Pure Mathematics 1 Practice Paper M7 MARK SCHEME

Q1.

Question number	Scheme	Marks
	(a) $2x^2 - x(x-4) = 8$	M1
	$x^2 + 4x - 8 = 0$ (*)	A1cso (2)
	(b) $x = \dfrac{-4 \pm \sqrt{4^2 - (4 \times 1 \times -8)}}{2}$ or $(x+2)^2 \pm 4 - 8 = 0$	M1
	$x = -2 \pm$ (any correct expression)	A1
	$\sqrt{48} = \sqrt{16}\sqrt{3} = 4\sqrt{3}$ or $\sqrt{12} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$	B1
	$y = (-2 \pm 2\sqrt{3}) - 4$ M: Attempt at least one y value	M1
	$\underline{x = -2 + 2\sqrt{3},\ \ y = -6 + 2\sqrt{3}}$ $\underline{x = -2 - 2\sqrt{3},\ \ y = -6 - 2\sqrt{3}}$	A1 (5)
		7

(a)	M1 for correct attempt to form an equation in x only. Condone sign errors/slips but attempt at this line must be seen. E.g. $2x^2 - x^2 \pm 4x = 8$ is OK for M1.
	A1cso for correctly simplifying to printed form. No incorrect working seen. The $= 0$ <u>is</u> required.
	These two marks can be scored in part (b). For multiple attempts pick best.
(b)	1st M1 for use of correct formula. If formula is not quoted then a fully correct substitution is required. Condone missing $x =$ or just + or – instead of $\pm$ for M1.
	For completing the square must have as printed or better.
	If they have $x^2 - 4x - 8 = 0$ then M1 can be given for $(x-2)^2 \pm 4 - 8 = 0$.
	1st A1 for $-2 \pm$ any correct expression. (The $\pm$ is required but $x =$ is not)
	B1 for simplifying the surd e.g. $\sqrt{48} = 4\sqrt{3}$. Must reduce to $b\sqrt{3}$ so $\sqrt{16}\sqrt{3}$ or $\sqrt{4}\sqrt{3}$ are OK.
	2nd M1 for attempting to find at least one y value. Substitution into one of the given equations and an attempt to solve for y.
	2nd A1 for correct y answers. Pairings need <u>not</u> be explicit but they must say which is x and which y.
	Mis-labelling x and y loses final A1 only.

Q2.

Question number	Scheme	Marks
	(a) Attempt to use discriminant b^2-4ac	M1
	$k^2-4(k+3)>0 \Rightarrow k^2-4k-12>0$ (*)	A1cso (2)
	(b) $k^2-4k-12=0 \Rightarrow$	
	$(k\pm a)(k\pm b)$, with $ab=12$ or $(k=)\dfrac{4\pm\sqrt{4^2--4\times12}}{2}$ or $(k-2)^2\pm2^2-12$	M1
	$k=-2$ and 6 (both)	A1
	$\underline{k<-2,\ \ k>6}$ or $\underline{(-\infty,-2);(6,\infty)}$ M: choosing "outside"	M1 A1ft (4)
		6
(a)	M1 for use of b^2-4ac, one of b or c must be correct. Or full attempt using completing the square that leads to a 3TQ in k e.g. $\left(\left[x+\dfrac{k}{2}\right]^2=\right)\dfrac{k^2}{4}-(k+3)$ A1cso Correct argument to printed result. Need to state (or imply) that $b^2-4ac>0$ and no incorrect working seen. Must have >0. If >0 just appears with $k^2-4(k+3)>0$ that is OK. If >0 appears on last line only with no explanation give A0. b^2-4ac followed by $k^2-4k-12>0$ only is insufficient so M0A0 e.g. $k^2-4\times1\times k+3$ (missing brackets) can get M1A0 but $k^2+4(k+3)$ is M0A0 (wrong formula) Using $\sqrt{b^2-4ac}>0$ is M0.	
(b)	1st M1 for attempting to find critical regions. Factors, formula or completing the square. 1st A1 for $k=6$ and -2 only 2nd M1 for choosing the outside regions 2nd A1f.t. as printed or f.t. their (non identical) critical values $6<k<-2$ is M1A0 but ignore if it follows a correct version $-2<k<6$ is M0A0 whatever their diagram looks like Condone use of x instead of k for critical values and final answers in (b). Treat this question as 3 two mark parts. If part (a) is seen in (b) or vice versa marks can be awarded.	

Q3.

Question number	Scheme	Marks
	(a) Translation parallel to x-axis	M1
	Top branch intersects +ve y-axis Lower branch has no intersections No obvious overlap	A1
	$\left(0, \frac{3}{2}\right)$ or $\frac{3}{2}$ marked on y- axis	B1 (3)
	(b) $x = -2, \quad y = 0$	B1, B1 (2)
S.C.	[Allow ft on first B1 for $x = 2$ when translated "the wrong way" but must be compatible with their sketch.]	
		5
(a)	M1 for a horizontal translation – two branches with one branch cutting y – axis only. If one of the branches cuts both axes (translation up and across) this is M0. A1 for a horizontal translation to left. Ignore any figures on axes for this mark. B1 for correct intersection on positive y-axis. More than 1 intersection is B0. x=0 and y = 1.5 in a table alone is insufficient unless intersection of their sketch is with +ve y-axis. A point marked on the graph overrides a point given elsewhere.	
(b)	1st B1 for $x = -2$. NB $x \neq -2$ is B0. Can accept $x = +2$ if this is compatible with their sketch. Usually they will have M1A0 in part (a) (and usually B0 too) 2nd B1 for $y = 0$.	
S.C.	If x = -2 and y =0 and some other asymptotes are also given award B1B0 The asymptote equations should be clearly stated in part (b). Simply marking x =-2 or y =0 on the sketch is insufficient <u>unless</u> they are clearly marked "asymptote x = -2" etc.	

