Pure Mathematics 1 Practice Paper J10 MARK SCHEME

Question 1

Question number	Scheme	Marks
	(a) $(x+2k)^2$ or $\left(x+\dfrac{4k}{2}\right)^2$	M1
	$(x \pm F)^2 \pm G \pm 3 \pm 11k$ (where F and G are <u>any</u> functions of k, not involving x)	M1
	$(x+2k)^2 - 4k^2 + (3+11k)$ Accept unsimplified equivalents such as	A1
	$\left(x+\dfrac{4k}{2}\right)^2 - \left(\dfrac{4k}{2}\right)^2 + 3 + 11k$, and i.s.w. if necessary.	(3)
	(b) Accept part (b) solutions seen in part (a).	
	"$4k^2 - 11k - 3$"=0 $(4k+1)(k-3)=0$ $k = ...,$	M1
	[Or, 'starting again', $b^2 - 4ac = (4k)^2 - 4(3+11k)$ and proceed to $k = ...$]	
	$-\dfrac{1}{4}$ and 3 (Ignore any inequalities for the first 2 marks in (b)).	A1
	Using $b^2 - 4ac < 0$ for no real roots, i.e. "$4k^2 - 11k - 3$"<0, to establish inequalities involving their <u>two</u> critical values m and n (<u>even if the inequalities are wrong</u>, e.g. $k < m, k < n$).	M1
	$-\dfrac{1}{4} < k < 3$ (See conditions below) Follow through their critical values.	A1ft
	The final A1ft is still scored if the answer $m < k < n$ follows $k < m, k < n$. <u>Using x instead of k in the final answer</u> loses only the 2nd A mark, (condone use of x in earlier working).	(4)
	(c) Shape $\smile$ (seen in (c))	B1

	Minimum in correct quadrant, <u>not</u> touching the
	                        x-axis, <u>not</u> on the y-axis, and there must
	                        be no other minimum or maximum.
		B1

	(0, 14) or 14 on y-axis. Allow (14, 0) marked on y-axis.	B1
	n.b. Minimum is at $(-2, 10)$, (but there is no mark for this).	(3)
		[10]

	(b) 1st M: Forming and solving a 3-term quadratic in k (usual rules.. see general principles at end of scheme). The quadratic must come from "$b^2 - 4ac$", or from the "q" in part (a).
	     Using <u>wrong discriminant</u>, e.g. "$b^2 + 4ac$" will score <u>no marks</u> in part (b).
	 2nd M: As defined in main scheme above.
	 2nd A1ft: $m < k < n$, where $m < n$, for their critical values m and n.
	     Other possible forms of the answer (in each case $m < n$):
	     (i) $n > k > m$
	     (ii) $k > m$ <u>and</u> $k < n$
	     In this case the word "and" must be seen (implying intersection).
	     (iii) $k \in (m, n)$ (iv) $\{k : k > m\} \cap \{k : k < n\}$
	     <u>Not</u> just a number line.
	     <u>Not</u> just $k > m$, $k < n$ (without the word "and").
	 (c) Final B1 is dependent upon a sketch having been attempted in part (c).	

