

Pure Mathematics 1 Practice Paper M8 **MARK SCHEME**

Q1.

Question Number	Scheme	Marks
(a)	$(1+ax)^{10} = 1+10ax.....$ $+ \frac{10 \times 9}{2}(ax)^2 + \frac{10 \times 9 \times 8}{6}(ax)^3$ $+ 45(ax)^2, +120(ax)^3$ or $+45a^2x^2, +120a^3x^3$	B1 M1 A1 A1 (4)
(b)	$120a^3 = 2 \times 45a^2$ $a = \frac{3}{4}$ or equiv. (e.g. $\frac{90}{120}, 0.75$)	M1 A1 (2)
		(6 marks)

Q2.

Question Number	Scheme	Marks
(a)	$QR = \sqrt{(7-1)^2 + (0-3)^2}$ $= \sqrt{36+9}$ or $\sqrt{45}$ $= 3\sqrt{5}$ or $a = 3$	M1 A1 A1 (3)
(b)	Gradient of QR (or l_1) = $\frac{3-0}{1-7}$ or $\frac{3}{-6}, = -\frac{1}{2}$ Gradient of l_2 is $-\frac{1}{-\frac{1}{2}}$ or 2	M1 A1 M1
	Equation for l_2 is: $y-3 = 2(x-1)$ or $\frac{y-3}{x-1} = 2$ [or $y = 2x + 1$]	M1 A1 ft (5)
(c)	P is $(0, 1)$ (allow " $x = 0, y = 1$ " but it must be clearly identifiable as P)	B1 (1)
(d)	$PQ = \sqrt{(1-x_p)^2 + (3-y_p)^2}$ $PQ = \sqrt{1^2 + 2^2} = \sqrt{5}$	M1 A1
	Area of triangle is $\frac{1}{2}QR \times PQ = \frac{1}{2}3\sqrt{5} \times \sqrt{5}, = \frac{15}{2}$ or 7.5	M1 A1 (4)
		(13 marks)

Q3.

Question Number	Scheme	Marks
(a)		B1 M1 A1 (3)
(b)	$2x + 5 = \frac{3}{x}$ $2x^2 + 5x - 3 [= 0] \quad \text{or} \quad 2x^2 + 5x = 3$ $(2x - 1)(x + 3) [= 0]$ $x = -3 \text{ or } \frac{1}{2}$ $y = \frac{3}{-3} \text{ or } 2 \times (-3) + 5 \quad \text{or} \quad y = \frac{3}{\frac{1}{2}} \text{ or } 2 \times \left(\frac{1}{2}\right) + 5$ <p>Points are <u>$(-3, -1)$ and $(\frac{1}{2}, 6)$</u> (correct pairings)</p>	M1 A1 M1 A1 M1 A1 ft (6) (9 marks)

Q4.

Question Number	Scheme	Marks
(a)	$(x^2 + 3)^2 = x^4 + 3x^2 + 3x^2 + 3^2$ $\frac{(x^2 + 3)^2}{x^2} = \frac{x^4 + 6x^2 + 9}{x^2} = x^2 + 6 + 9x^{-2} \quad (*)$	M1 A1 cso (2)
(b)	$y = \frac{x^3}{3} + 6x + \frac{9}{-1}x^{-1} (+c)$ $20 = \frac{27}{3} + 6 \times 3 - \frac{9}{3} + c$ $c = -4$ $[y =] \frac{x^3}{3} + 6x - 9x^{-1} - 4$	M1 A1 A1 M1 A1 A1 ft (6) (8 marks)

Q5.

Question Number	Scheme	Marks
(a)	$(8-3)^2 + (3-1)^2$ or $\sqrt{(8-3)^2 + (3-1)^2}$ $(x \pm 3)^2 + (y \pm 1)^2 = k$ or $(x \pm 1)^2 + (y \pm 3)^2 = k$ (k a positive value) $(x-3)^2 + (y-1)^2 = 29$	M1 A1 M1 A1 (4)
(b)	Gradient of radius = $\frac{2}{5}$ (or exact equivalent) Gradient of tangent = $-\frac{5}{2}$ $y-3 = -\frac{5}{2}(x-8)$ $5x+2y-46=0$ or equivalent	B1 M1 M1 A1 ft A1 (5)
		(9 marks)

Q6.

Question Number	Scheme	Marks
(a)	$\left[\frac{dy}{dx} = \right] 3kx^2 - 2x + 1$	M1 A1 (2)
(b)	Gradient of line is $\frac{7}{2}$ When $x = -\frac{1}{2}$: $3k \times \left(\frac{1}{4}\right) - 2 \times \left(-\frac{1}{2}\right) + 1 = \frac{7}{2}$ $\frac{3k}{4} = \frac{3}{2} \Rightarrow k = 2$	B1 M1 A1 A1 (4)
(c)	$x = -\frac{1}{2} \Rightarrow y = k \times \left(-\frac{1}{8}\right) - \left(\frac{1}{4}\right) - \frac{1}{2} - 5 = -6$	M1 A1 (2)
		(8 marks)

Q7.

