Name:

Total Marks:

Pure Mathematics 1

Advanced Subsidiary

Practice Paper J8

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE AS Level Specifications
- There are 12 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

The point *A* (–6, 4) and the point *B* (8, –3) lie on the line *L*.

(a) Find an equation for *L* in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers. **(4)**

(b) Find the distance *AB*, giving your answer in the form $k\sqrt{5}$, where *k* is an integer. **(3)**

(Total 7 marks)

Question 2

A circle *C* has centre *M* (6, 4) and radius 3.

(a) Write down the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = r^2$$ **(2)**

Figure 3

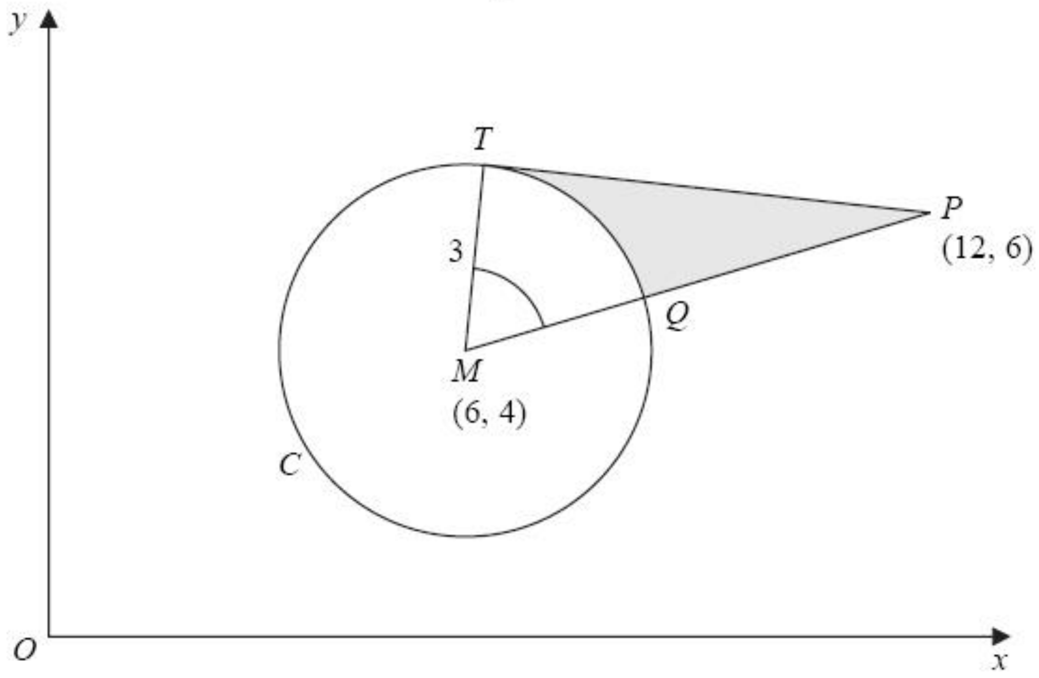


Figure 3 shows the circle *C*. The point *T* lies on the circle and the tangent at *T* passes through the point *P* (12, 6). The line *MP* cuts the circle at *Q*.

(b) Show that the angle *TMQ* is 61.8835 degrees to 4 decimal places. **(4)**

The shaded region *TPQ* is bounded by the straight lines *TP*, *QP* and the arc *TQ*, as shown in Figure 3.

Given that the area of sector MTQ is 4.8446

(c) Find the area of the shaded region *TPQ*. Give your answer to 3 decimal places. **(4)**

(Total 9 marks)

Question 3

Figure 4

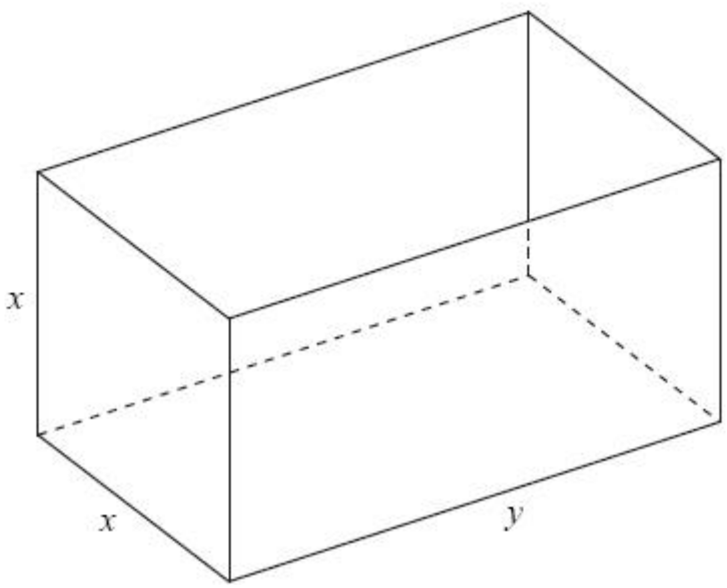


Figure 4 shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle *x* metres by *y* metres. The height of the tank is *x* metres.

