

Integration by Substitution- Edexcel Past Exam Questions

1. Use the substitution $x = \sin \theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{1}{(1-x^2)^{\frac{3}{2}}} dx. \quad (7)$$

June 05 Q4

2. Using the substitution $u^2 = 2x - 1$, or otherwise, find the exact value of

$$\int_1^5 \frac{3x}{\sqrt{2x-1}} dx. \quad (8)$$

Jan 06 Q3

3.
$$I = \int_0^5 e^{\sqrt{3x+1}} dx.$$

- (a) Given that $y = e^{\sqrt{3x+1}}$, copy and complete the table with the values of y corresponding to $x = 2, 3$ and 4 .

x	0	1	2	3	4	5
y	e^1	e^2				e^4

(2)

- (b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the original integral I , giving your answer to 4 significant figures. **(3)**

- (c) Use the substitution $t = \sqrt{3x+1}$ to show that I may be expressed as $\int_a^b kte^t dt$, giving the values of a, b and k . **(5)**

- (d) Use integration by parts to evaluate this integral, and hence find the value of I correct to 4 significant figures, showing all the steps in your working. **(5)**

Jan 07 Q8

4. Use the substitution $u = 2^x$ to find the exact value of

$$\int_0^1 \frac{2^x}{(2^x+1)^2} dx. \quad (6)$$

June 07 Q2

5. (a) Using the substitution $x = 2 \cos u$, or otherwise, find the exact value of

$$\int_1^{\sqrt{2}} \frac{1}{x^2 \sqrt{4-x^2}} dx. \quad (7)$$

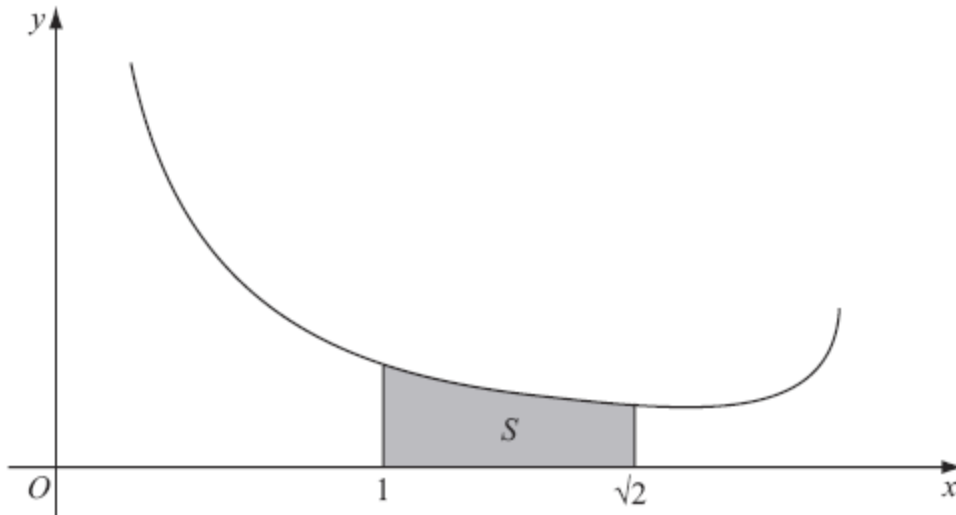


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \frac{4}{x(4-x^2)^{\frac{1}{4}}}$, $0 < x < 2$.

The shaded region S , shown in Figure 3, is bounded by the curve, the x -axis and the lines with equations $x = 1$ and $x = \sqrt{2}$. The shaded region S is rotated through 2π radians about the x -axis to form a solid of revolution.

- (b) Using your answer to part (a), find the exact volume of the solid of revolution formed. (3)

Jan 10 Q8

6. Using the substitution $u = \cos x + 1$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{\cos x + 1} \sin x \, dx = e(e-1). \quad (6)$$

June 10 Q2



7.

$$I = \int_2^5 \frac{1}{4 + \sqrt{x-1}} dx.$$

- (a) Given that $y = \frac{1}{4 + \sqrt{x-1}}$, copy and complete the table below with values of y corresponding to $x = 3$ and $x = 5$. Give your values to 4 decimal places.

x	2	3	4	5
y	0.2		0.1745	

(2)

- (b) Use the trapezium rule, with all of the values of y in the completed table, to obtain an estimate of I , giving your answer to 3 decimal places. (4)
- (c) Using the substitution $x = (u - 4)^2 + 1$, or otherwise, and integrating, find the exact value of I . (8)

Jan 11 Q7

8.