Question 2

Question number	Scheme	Marks
	(a) $x(x^2-4)$ Factor x seen in a <u>correct</u> factorised form of the expression. $= x(x-2)(x+2)$ M: Attempt to factorise quadratic (general principles). Accept $(x-0)$ or $(x+0)$ instead of x at any stage. Factorisation must be seen in part (a) to score marks.	B1 M1 A1 (3)
	(b) Shape $\sim$ (2 turning points required) Through (or touching) origin Crossing x-axis or "stopping at x-axis" (<u>not</u> a turning point) at $(-2, 0)$ and $(2, 0)$. Allow -2 and 2 on x-axis. Also allow $(0, -2)$ and $(0, 2)$ if marked on x-axis. Ignore extra intersections with x-axis.	B1 B1 B1 (3)
	(c) Either $y = 3$ (at $x = -1$) <u>or</u> $y = 15$ (at $x = 3$) Allow if seen elsewhere. Gradient $= \dfrac{"15-3"}{3-(-1)}$ $(=3)$ Attempt correct grad. formula with their y values. For gradient M mark, if correct formula not seen, allow one slip, e.g. $\dfrac{"15-3"}{3-1}$ $y-"15"=m(x-3)$ or $y-"3"=m(x-(-1))$, with any value for m. $y-15=3(x-3)$ or the <u>correct</u> equation in <u>any</u> form, e.g. $y-3=\dfrac{15-3}{3-(-1)}(x-(-1))$, $\dfrac{y-3}{x+1}=\dfrac{15-3}{3+1}$ $y=3x+6$	B1 M1 M1 A1 A1 (5)
	(d) $AB = \sqrt{("15-3")^2+(3-(-1))^2}$ (With their <u>non-zero</u> y values)... Square root is required. $= \sqrt{160}$ $\left(=\sqrt{16}\sqrt{10}\right) = 4\sqrt{10}$ (Ignore $\pm$ if seen) ($\sqrt{16}\sqrt{10}$ need not be seen).	M1 A1 (2) **[13]**

	(a) $x^3-4x \rightarrow x(x^2-4) \rightarrow (x-2)(x+2)$ scores B1 M1 A0.
	  $x^3-4x \rightarrow x^2-4 \rightarrow (x-2)(x+2)$ scores B0 M1 A0 (dividing by x).
	  $x^3-4x \rightarrow x(x^2-4x) \rightarrow x^2(x-4)$ scores B0 M1 A0.
	  $x^3-4x \rightarrow x(x^2-4) \rightarrow x(x-2)^2$ scores B1 M1 A0
	  Special cases: $x^3-4x \rightarrow (x-2)(x^2+2x)$ scores B0 M1 A0.
	  $x^3-4x \rightarrow x(x-2)^2$ (with no intermediate step seen) scores B0 M1 A0
	(b) The 2nd and 3rd B marks are not dependent upon the 1st B mark, but <u>are</u>
	  dependent upon a sketch having been attempted.
	(c) 1st M: May be implicit in the equation of the line, e.g. $\dfrac{y-"15"}{3-"15"}=\dfrac{x-"3"}{-1-"3"}$
	  2nd M: An equation of a line through $(3, "15")$ or $(-1, "3")$ <u>in any form</u>,
	    with any gradient (except 0 or ∞).
	  2nd M: Alternative is to use one of the points in $y = mx + c$ to <u>find a value</u>
	  for c, in which case $y = 3x + c$ leading to $c = 6$ is sufficient for both A marks.
	  1st A1: <u>Correct</u> equation in <u>any</u> form.	

