\pi$.

Question 14: Jan 10 Q8

Question Number	Scheme		Marks
	$\operatorname{cosec}^2 2x - \cot 2x = 1, \quad (\text{eqn } *) \quad 0 \le x \le 180^\circ$		
	Using $\operatorname{cosec}^2 2x = 1 + \cot^2 2x$ gives $1 + \cot^2 2x - \cot 2x = 1$	Writing down or using $\operatorname{cosec}^2 2x = \pm 1 \pm \cot^2 2x$ or $\operatorname{cosec}^2\theta = \pm 1 \pm \cot^2\theta$.	M1
	$\underline{\cot^2 2x - \cot 2x} = 0 \quad \text{or} \quad \cot^2 2x = \cot 2x$	For either $\underline{\cot^2 2x - \cot 2x}\ \{=0\}$ or $\cot^2 2x = \cot 2x$	A1
	$\cot 2x(\cot 2x - 1) = 0 \quad \text{or} \quad \cot 2x = 1$	Attempt to factorise or solve a quadratic (See rules for factorising quadratics) or cancelling out $\cot 2x$ from both sides.	dM1
	$\cot 2x = 0 \quad \text{or} \quad \cot 2x = 1$	Both $\cot 2x = 0$ and $\cot 2x = 1$.	A1
	$\cot 2x = 0 \Rightarrow (\tan 2x \to \infty) \Rightarrow 2x = 90, 270$ $\Rightarrow x = 45, 135$ $\cot 2x = 1 \Rightarrow \tan 2x = 1 \Rightarrow 2x = 45, 225$ $\Rightarrow x = 22.5, 112.5$	Candidate attempts to divide at least one of their principal angles by 2. This will be usually implied by seeing $x = 22.5$ resulting from $\cot 2x = 1$.	ddM1
	Overall, $x = \{22.5, 45, 112.5, 135\}$	***Both*** $x = 22.5$ and $x = 112.5$ ***Both*** $x = 45$ and $x = 135$	A1 B1
			[7]

If there are any EXTRA solutions inside the range $0 \le x \le 180^\circ$ and the candidate would otherwise score FULL MARKS then withhold the final accuracy mark (the sixth mark in this question).
Also ignore EXTRA solutions outside the range $0 \le x \le 180^\circ$.

Question 15: June 10 Q1

Question Number	Scheme	Marks
(a)	$\dfrac{2\sin\theta\cos\theta}{1+2\cos^2\theta-1}$	M1
	$\dfrac{\cancel{2}\sin\theta\cancel{\cos\theta}}{\cancel{2}\cos\theta\cancel{\cos\theta}} = \tan\theta$ (as required) **AG**	A1 cso
		(2)
(b)	$2\tan\theta = 1 \Rightarrow \tan\theta = \dfrac{1}{2}$	M1
	θ_1 = awrt 26.6°	A1
	θ_2 = awrt -153.4°	A1$\sqrt{}$
		(3) **[5]**
	(a) M1: Uses **both** a correct identity for $\sin 2\theta$ **and** a correct identity for $\cos 2\theta$. Also allow a candidate writing $1+\cos 2\theta = 2\cos^2\theta$ on the denominator. Also note that angles **must be consistent** in when candidates apply these identities. A1: Correct proof. No errors seen. (b) 1st M1 for either $2\tan\theta = 1$ or $\tan\theta = \dfrac{1}{2}$, seen or implied. A1: awrt 26.6 A1$\sqrt{}$: awrt -153.4° or $\theta_2 = -180^\circ + \theta_1$ **Special Case**: For candidate solving, $\tan\theta = k$, where $k \neq \frac{1}{2}$, to give θ_1 and $\theta_2 = -180^\circ + \theta_1$, then award M0A0B1 in part (b). **Special Case:** Note that those candidates who writes $\tan\theta = 1$, and gives ONLY two answers of 45° and -135° that are inside the range will be awarded SC M0A0B1.	

