Integration using Partial Fractions - Edexcel Past Exam Questions

1. (*a*) Express $\frac{5x+3}{(2x-3)(x+2)}$ in partial fractions. **(3)**

(*b*) Hence find the exact value of $\int_2^6 \frac{5x+3}{(2x-3)(x+2)}\,\mathrm{d}x$, giving your answer as a single logarithm. **(5)**

Jun 05 Q3

---

2. (*a*) Express $\frac{2x-1}{(x-1)(2x-3)}$ in partial fractions. **(3)**

(*b*) Given that $x \geq 2$, find the general solution of the differential equation

$$(2x-3)(x-1)\frac{\mathrm{d}y}{\mathrm{d}x} = (2x-1)y. \quad \textbf{(5)}$$

(*c*) Hence find the particular solution of this differential equation that satisfies $y = 10$ at $x = 2$, giving your answer in the form $y = \mathrm{f}(x)$. **(4)**

Jan 07 Q4

---

3. $$\frac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv A + \frac{B}{(2x+1)} + \frac{C}{(2x-1)}.$$

(*a*) Find the values of the constants *A*, *B* and *C*. **(4)**

(*b*) Hence show that the exact value of $\int_1^2 \frac{2(4x^2+1)}{(2x+1)(2x-1)}\,\mathrm{d}x$ is $2 + \ln k$, giving the value of the constant *k*. **(6)**

June 07 Q4

---

4. (*a*) Express $\frac{2}{4-y^2}$ in partial fractions. **(3)**

(*b*) Hence obtain the solution of

$$2\cot x \frac{\mathrm{d}y}{\mathrm{d}x} = (4-y^2)$$

for which $y = 0$ at $x = \frac{\pi}{3}$, giving your answer in the form $\sec^2 x = \mathrm{g}(y)$. **(8)**

June 08 Q7

5.

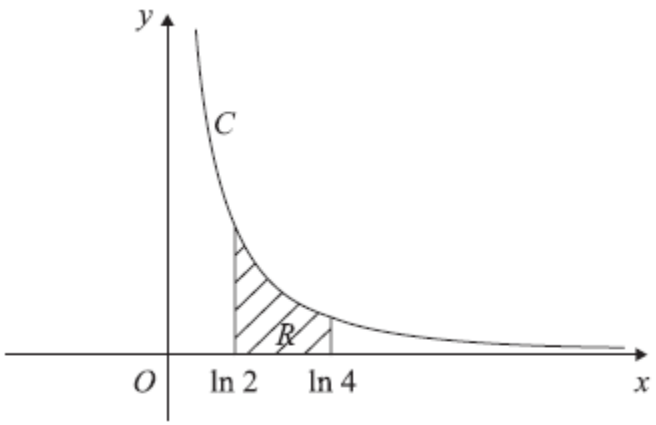

Figure 3

The curve C has parametric equations

$$x = \ln(t + 2), \quad y = \frac{1}{(t + 1)}, \quad t > -1.$$

The finite region R between the curve C and the x-axis, bounded by the lines with equations $x = \ln 2$ and $x = \ln 4$, is shown shaded in Figure 3.

(a) Show that the area of R is given by the integral

$$\int_0^2 \frac{1}{(t + 1)(t + 2)}\,\mathrm{d}t\,. \qquad \textbf{(4)}$$

(b) Hence find an exact value for this area. **(6)**

(c) Find a cartesian equation of the curve C, in the form $y = \mathrm{f}(x)$. **(4)**

(d) State the domain of values for x for this curve. **(1)**

Jan 08 Q7

---

6. $$\mathrm{f}(x) = \frac{4 - 2x}{(2x + 1)(x + 1)(x + 3)} = \frac{A}{(2x + 1)} + \frac{B}{(x + 1)} + \frac{C}{(x + 3)}\,.$$

(a) Find the values of the constants A, B and C. **(4)**

(b) (i) Hence find $\int \mathrm{f}(x)\ \mathrm{d}x$. **(3)**

(ii) Find $\int_0^2 \mathrm{f}(x)\ \mathrm{d}x$ in the form $\ln k$, where k is a constant. **(3)**

June 09 Q3

---

7. (*a*) Express $\dfrac{5}{(x-1)(3x+2)}$ in partial fractions. **(3)**

(*b*) Hence find $\displaystyle\int \frac{5}{(x-1)(3x+2)}\,\mathrm{d}x$, where $x > 1$. **(3)**

(*c*) Find the particular solution of the differential equation

$$(x-1)(3x+2)\frac{\mathrm{d}y}{\mathrm{d}x} = 5y, \quad x > 1,$$

for which $y = 8$ at $x = 2$. Give your answer in the form $y = \mathrm{f}(x)$. **(6)**

Jan 11 Q3

---

8. (*a*) Express $\dfrac{1}{P(5-P)}$ in partial fractions. **(3)**

A team of conservationists is studying the population of meerkats on a nature reserve. The population is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{15}P(5-P), \quad t \geq 0,$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that when $t = 0$, $P = 1$,

(*b*) solve the differential equation, giving your answer in the form,

$$P = \frac{a}{b + c\mathrm{e}^{-\frac{1}{3}t}}$$

where a, b and c are integers. **(8)**

(*c*) Hence show that the population cannot exceed 5000. **(1)**

Jan 12 Q8