Question 3

Question Number	Scheme	Marks
(a)	$N\,(2,\,-1)$	B1, B1 (2)
(b)	$r=\sqrt{\dfrac{169}{4}}=\dfrac{13}{2}=6.5$	B1 (1)
(c)	Complete Method to find x coordinates, $x_2-x_1=12$ and $\dfrac{x_1+x_2}{2}=2$ then solve To obtain $x_1=-4,\ x_2=8$ Complete Method to find y coordinates, using equation of circle or Pythagoras i.e. let d be the distance below N of A then $d^2=6.5^2-6^2 \Rightarrow d=2.5 \Rightarrow y=..$ So $y_2=y_1=-3.5$	M1 A1ft A1ft M1 A1 (5)
(d)	Let $A\hat{N}B=2\theta \Rightarrow \sin\theta=\dfrac{6}{\text{"}6.5\text{"}} \Rightarrow \theta=(67.38)...$ So angle ANB is 134.8 $*$	M1 A1 (2)
(e)	AP is perpendicular to AN so using triangle ANP $\tan\theta=\dfrac{AP}{\text{"}6.5\text{"}}$ Therefore $AP=15.6$	M1 A1cao (2) [12]
(a) (b)	B1 for 2 (α) , B1 for -1 B1 for 6.5 o.e.	
(c)	1st M1 for finding x coordinates – may be awarded if either x co-ord is correct A1ft,A1ft are for $\alpha-6$ and $\alpha+6$ if x coordinate of N is α 2nd M1 for a method to find y coordinates – may be given if y co-ordinate is correct A marks is for -3.5 only.	
(d)	M1 for a full method to find θ or angle ANB (eg sine rule or cosine rule directly or finding another angle and using angles of triangle.) **ft their 6.5 from radius or wrong y.** (cos ANB = $\dfrac{\text{"}6.5\text{"}^2+\text{"}6.5\text{"}^2-12^2}{2\times\text{"}6.5\text{"}\times\text{"}6.5\text{"}}$ =-0.704) A1 is a printed answer and must be 134.8 – do not accept 134.76.	
(e)	M1 for a full method to find AP **Alternative Methods** N.B. May use triangle AXP where X is the mid point of AB. Or may use triangle ABP. From circle theorems may use angle $BAP=67.38$ or some variation. Eg $\dfrac{AP}{\sin 67.4}=\dfrac{12}{\sin 45.2}$, $AP=\dfrac{6}{\sin 22.6}$ or $AP=\dfrac{6}{\cos 67.4}$ are each worth M1	

Question 4

Question Number	Scheme	Marks
(a)	$\left[y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}} - 10 \right]$	
	$[y' =] \qquad 6x^{-\frac{1}{2}} - \dfrac{3}{2}x^{\frac{1}{2}}$	M1 A1
	Puts their $\quad \dfrac{6}{x^{\frac{1}{2}}} - \dfrac{3}{2}x^{\frac{1}{2}} = 0$	M1
	So $x = \quad \dfrac{12}{3} = 4$ (If $x = 0$ appears also as solution then lose A1)	M1, A1
	$x = 4, \implies y = 12 \times 2 - 4^{\frac{3}{2}} - 10$, so $y = 6$	dM1,A1 (7)
(b)	$y'' = -3x^{-\frac{3}{2}} - \dfrac{3}{4}x^{-\frac{1}{2}}$	M1A1 (2)
(c)	[Since $x > 0$] It is a maximum	B1 (1) [10]

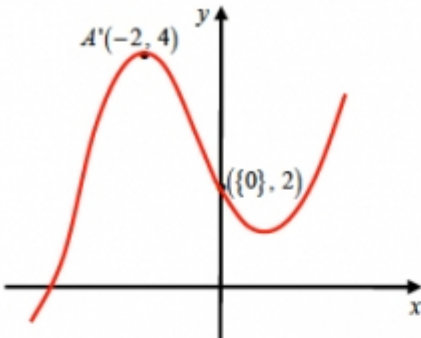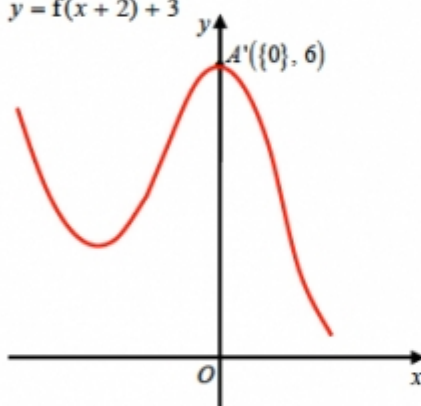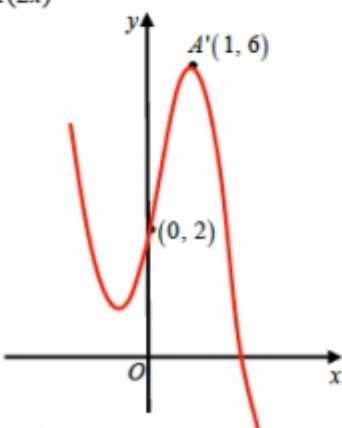|		
(a)	1st M1 for an attempt to differentiate a fractional power $x^n \to x^{n-1}$ A1 a.e.f – can be unsimplified 2nd M1 for forming a suitable equation using their $y' = 0$ 3rd M1 for correct processing of fractional powers leading to $x = \dots$ (Can be implied by $x = 4$) A1 is for $x = 4$ only. If $x = 0$ also seen and not discarded they lose this mark only. 4th M1 for substituting their value of x back into y to find y value. Dependent on three previous M marks. Must see evidence of the substitution with attempt at fractional powers to give M1A0, but $y = 6$ can imply M1A1	
(b)	M1 for differentiating their y' again A1 should be simplified	
(c)	B1 . Clear conclusion needed and must follow correct y'' It is dependent on previous A mark (Do not need to have found x earlier). (Treat parts (a),(b) and (c) together for award of marks)	