The capacity of the tank is 100 m^3.

(a) Show that the area A m^2 of the sheet metal used to make the tank is given by

$$A = \frac{300}{x} + 2x^2.$$

(4)

(b) Use calculus to find the value of x for which A is stationary.

(4)

(c) Prove that this value of x gives a minimum value of A.

(2)

(d) Calculate the minimum area of sheet metal needed to make the tank.

(2)

(Total 12 marks)

Question 4

The curve C has equation

$$y = (x + 3)(x - 1)^2.$$

(a) Sketch C showing clearly the coordinates of the points where the curve meets the coordinate axes.

(4)

(b) Show that the equation of C can be written in the form

$$y = x^3 + x^2 - 5x + k,$$

where k is a positive integer, and state the value of k.

(2)

There are two points on C where the gradient of the tangent to C is equal to 3.

(c) Find the x-coordinates of these two points.

(6)

(Total 12 marks)

Question 5

Given that a and b are positive constants, solve the simultaneous equations

$$a = 3b,$$

$$\log_3 a + \log_3 b = 2.$$

Give your answers as exact numbers.

(6)

(Total 6 marks)

Question 6

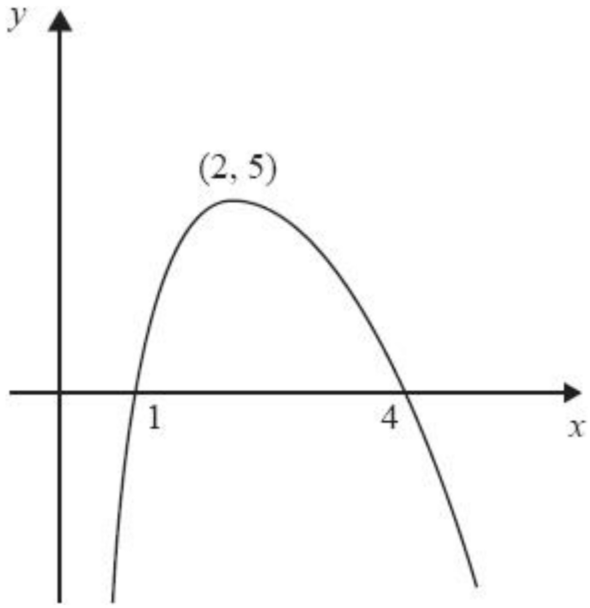


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$. The curve crosses the x-axis at the points (1, 0) and (4, 0). The maximum point on the curve is (2, 5).

In separate diagrams sketch the curves with the following equations.

On each diagram show clearly the coordinates of the maximum point and of each point at which the curve crosses the x-axis.

(a) $y = 2\mathrm{f}(x)$,

(3)

(b) $y = \mathrm{f}(-x)$.

(3)

The maximum point on the curve with equation $y = \mathrm{f}(x + a)$ is on the y-axis.

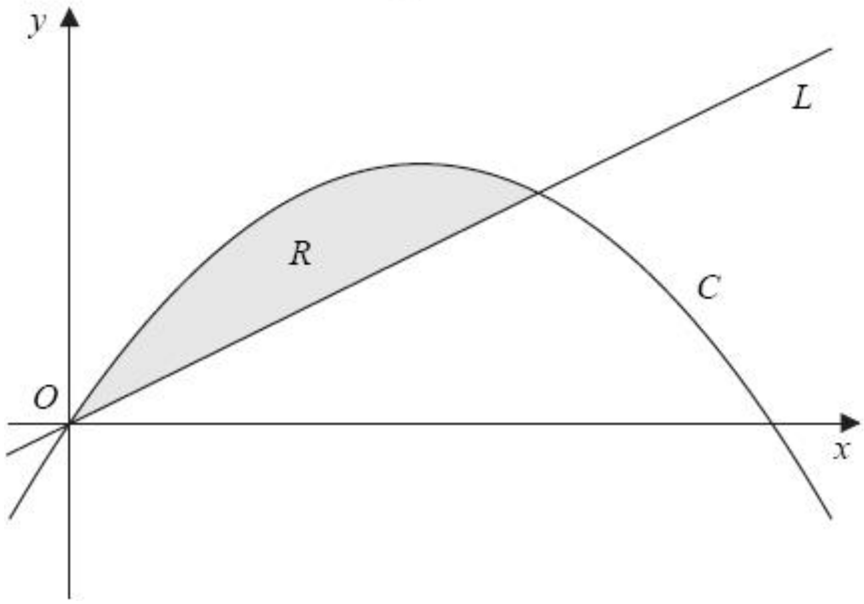
(c) Write down the value of the constant a.