Question Number	Scheme	Marks
(a)	$\left(\frac{dy}{dx} = 8 + 2x - 3x^2\right)$ $3x^2 - 2x - 8 = 0 \quad (3x+4)(x-2) = 0 \quad x = 2$	M1 A1 A1 cso (3)
(b)	<p>Area of triangle = $\frac{1}{2} \times 2 \times 22$</p> $\int 10 + 8x + x^2 - x^3 \, dx = 10x + \frac{8x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$ $\left[10x + \frac{8x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}\right]_0^2 = \dots \left(= 20 + 16 + \frac{8}{3} - 4\right)$ <p>Area of R = $34\frac{2}{3} - 22 = \frac{38}{3} \left(= 12\frac{2}{3}\right)$ (Or 12.6)</p>	M1 A1 M1 A1 A1 M1 M1 A1 (8) (11 marks)

Q8.

Question Number	Scheme	Marks
(a)	$x = \frac{\log 7}{\log 5} \quad \text{or} \quad x = \log_5 7$	M1
	1.21	A1 (2)
(b)	$(5^x - 7)(5^x - 5)$ $(5^x = 7 \quad \text{or} \quad 5^x = 5) \quad x = 1.2 \text{ (awrt)}$ $x = 1$	M1 A1 A1 ft B1 (4) (6 marks)

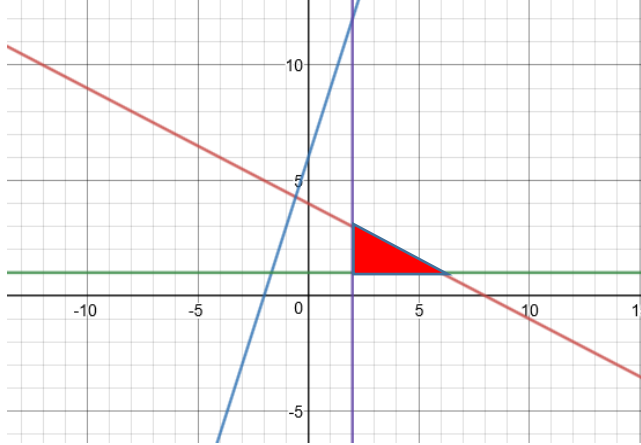
Q9.

Question Number	Scheme	Marks
(a)	45 (α) 180 - α , Add 20 (for at least one angle) 65 155	B1 M1 M1 A1 (4)
(b)	120 or 240 (β): 360 - β , 360 + β Dividing by 3 (for at least one angle) 40 80 160 200 280 320	B1 M1 M1 M1 A1 A1 (6) (10 marks)

Q10.

Q	Scheme	Marks
(a)	$\vec{BC} = \vec{BA} + \vec{AC}$ $= \begin{pmatrix} -4 \\ -6 \end{pmatrix} + \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ $= \begin{pmatrix} 2 \\ -4 \end{pmatrix}$	M1 A1
(b)	$AB = \sqrt{4^2 + 6^2} = \sqrt{52}$ $AC = \sqrt{6^2 + 2^2} = \sqrt{40}$ $BC = \sqrt{2^2 + (-4)^2} = \sqrt{20}$ (M1 awarded for any 2 calculated length) Use of cosine rule $(\sqrt{20})^2 = (\sqrt{56})^2 + (\sqrt{40})^2 - 2(\sqrt{48})(\sqrt{40}) \cos Q$ $Q = \cos^{-1}\left(\frac{76}{16\sqrt{30}}\right)$ $Q = 29.86$ (2 d.p)	M1 M1A1 A1
(c)	Use of sine rule Area = $\frac{1}{2} \times \sqrt{48} \times \sqrt{40} \times \sin 29.86 \dots$ = 10.91 (2 d.p)	M1 A1

Q11.

Q11	Scheme	Marks
(a)	Drawing the line $2y + x = 8$ Drawing the line $y = 3x + 6$ Drawing the line $y = 1$ Drawing the line $x = 2$	M1 M1 M1 M1
(b)	 <p style="margin-left: 200px;">Area of triangle = $\frac{1}{2} \times 4 \times 2 = 4 \text{ sq units}$</p>	M1 A1

Q12.

Question Number	Scheme	Marks
(a)	$e^{2x+1} = 2$ $2x = \ln 2$ $x = \frac{1}{2} \ln 2$	M1 A1

Question Number	Scheme	Marks
(b)	$\frac{dy}{dx} = 8e^{2x}$ $x = \frac{1}{2}(\ln 2) \Rightarrow \frac{dy}{dx} = 16$ $y - 8 = 16\left(x - \frac{1}{2}(\ln 2)\right)$ $y = 16x + 8 - 8\ln 2$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1 (4)</p> <p>(6 marks)</p>