Question 5

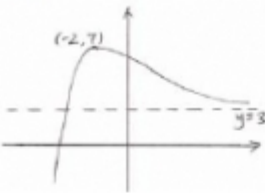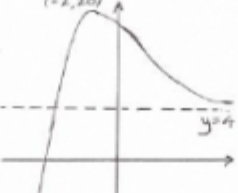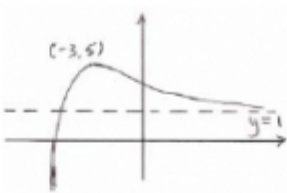| Question Number	Scheme	Marks	
(a)	$\log_x 64 = 2 \implies 64 = x^2$	M1	
	$\qquad\qquad$ So $x = 8$	A1	(2)
(b)	$\log_2 (11 - 6x) = \log_2 (x-1)^2 + 3$	M1	
	$\log_2 \left[\dfrac{11-6x}{(x-1)^2} \right] = 3$	M1	
	$\dfrac{11-6x}{(x-1)^2} = 2^3$	M1	
	$\{11 - 6x = 8(x^2 - 2x + 1)\}$ and so $0 = 8x^2 - 10x - 3$	A1	
	$0 = (4x+1)(2x-3) \implies x = ...$	dM1	
	$x = \dfrac{3}{2}, \left[-\dfrac{1}{4} \right]$	A1	(6)
			[8]
(a)	M1 for getting out of logs A1 Do not need to see $x = -8$ appear and get rejected. Ignore $x = -8$ as extra solution. $x = 8$ with no working is M1 A1		
(b)	1st M1 for using the $n\log x$ rule 2nd M1 for using the $\log x - \log y$ rule or the $\log x + \log y$ rule as appropriate 3rd M1 for using 2 to the power – need to see 2^3 or 8 (May see $3 = \log_2 8$ used) **If all three M marks have been earned and logs are still present in equation** **do not give final M1. So solution stopping at** $\log_2 \left[\dfrac{11-6x}{(x-1)^2} \right] = \log_2 8$ **would earn** **M1M1M0** 1st A1 for a correct 3TQ 4th dependent M1 for attempt to solve or factorize their 3TQ to obtain $x = ...$ (mark depends on three previous M marks) 2nd A1 for 1.5 (ignore -0.25) s.c 1.5 only – no working – is 0 marks		
(a)	**Alternatives** Change base : (i) $\dfrac{\log_2 64}{\log_2 x} = 2$, so $\log_2 x = 3$ and $x = 2^3$, is M1 or (ii) $\dfrac{\log_{10} 64}{\log_{10} x} = 2$, $\log x = \frac{1}{2}\log 64$ so $x = 64^{\frac{1}{2}}$ is M1 then $x = 8$ is A1 **BUT** $\log x = 0.903$ so $x = 8$ is M1A0 (loses accuracy mark) (iii) $\log_{64} x = \frac{1}{2}$ so $x = 64^{\frac{1}{2}}$ is M1 then $x = 8$ is A1		