(1)

(Total 7 marks)

Question 7

Figure 2



In Figure 2 the curve C has equation $y = 6x - x^2$ and the line L has equation $y = 2x$.

(a) Show that the curve C intersects the x-axis at $x = 0$ and $x = 6$. **(1)**

(b) Show that the line L intersects the curve C at the points (0, 0) and (4, 8). **(3)**

The region R, bounded by the curve C and the line L, is shown shaded in Figure 2.

(c) Use calculus to find the area of R. **(6)**

(Total 10 marks)

Question 8

(a) Show that the equation

$$3 \sin^2\theta - 2 \cos^2\theta = 1$$

can be written as

$$5 \sin^2\theta = 3.$$

(2)

(b) Hence solve, for $0° \leq \theta < 360°$, the equation

$$3 \sin^2\theta - 2 \cos^2\theta = 1,$$

giving your answers to 1 decimal place.

(7)

(Total 9 marks)

Question 9

Figure 1

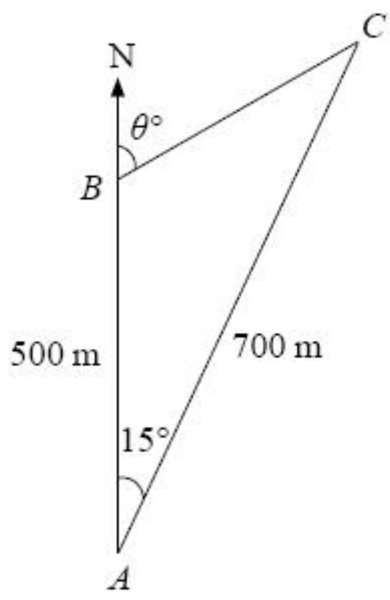


Figure 1 shows 3 yachts *A*, *B* and *C* which are assumed to be in the same horizontal plane. Yacht *B* is 500 m due north of yacht *A* and yacht *C* is 700 m from A. The bearing of *C* from *A* is 015°.

(a) Calculate the distance between yacht *B* and yacht *C*, in metres to 3 significant figures. **(3)**

The bearing of yacht *C* from yacht *B* is θ°, as shown in Figure 1.

(b) Calculate the value of θ. **(4)**

(Total 7 marks)

Question 10

The radioactive decay of a substance is given by

$$R = 1000e^{-ct}, \qquad t \geq 0.$$

where *R* is the number of atoms at time *t* years and *c* is a positive constant.

(a) Find the number of atoms when the substance started to decay. **(1)**

It takes 5730 years for half of the substance to decay.

(b) Find the value of *c* to 3 significant figures. **(4)**

(c) Calculate the number of atoms that will be left when $t = 22\,920$. **(2)**

(d) Sketch the graph of *R* against *t* . **(2)**

(Total 9 marks)

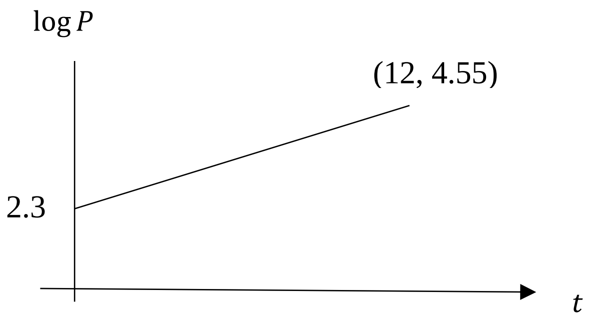
Question 11

Prove, from first principles, that the derivate of $3x^3$ is $9x^2$ **(3)**

(Total 3 marks)

Question 12

The graph represents the growth of a population of bacteria, P over t hours. The graph is modelled by the equation $P = ab^t$, where a and b are constants to be found.



The graph passes through the points (0, 2.3) and (12, 4.55)

(a) Write down the equation of the line **(2)**

(b) Using your answer to part **(a)** or otherwise, find the values of a and b giving your answers to 3 significant figures. **(4)**

(c) Interpret the meaning of the constant a in this model **(1)**

(d) Use your model to predict the population of bacteria to the nearest thousands after 20 hours. Comment on the validity of your answer. **(2)**

(Total 9 marks)

TOTAL FOR PAPER IS 100 MARKS