Integration by Parts - Edexcel Past Exam Questions MARK SCHEME

Question 1: June 07 Q3

Question Number	Scheme		Marks
(a)	$\left\{\begin{array}{l} u = x \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = \cos 2x \Rightarrow v = \frac{1}{2}\sin 2x \end{array}\right\}$		
	Int $= \int x\cos 2x \, dx = \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x.1 \, dx$	***(see note below)*** Use of 'integration by parts' formula in the correct direction. Correct expression.	M1 A1
	$= \frac{1}{2}x\sin 2x - \frac{1}{2}\left(-\frac{1}{2}\cos 2x\right) + c$	$\sin 2x \to -\frac{1}{2}\cos 2x$ or $\sin kx \to -\frac{1}{k}\cos kx$ with $k \neq 1, k > 0$	dM1
	$= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$	Correct expression with $+c$	A1 **[4]**
(b)	$\int x\cos^2 x \, dx = \int x\left(\frac{\cos 2x + 1}{2}\right) dx$	Substitutes **correctly** for $\cos^2 x$ in the given integral	M1
	$= \frac{1}{2}\int x\cos 2x \, dx + \frac{1}{2}\int x \, dx$		
	$= \underline{\frac{1}{2}\left(\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right)}; + \frac{1}{2}\int x \, dx$	$\frac{1}{2}$(their answer to (a)); or underlined expression	A1; √
	$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2 \; (+c)$	Completely correct expression with/without $+c$	A1 **[3]**
			7 marks
Notes:			
(b)	Int $= \int x\cos 2x \, dx = \frac{1}{2}x\sin 2x \pm \int \frac{1}{2}\sin 2x.1 \, dx$	This is acceptable for M1	M1
	$\left\{\begin{array}{l} u = x \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = \cos 2x \Rightarrow v = \lambda\sin 2x \end{array}\right\}$ Int $= \int x\cos 2x \, dx = \lambda x\sin 2x \pm \int \lambda\sin 2x.1 \, dx$	This is also acceptable for M1	M1

Aliter (b) **Way 2**	$\int x\cos^2 x\,dx = \int x\left(\frac{\cos 2x+1}{2}\right)dx$	Substitutes **correctly** for $\cos^2 x$ in the given integral …	M1
	$\left\{\begin{array}{ll} u = x & \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = \frac{1}{2}\cos 2x + \frac{1}{2} & \Rightarrow v = \frac{1}{4}\sin 2x + \frac{1}{2}x \end{array}\right\}$	… or $u = x$ and $\frac{dv}{dx} = \frac{1}{2}\cos 2x + \frac{1}{2}$	
	$= \frac{1}{4}x\sin 2x + \frac{1}{2}x^2 - \int\left(\frac{1}{4}\sin 2x + \frac{1}{2}x\right)dx$		
	$= \underline{\frac{1}{4}x\sin 2x} + \frac{1}{2}x^2 + \underline{\frac{1}{8}\cos 2x} - \frac{1}{4}x^2 + c$	$\frac{1}{2}$(their answer to (a)); or underlined expression	A1 $\sqrt{}$
	$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2\ (+c)$	Completely correct expression with/without $+c$	A1
			[3]
Aliter (b) **Way 3**	$\int x\cos 2x\,dx = \int x\left(2\cos^2 x - 1\right)dx$	Substitutes **correctly** for $\cos 2x$ in $\int x\cos 2x\,dx$	M1
	$\Rightarrow 2\int x\cos^2 x\,dx - \int x\,dx = \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$		
	$\Rightarrow \int x\cos^2 x\,dx = \underline{\frac{1}{2}\left(\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right)}; + \frac{1}{2}\int x\,dx$	$\frac{1}{2}$(their answer to (a)); or underlined expression	A1; $\sqrt{}$
	$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2\ (+c)$	Completely correct expression with/without $+c$	A1
			[3]
			7 marks