Question 6

Question Number	Scheme	Marks
(i)(a)	$\ln(3x - 7) = 5$ $e^{\ln(3x-7)} = e^5$ Takes e of both sides of the equation. This can be implied by $3x - 7 = e^5$.	M1
	$3x - 7 = e^5 \Rightarrow x = \dfrac{e^5 + 7}{3} \{= 51.804...\}$ Then rearranges to make x the subject. *Exact answer* of $\dfrac{e^5 + 7}{3}$.	dM1 A1 (3)
(b)	$3^x e^{7x+2} = 15$ $\ln\left(3^x e^{7x+2}\right) = \ln 15$ Takes ln (or logs) of both sides of the equation.	M1
	$\ln 3^x + \ln e^{7x+2} = \ln 15$ Applies the addition law of logarithms.	M1
	$x\ln 3 + 7x + 2 = \ln 15$ $x\ln 3 + 7x + 2 = \ln 15$	A1 oe
	$x(\ln 3 + 7) = -2 + \ln 15$ Factorising out at least two x terms on one side and collecting number terms on the other side.	ddM1
	$x = \dfrac{-2 + \ln 15}{7 + \ln 3} \{= 0.0874...\}$ *Exact answer* of $\dfrac{-2 + \ln 15}{7 + \ln 3}$	A1 oe (5)

Question 7

Question Number	Scheme	Marks
(i)	$y = f(-x) + 1$ $A'(-2, 4)$ $(\{0\}, 2)$ Shape of $\bigwedge\!\!\!\bigvee$ and must have a maximum in quadrant 2 and a minimum in quadrant 1 or on the positive y-axis. **B1** Either $(\{0\}, 2)$ or $A'(-2, 4)$ **B1** Both $(\{0\}, 2)$ and $A'(-2, 4)$ **B1**	**(3)**
(ii)	$y = f(x + 2) + 3$ $A'(\{0\}, 6)$ Any translation $\bigvee\!\!\!\bigwedge$ of the original curve. **B1** The *translated maximum* has either x-coordinate of 0 (can be implied) or y-coordinate of 6. **B1** The translated curve has maximum $(\{0\}, 6)$ and is in the correct position on the Cartesian axes. **B1**	**(3)**
(iii)	$y = 2f(2x)$ $A'(1, 6)$ $(0, 2)$ Shape of $\bigvee\!\!\!\bigwedge$ with a minimum in quadrant 2 and a maximum in quadrant 1. **B1** Either $(\{0\}, 2)$ or $A'(1, 6)$ **B1** Both $(\{0\}, 2)$ and $A'(1, 6)$ **B1**	**(3)** **[9]**

Question 8

Question number	Scheme	Marks
	(a) (b) (c) 	
	(a) $(-2, 7)$, $y = 3$ (Marks are dependent upon a sketch being attempted) See conditions below.	B1, B1 (2)
	(b) $(-2, 20)$, $y = 4$ (Marks are dependent upon a sketch being attempted) See conditions below.	B1, B1 (2)
	(c) Sketch: Horizontal translation (either way)… (There must be evidence that $y = 5$ at the max and that the asymptote is still $y = 1$) $(-3, 5)$, $y = 1$	B1 B1, B1 (3) **[7]**
	Parts (a) and (b): (i) If <u>only one</u> of the B marks is scored, there is <u>no penalty</u> for a wrong sketch. (ii) If both the maximum and the equation of the asymptote are correct, the sketch must be "correct" to score B1 B1. If the sketch is "wrong", award B1 B0. The (generous) conditions for a "correct" sketch are that the maximum must be in the 2nd quadrant and that the curve must not cross the positive x-axis… ignore other "errors" such as "curve appearing to cross its asymptote" and "curve appearing to have a minimum in the 1st quadrant". **Special case:** (b) Stretch $\frac{1}{4}$ instead of 4: Correct shape, with $\left(-2, \frac{5}{4}\right)$, $y = \frac{1}{4}$: B1 B0. **Coordinates of maximum:** If the coordinates are the wrong way round (e.g. $(7, -2)$ in part (a)), or the coordinates are just shown as values on the x and y axes, penalise <u>only once in the whole question</u>, at first occurrence. **Asymptote marks:** If the <u>equation</u> of the asymptote is not given, e.g. in part (a), 3 is marked on the y-axis but $y = 3$ is not seen, penalise <u>only once in the whole question</u>, at first occurrence. <u>Ignore</u> extra asymptotes stated (such as $x = 0$).	