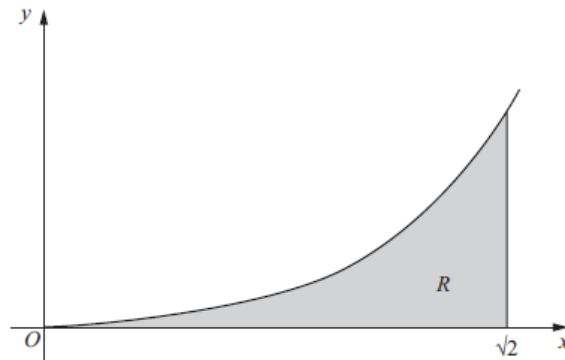


Figure 2

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln(x^2 + 2)$, $x \geq 0$.

The finite region R , shown shaded in Figure 2, is bounded by the curve, the x -axis and the line $x = \sqrt{2}$.

The table below shows corresponding values of x and y for $y = x^3 \ln(x^2 + 2)$.

x	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{4}$	$\sqrt{2}$
y	0		0.3240		3.9210

- (a) Complete the table above giving the missing values of y to 4 decimal places. (2)
- (b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R , giving your answer to 2 decimal places. (3)
- (c) Use the substitution $u = x^2 + 2$ to show that the area of R is

$$\frac{1}{2} \int_2^4 (u - 2) \ln u \, du . \tag{4}$$

- (d) Hence, or otherwise, find the exact area of R . (6)

June 11 Q4

9.

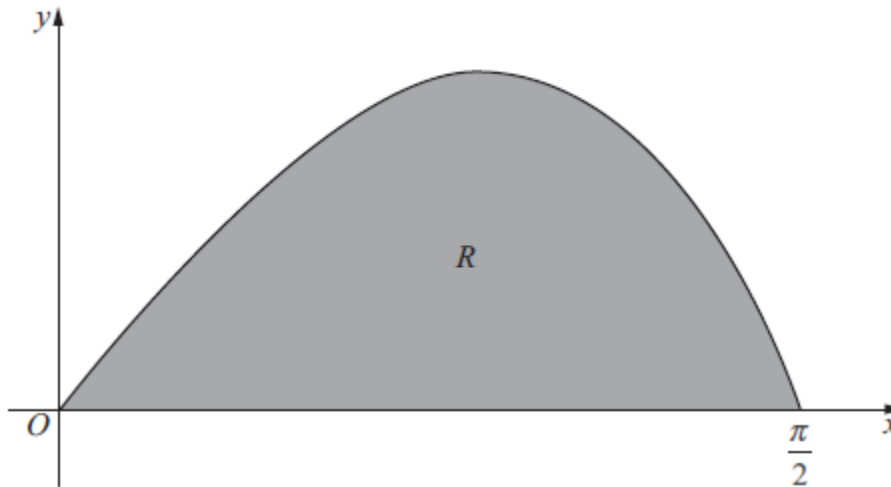

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \frac{2 \sin 2x}{(1 + \cos x)}$, $0 \leq x \leq \frac{\pi}{2}$.

The finite region R , shown shaded in Figure 3, is bounded by the curve and the x -axis.

The table below shows corresponding values of x and y for $y = \frac{2 \sin 2x}{(1 + \cos x)}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

- (a) Complete the table above giving the missing value of y to 5 decimal places. (1)
- (b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R , giving your answer to 4 decimal places. (3)
- (c) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2 \sin 2x}{(1 + \cos x)} dx = 4 \ln(1 + \cos x) - 4 \cos x + k,$$

where k is a constant. (5)

- (d) Hence calculate the error of the estimate in part (b), giving your answer to 2 significant figures. (3)

Jan 12 Q6