Question 2: Jan 08 Q4

Question Number	Scheme		Marks
(i)	$\int \ln\left(\frac{x}{2}\right)\,dx = \int 1.\ln\left(\frac{x}{2}\right)\,dx \Rightarrow \left\{ \begin{array}{l} u = \ln\left(\frac{x}{2}\right) \Rightarrow \frac{du}{dx} = \frac{\frac{1}{2}}{\frac{x}{2}} = \frac{1}{x} \\ \frac{dv}{dx} = 1 \Rightarrow v = x \end{array} \right\}$		
	$\int \ln\left(\frac{x}{2}\right)\,dx = x\ln\left(\frac{x}{2}\right) - \int x.\frac{1}{x}\,dx$	Use of 'integration by parts' formula in the correct direction. Correct expression.	M1 A1
	$= x\ln\left(\frac{x}{2}\right) - \int \underline{1}\,dx$	An attempt to multiply x by a candidate's $\frac{a}{x}$ or $\frac{1}{bx}$ or $\frac{1}{x}$.	<u>dM1</u>
	$= x\ln\left(\frac{x}{2}\right) - x + c$	Correct integration with $+ c$	A1 aef [4]
(ii)	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\,dx$		
	$\left[\text{NB}: \underline{\cos 2x = \pm 1 \pm 2\sin^2 x} \text{ gives } \sin^2 x = \frac{1-\cos 2x}{2}\right]$	Consideration of double angle formula for $\sin^2 x$	M1
	$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1-\cos 2x}{2}\,dx = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\underline{1-\cos 2x}\right)\,dx$		
	$= \frac{1}{2}\left[\underline{x - \frac{1}{2}\sin 2x}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$	<u>Integrating to give</u> $\underline{\pm ax \pm b\sin 2x}$; Correct result of anything equivalent to $\frac{1}{2}x - \frac{1}{4}\sin 2x$	dM1 A1
	$= \frac{1}{2}\left[\left(\frac{\pi}{2} - \frac{\sin(\pi)}{2}\right) - \left(\frac{\pi}{4} - \frac{\sin(\frac{\pi}{2})}{2}\right)\right]$ $= \frac{1}{2}\left[\left(\frac{\pi}{2} - 0\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right)\right]$	Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round.	ddM1
	$= \frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right) = \frac{\pi}{8} + \frac{1}{4}$	$\underline{\frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right)}$ or $\underline{\frac{\pi}{8} + \frac{1}{4}}$ Candidate must collect their π term and constant term together for A1	A1 aef [5]
			9 marks

Question Number	Scheme		Marks
Aliter (i) Way 2	$\int \ln\left(\frac{x}{2}\right) dx = \int (\ln x - \ln 2)\, dx = \int \ln x\, dx - \int \ln 2\, dx$		
	$\int \ln x\, dx = \int 1.\ln x\, dx \Rightarrow \begin{Bmatrix} u = \ln x & \Rightarrow & \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = 1 & \Rightarrow & v = x \end{Bmatrix}$		
	$\int \ln x\, dx = x\ln x - \int x.\frac{1}{x}\, dx$	Use of 'integration by parts' formula in the correct direction.	M1
	$= x\ln x - x + c$	Correct integration of $\ln x$ with or without $+ c$	A1
	$\int \ln 2\, dx = x\ln 2 + c$	Correct integration of $\ln 2$ with or without $+ c$	M1
	Hence, $\int \ln\left(\frac{x}{2}\right) dx = x\ln x - x - x\ln 2 + c$	Correct integration with $+ c$	A1 aef
			[4]

Question Number	Scheme		Marks
Aliter (ii) **Way 2**	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x . \sin x \, dx$ and $I = \int \sin^2 x \, dx$		
	$\left\{ \begin{array}{l} u = \sin x \quad \Rightarrow \frac{du}{dx} = \cos x \\ \frac{dv}{dx} = \sin x \quad \Rightarrow v = -\cos x \end{array} \right\}$		
	$\therefore I = \underline{\left\{ -\sin x \cos x + \int \cos^2 x \, dx \right\}}$	An attempt to use the correct by parts formula.	M1
	$\therefore I = \left\{ -\sin x \cos x + \int (1 - \sin^2 x) dx \right\}$		
	$\int \sin x \, dx = \left\{ -\sin x \cos x + \int 1 \, dx - \int \sin^2 x \, dx \right\}$		
	$2 \int \sin^2 x \, dx = \left\{ -\sin x \cos x + \int 1 \, dx \right\}$	For the LHS becoming $2I$	dM1
	$2 \int \sin^2 x \, dx = \{ -\sin x \cos x + x \}$		
	$\int \sin^2 x \, dx = \{ \underline{-\tfrac{1}{2} \sin x \cos x + \tfrac{x}{2}} \}$	<u>Correct integration</u>	A1
	$\therefore \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx = \left[\left(-\tfrac{1}{2}\sin(\tfrac{\pi}{2})\cos(\tfrac{\pi}{2}) + \tfrac{(\frac{\pi}{2})}{2} \right) - \left(-\tfrac{1}{2}\sin(\tfrac{\pi}{4})\cos(\tfrac{\pi}{4}) + \tfrac{(\frac{\pi}{4})}{2} \right) \right]$ $= \left[(0 + \tfrac{\pi}{4}) - (-\tfrac{1}{4} + \tfrac{\pi}{8}) \right]$	Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round.	ddM1
	$= \tfrac{\pi}{8} + \tfrac{1}{4}$	$\underline{\tfrac{1}{8}(\pi + 2)}$ or $\underline{\tfrac{\pi}{8} + \tfrac{1}{4}}$ Candidate must collect their pi term and constant term together for A1	A1 aef **[5]**