Question 9

Question Number	Scheme	Marks
(a)	**Puts** $y = 0$ and attempts to solve quadratic e.g. $(x-4)(x-1) = 0$ Points are $(1,0)$ and $(4, 0)$	M1 A1 (2)
(b)	$x = 5$ gives $y = 25 - 25 + 4$ and so $(5, 4)$ lies on the curve	B1cso (1)
(c)	$\int\left(x^2 - 5x + 4\right)dx = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x \quad (+c)$	M1A1 (2)
(d)	Area of triangle $= \frac{1}{2} \times 4 \times 4 = 8$ or $\int(x-1)dx = \frac{1}{2}x^2 - x$ with limits 1 and 5 to give 8	B1
	Area under the curve $= \int_4^5 \qquad \frac{1}{3} \times 5^3 - \frac{5}{2} \times 5^2 + 4 \times 5 \quad \left[= -\frac{5}{6} \right]$	M1
	$\frac{1}{3} \times 4^3 - \frac{5}{2} \times 4^2 + 4 \times 4 \quad \left[= -\frac{8}{3} \right]$	M1
	$\int_4^5 = -\frac{5}{6} - \frac{8}{3} = \frac{11}{6}$ or equivalent (allow 1.83 or 1.8 here)	A1 cao
	Area of $R = 8 - \frac{11}{6} = 6\frac{1}{6}$ or $\frac{37}{6}$ or 6.16^r (not 6.17)	A1 cao (5) [10]
(a)	M1 for attempt to find L and M A1 Accept $x = 1$ and $x = 4$, then isw or accept $L = (1,0)$, $M = (4,0)$ Do not accept $L = 1$, $M = 4$ nor $(0, 1)$, $(0, 4)$ (unless subsequent work) Do not need to distinguish L and M. Answers imply M1A1.	
(b)	See substitution, working should be shown, need conclusion which could be just $y = 4$ or a tick. Allow $y = 25 - 25 + 4 = 4$ But not $25 - 25 + 4 = 4$. ($y = 4$ may appear at start) Usually $0 = 0$ or $4 = 4$ is B0	
(c)	M1 for attempt to integrate $x^2 \to kx^3, x \to kx^2$ or $4 \to 4x$ A1 for correct integration of all three terms (do not need constant) isw. Mark correct work when seen. So e.g. $\frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x$ is A1 then $2x^3 - 15x^2 + 24x$ would be ignored as subsequent work.	
(d)	B1 for this triangle only (not triangle LMN) 1st M1 for substituting 5 into their changed function 2nd M1 for substituting 4 into their changed function	
(d)	Alternative method: $\int_1^5 (x-1) - (x^2 - 5x + 4)dx + \int_1^4 x^2 - 5x + 4 \, dx$ can lead to correct answer Constructs $\int_1^5 (x-1) - (x^2 - 5x + 4)dx$ is B1 M1 for substituting 5 and 1 and subtracting in first integral M1 for substituting 4 and 1 and subtracting in second integral A1 for answer to first integral i.e. $\frac{32}{3}$ (allow 10.7) and A1 for final answer as before..	
(d)	Another alternative $\int_4^5 (x-1) - (x^2 - 5x + 4)dx + area\ of\ triangle\ LMP$ Constructs $\int_4^5 (x-1) - (x^2 - 5x + 4)dx$ is B1 M1 for substituting 5 and 4 and subtracting in first integral M1 for complete method to find area of triangle (4.5) A1 for answer to first integral i.e. $\frac{5}{3}$ and A1 for final answer as before.	
(d)	Could also use $\int_4^5 (4x - 16) - (x^2 - 5x + 4)dx + area\ of\ triangle\ LMN$ Similar scheme to previous one. Triangle has area 6 A1 for finding Integral has value $\frac{1}{6}$ and A1 for final answer as before.	