Question 16: Jan 11 Q1

Question Number	Scheme		Marks
(a)	$7\cos x - 24\sin x = R\cos(x+\alpha)$		
	$7\cos x - 24\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$		
	Equate $\cos x$: $7 = R\cos\alpha$ Equate $\sin x$: $24 = R\sin\alpha$		
	$R = \sqrt{7^2+24^2}$; $= 25$	$R = 25$	B1
	$\tan\alpha = \frac{24}{7} \Rightarrow \alpha = 1.287002218...^c$	$\tan\alpha = \frac{24}{7}$ or $\tan\alpha = \frac{7}{24}$ awrt 1.287	M1 A1
	Hence, $7\cos x - 24\sin x = 25\cos(x+1.287)$		(3)
(b)	Minimum value = $\underline{-25}$	-25 or $-R$	B1ft (1)
(c)	$7\cos x - 24\sin x = 10$		
	$25\cos(x+1.287) = 10$		
	$\cos(x+1.287) = \frac{10}{25}$	$\cos(x \pm \text{their } \alpha) = \frac{10}{(\text{their } R)}$	M1
	PV $= 1.159279481...^c$ or $66.42182152...^\circ$	For applying $\cos^{-1}\left(\frac{10}{\text{their } R}\right)$	M1
	So, $x + 1.287 = \{1.159279...^c, 5.123906...^c, 7.442465...^c\}$	either 2π + or − their PV^c or 360° + or − their PV$^\circ$	M1
	gives, $x = \{3.836906..., 6.155465...\}$	awrt 3.84 OR 6.16 awrt 3.84 AND 6.16	A1 A1 (5) **[9]**

Question 17: Jan 11 Q3

Question Number	Scheme		Marks
	$2\cos 2\theta = 1 - 2\sin\theta$		
	$2(1 - 2\sin^2\theta) = 1 - 2\sin\theta$	Substitutes either $1 - 2\sin^2\theta$ or $2\cos^2\theta - 1$ or $\cos^2\theta - \sin^2\theta$ for $\cos 2\theta$.	M1
	$2 - 4\sin^2\theta = 1 - 2\sin\theta$		
	$4\sin^2\theta - 2\sin\theta - 1 = 0$	Forms a "quadratic in sine" = 0	M1(*)
	$\sin\theta = \dfrac{2 \pm \sqrt{4 - 4(4)(-1)}}{8}$	Applies the quadratic formula See notes for alternative methods.	M1
	PVs: $\alpha_1 = 54^\circ$ or $\alpha_2 = -18^\circ$	Any one correct answer	A1
	$\theta = \{54, 126, 198, 342\}$	180-their pv All four solutions correct.	dM1(*) A1
			[6]

Question 18: June 11 Q6

Question Number	Scheme	Marks
(a)	$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \frac{1-\cos 2\theta}{\sin 2\theta}$	M1
	$= \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$	M1A1
	$= \frac{\sin\theta}{\cos\theta} = \tan\theta$ cso	A1* (4)
(b)(i)	$\tan 15° = \frac{1}{\sin 30°} - \frac{\cos 30°}{\sin 30°}$	M1
	$\tan 15° = \frac{1}{\frac{1}{2}} - \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2 - \sqrt{3}$ cso	dM1 A1* (3)

(b)(ii)	$\tan 2x = 1$	M1
	$2x = 45°$	A1
	$2x = 45° + 180°$	M1
	$x = 22.5°, 112.5°, 202.5°, 292.5°$	A1(any two) A1 (5)
	Alt for (b)(i) $\tan 15° = \tan(60° - 45°) \quad or \quad \tan(45° - 30°)$	12 Marks
	$\tan 15° = \dfrac{\tan 60 - \tan 45}{1 + \tan 60 tan 45} \; or \; \dfrac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$	M1
	$\tan 15° = \dfrac{\sqrt{3} - 1}{1 + \sqrt{3}} \; or \; \dfrac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$	M1
	Rationalises to produce $\tan 15° = 2 - \sqrt{3}$	A1*

Question 19: June 11 Q8

Question Number	Scheme	Marks
(a)	$R^2 = 2^2 + 3^2$ $R = \sqrt{13}$ or 3.61	M1 A1
	$\tan\alpha = \dfrac{3}{2}$ $\alpha = 0.983 \ldots$	M1 A1 (4)