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Question 3: June 08 Q2

Question Number	Scheme	Marks
(a)	$\left\{\begin{matrix} u = x & \Rightarrow & \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^x & \Rightarrow & v = e^x \end{matrix}\right\}$	
	$\int xe^x \, dx = xe^x - \int e^x.1 \, dx$	M1 A1
	$= xe^x - \int e^x \, dx$	
	$= xe^x - e^x \ (+c)$	A1 (3)
(*b*)	$\left\{\begin{matrix} u = x^2 & \Rightarrow & \frac{du}{dx} = 2x \\ \frac{dv}{dx} = e^x & \Rightarrow & v = e^x \end{matrix}\right\}$	
	$\int x^2e^x \, dx = x^2 e^x - \int e^x.2x \, dx$	M1 A1
	$= x^2e^x - 2\int xe^x \, dx$	
	$= x^2e^x - 2\left(xe^x - e^x\right) + c$	A1 (3)
		(6 marks)

Question 4: Jan 09 Q6

Question Number	Scheme		Marks
(a)	$\int \tan^2 x \, dx$		
	$\left[NB: \sec^2 A = 1 + \tan^2 A \text{ gives } \underline{\tan^2 A = \sec^2 A - 1}\right]$	The correct underlined identity.	M1 oe
	$= \int \sec^2 x - 1 \, dx$		
	$= \underline{\tan x - x} \, (+c)$	Correct integration with/without + c	A1
			(2)
(b)	$\int \frac{1}{x^3} \ln x \, dx$		
	$\left\{\begin{matrix} u = \ln x & \Rightarrow & \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x^{-3} & \Rightarrow & v = \frac{x^{-2}}{-2} = \frac{-1}{2x^2} \end{matrix}\right\}$		
	$= -\frac{1}{2x^2} \ln x - \int -\frac{1}{2x^2} \cdot \frac{1}{x} \, dx$	Use of 'integration by parts' formula in the correct direction. Correct direction means that $u = \ln x$.	M1
		Correct expression.	A1
	$= -\frac{1}{2x^2} \ln x + \frac{1}{2} \int \frac{1}{x^3} \, dx$	An attempt to multiply through $\frac{k}{x^n}$, $n \in \square$, $n \ldots 2$ by $\frac{1}{x}$ and an attempt to ...	
	$= \underline{-\frac{1}{2x^2} \ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)} (+c)$	... "integrate" (process the result);	M1
		correct solution with/without + c	A1 oe
			(4)

Question Number	Scheme		Marks
(c)	$\int \frac{e^{3x}}{1+e^x} \, dx$		
	$\left\{u = 1 + e^x \Rightarrow \underline{\frac{du}{dx} = e^x}, \underline{\frac{dx}{du} = \frac{1}{e^x}}, \underline{\frac{dx}{du} = \frac{1}{u-1}}\right\}$	Differentiating to find any one of the three underlined	B1
	$= \int \frac{e^{2x} \cdot e^x}{1+e^x} \, dx = \int \frac{(u-1)^2 \cdot e^x}{u} \cdot \frac{1}{e^x} \, du$	Attempt to substitute for $e^{2x} = f(u)$, their $\frac{dx}{du} = \frac{1}{e^x}$ and $u = 1 + e^x$	
	or $= \int \frac{(u-1)^3}{u} \cdot \frac{1}{(u-1)} \, du$	or $e^{3x} = f(u)$, their $\frac{dx}{du} = \frac{1}{u-1}$ and $u = 1 + e^x$.	M1*
	$= \int \frac{(u-1)^2}{u} \, du$	$\underline{\int \frac{(u-1)^2}{u} \, du}$	A1
	$= \int \frac{u^2 - 2u + 1}{u} \, du$		
	$= \int u - 2 + \frac{1}{u} \, du$	An attempt to multiply out their numerator to give at least three terms and divide through each term by u	dM1*
	$= \frac{u^2}{2} - 2u + \ln u \; (+c)$	Correct integration with/without +c	A1
	$= \frac{(1+e^x)^2}{2} - 2(1+e^x) + \ln(1+e^x) + c$	Substitutes $u = 1 + e^x$ back into their integrated expression with at least two terms.	dM1*
	$= \frac{1}{2} + e^x + \frac{1}{2}e^{2x} - 2 - 2e^x + \ln(1+e^x) + c$		
	$= \frac{1}{2} + e^x + \frac{1}{2}e^{2x} - 2 - 2e^x + \ln(1+e^x) + c$		
	$= \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) - \frac{3}{2} + c$		
	$= \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k$ **AG**	$\underline{\frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k}$ must use a $+c$ and "$-\frac{3}{2}$" combined.	A1 cso
			(7)
			[13]