Question 10

Question Number	Scheme	Marks
(a)	$5\sin x = 1 + 2\left(1 - \sin^2 x\right)$ $2\sin^2 x + 5\sin x - 3 = 0$ \qquad (*)	M1 A1cso (2)
(b)	$(2s - 1)(s + 3) = 0$ **giving** $s =$ $\left[\sin x = -3 \text{ has no solution}\right]$ **so** $\sin x = \tfrac{1}{2}$ $\therefore \quad x = 30,\ 150$	M1 A1 B1, B1ft (4) **[6]**

(a)

M1 for a correct method to change $\cos^2 x$ into $\sin^2 x$ (must use $\cos^2 x = 1 - \sin^2 x$)

A1 need 3 term quadratic printed in any order with =0 included

(b) M1 for attempt to solve given quadratic (usual rules for solving quadratics) (can use any variable here, s, y, x, or $\sin x$)

A1 requires no incorrect work seen and is for $\sin x = \tfrac{1}{2}$ or $x = \sin^{-1}\tfrac{1}{2}$

$y = \tfrac{1}{2}$ is A0 (unless followed by $x = 30$)

B1 for 30 (α) not dependent on method

2nd B1 for $180 - \alpha$ \qquad provided in required range (otherwise $540 - \alpha$)

Extra solutions outside required range: Ignore

Extra solutions inside required range: Lose final B1

Answers in radians: Lose final B1

S.C. Merely writes down two correct answers is M0A0B1B1

Or $\sin x = \tfrac{1}{2}$ $\therefore$ $x = 30,\ 150$ is **M1A1B1B1**

Just gives one answer : 30 only is M0A0B1B0 or 150 only is M0A0B0B1

NB Common error is to factorise wrongly giving $(2\sin x + 1)(\sin x - 3) = 0$

$\left[\sin x = 3 \text{ gives no solution}\right]$ $\sin x = -\tfrac{1}{2}$ $\Rightarrow$ $x = 210,\ 330$

This earns M1 A0 B0 B1ft

Another common error is to factorise correctly $(2\sin x - 1)(\sin x + 3) = 0$ and follow this

with $\sin x = \dfrac{1}{2}$, $\sin x = 3$ then $x = 30°, 150°$

This would be M1 A0 B1 B1

Q11	Scheme		Marks
(i)	$AC^2 = (3-x)^2 + (x-4)^2 - 2(3-x)(x-4)\cos 120$ $\quad = 9 - 6x + x^2 + x^2 - 8x + 16 + 3x - 12 - x^2 + 4x$ $\quad = x^2 - 7x + 13$	M1 for attempt to use cosine rule	M1A1 A1 A1
(ii)	$AC^2 = \left(x - \dfrac{7}{2}\right)^2 - \dfrac{49}{4} + 13$ $\quad = \left(x - \dfrac{7}{2}\right)^2 + \dfrac{3}{4}$ Value of x for minimum value of $AC = \dfrac{7}{2}$	M1 for attempt for using completing the square A1 for any two correct terms i.e $-\dfrac{7}{2}$ or $-\dfrac{49}{4}$ or 13	M1A1 A1 A1 **(7)**