Question 5: June 09 Q6

Question Number	Scheme	Marks
Q (a)	$\int \sqrt{(5-x)}\,dx = \int (5-x)^{\frac{1}{2}}\,dx = \dfrac{(5-x)^{\frac{3}{2}}}{-\frac{3}{2}} \quad (+C)$	M1 A1 (2)
	$\left(= -\frac{2}{3}(5-x)^{\frac{3}{2}} + C\right)$	
(b)	(i) $\int (x-1)\sqrt{(5-x)}\,dx = -\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} + \frac{2}{3}\int (5-x)^{\frac{3}{2}}\,dx$	M1 A1ft
	$= \quad \ldots \quad + \frac{2}{3} \times \dfrac{(5-x)^{\frac{5}{2}}}{-\frac{5}{2}} \quad (+C)$	M1
	$= -\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \frac{4}{15}(5-x)^{\frac{5}{2}} \quad (+C)$	A1 (4)
	(ii) $\left[-\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \frac{4}{15}(5-x)^{\frac{5}{2}}\right]_1^5 = (0-0) - \left(0 - \frac{4}{15} \times 4^{\frac{5}{2}}\right)$	
	$= \frac{128}{15}\left(= 8\frac{8}{15} \approx 8.53\right)$ awrt 8.53	M1 A1 (2)
		[8]
	Alternatives for (b) *and* (c)	
	(b) $u^2 = 5 - x \Rightarrow 2u\frac{du}{dx} = -1 \quad \left(\Rightarrow \frac{dx}{du} = -2u\right)$	
	$\int (x-1)\sqrt{(5-x)}\,dx = \int (4-u^2)u\frac{dx}{du}\,du = \int (4-u^2)u(-2u)\,du$	M1 A1
	$= \int (2u^4 - 8u^2)\,du = \frac{2}{5}u^5 - \frac{8}{3}u^3 \quad (+C)$	M1
	$= \frac{2}{5}(5-x)^{\frac{5}{2}} - \frac{8}{3}(5-x)^{\frac{3}{2}} \quad (+C)$	A1
	(c) $x = 1 \Rightarrow u = 2, \quad x = 5 \Rightarrow u = 0$	
	$\left[\frac{2}{5}u^5 - \frac{8}{3}u^3\right]_2^0 = (0-0) - \left(\frac{64}{5} - \frac{64}{3}\right)$	M1
	$= \frac{128}{15}\left(= 8\frac{8}{15} \approx 8.53\right)$ awrt 8.53	A1 (2)

Question 6: Jan 10 Q2

	(c)(i) $\int x\ln x\,dx = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\times\frac{1}{x}\,dx$	M1 A1
	$= \frac{x^2}{2}\ln x - \int \frac{x}{2}\,dx$	
	$= \frac{x^2}{2}\ln x - \frac{x^2}{4}\ (+C)$	M1 A1

Question 7: June 10 Q6

Question Number	Scheme		Marks	
	(a) $f(\theta) = 4\cos^2\theta - 3\sin^2\theta$			
	$= 4\left(\frac{1}{2}+\frac{1}{2}\cos 2\theta\right) - 3\left(\frac{1}{2}-\frac{1}{2}\cos 2\theta\right)$		M1 M1	
	$= \frac{1}{2}+\frac{7}{2}\cos 2\theta$ ✱	cso	A1	(3)
	(b) $\int \theta\cos 2\theta\,d\theta = \frac{1}{2}\theta\sin 2\theta - \frac{1}{2}\int \sin 2\theta\,d\theta$		M1 A1	
	$= \frac{1}{2}\theta\sin 2\theta + \frac{1}{4}\cos 2\theta$		A1	
	$\int \theta f(\theta)\,d\theta = \frac{1}{4}\theta^2 + \frac{7}{4}\theta\sin 2\theta + \frac{7}{8}\cos 2\theta$		M1 A1	
	$[\ \ldots\ ]_0^{\frac{\pi}{2}} = \left[\frac{\pi^2}{16}+0-\frac{7}{8}\right] - \left[0+0+\frac{7}{8}\right]$		M1	
	$= \frac{\pi^2}{16} - \frac{7}{4}$		A1	(7)
				[10]

Question 8: Jan 11 Q1

Question Number	Scheme	Marks
	$\int x\sin 2x\,dx = -\frac{x\cos 2x}{2} + \int \frac{\cos 2x}{2}\,dx$	M1 A1 A1
	$= \ \ldots\ + \frac{\sin 2x}{4}$	M1
	$[\ \ldots\ ]_0^{\frac{\pi}{2}} = \frac{\pi}{4}$	M1 A1
		[6]