Q4.

Question number	Scheme	Marks
	(a) $y=-\frac{3}{2}x(+4)$ Gradient $=-\frac{3}{2}$	M1 A1 (2)
	(b) $3x+2=-\frac{3}{2}x+4$ $x=\ldots,\ \underline{\frac{4}{9}}$	M1, A1
	$y=3\left(\frac{4}{9}\right)+2=\underline{\frac{10}{3}}\ \left(=3\frac{1}{3}\right)$	A1 (3)
	(c) Where $y=1$, $l_1: x_A=-\frac{1}{3}$ $l_2: x_B=2$ M: Attempt one of these	M1 A1
	Area $=\frac{1}{2}(x_B-x_A)(y_P-1)$	M1
	$=\frac{1}{2}\times\frac{7}{3}\times\frac{7}{3}=\underline{\frac{49}{18}=2\frac{13}{18}}$ o.e.	A1 (4)
		9

(a) M1 for an attempt to write $3x+2y-8=0$ in the form $y=mx+c$

or a full method that leads to $m=$, e.g find 2 points, and attempt gradient using $\frac{y_2-y_1}{x_2-x_1}$

e.g. finding $y=-1.5x+4$ alone can score M1 (even if they go on to say $m=4$)

A1 for $m=-\frac{3}{2}$ (can ignore the $+c$) or $\frac{dy}{dx}=-\frac{3}{2}$

(b) M1 for forming a suitable equation in one variable and attempting to solve leading to $x=$ ..or $y=$

1st A1 for any exact correct value for x

2nd A1 for any exact correct value for y

(These 3 marks can be scored anywhere, they may treat (a) and (b) as a single part)

(c)

1st M1 for attempting the x coordinate of A or B. One correct value seen scores M1.

1st A1 for $x_A=-\frac{1}{3}$ and $x_B=2$

2nd M1 for a full method for the area of the triangle – follow through their x_A, x_B, y_P.

e.g. determinant approach $\frac{1}{2}\begin{vmatrix} 2 & -\frac{1}{3} & \frac{4}{9} & 2 \\ 1 & 1 & \frac{10}{3} & 1 \end{vmatrix}=\frac{1}{2}\left|2-\ldots-\left(-\frac{1}{3}\ldots\right)\right|$

2nd A1 for $\frac{49}{18}$ or an exact equivalent.

All accuracy marks require answers as single fractions or mixed numbers not necessarily in lowest terms.

Q5.

Question number	Scheme	Marks
	(a) Gradient of AM: $\frac{1-(-2)}{3-1}=\frac{3}{2}$ or $\frac{-3}{-2}$	B1
	Gradient of l: $=-\frac{2}{3}$ M: use of $m_1m_2=-1$, or equiv.	M1
	$y-1=-\frac{2}{3}(x-3)$ or $\frac{y-1}{x-3}=-\frac{2}{3}$ $[3y=-2x+9]$ (Any equiv. form)	M1 A1 (4)
	(b) $x=6$: $3y=-12+9=-3$ $y=-1$ (or show that for $y=-1$, $x=6$) (*) (A conclusion is not required).	B1 (1)
	(c) $(r^2=)$ $(6-1)^2+(-1-(-2))^2$ M: Attempt r^2 or r	M1 A1
	N.B. Simplification is not required to score M1 A1	
	$(x\pm6)^2+(y\pm1)^2=k$, $k\neq0$ (Value for k not needed, could be r^2 or r)	M1
	$(x-6)^2+(y+1)^2=26$ (or equiv.)	A1 (4)
	Allow $(\sqrt{26})^2$ or other exact equivalents for 26. (But… $(x-6)^2+(y--1)^2=26$ scores M1 A0)	
	(Correct answer with no working scores full marks)	
		9
	(a) 2nd M1: eqn. of a straight line through (3, 1) with any gradient except 0 or ∞.	
	Alternative: Using (3, 1) in $y=mx+c$ to find a value of c scores M1, but an equation (general or specific) must be seen.	
	Having coords the wrong way round, e.g. $y-3=-\frac{2}{3}(x-1)$, loses the 2nd M mark unless a correct general formula is seen, e.g. $y-y_1=m(x-x_1)$.	
	If the point $P(6,-1)$ is used to find the gradient of MP, maximum marks are (a) B0 M0 M1 A1 (b) B0.	
	(c) 1st M1: Condone one slip, numerical or sign, inside a bracket.	
	Must be attempting to use points $P(6,-1)$ and $A(1,-2)$, or perhaps P and B. (Correct coordinates for B are (5, 4)).	
	1st M alternative is to use a complete Pythag. method on triangle MAP, n.b. $MP=MA=\sqrt{13}$.	
	Special case: If candidate persists in using their value for the y-coordinate of P instead of the given -1, allow the M marks in part (c) if earned.	

Q6.

Question number	Scheme		Marks
	(a) $x=1:\ y=-5+4=\underline{-1}$, $x=2:\ y=-16+2=\underline{-14}$	(can be given in (b) or (c))	1st B1 for – 1 2nd B1 for - 14
	$PQ=\sqrt{(2-1)^2+(-14-(-1))^2}=\sqrt{170}$	(*)	M1 A1cso (4)
	(b) $y=x^3-6x^2+4x^{-1}$		M1
	$\frac{dy}{dx}=3x^2-12x-4x^{-2}$		M1 A1
	$x=1:\ \frac{dy}{dx}=3-12-4=-13$	M: Evaluate at one of the points	M1
	$x=2:\ \frac{dy}{dx}=12-24-1=-13$ $\therefore$Parallel	A: Both correct + conclusion	A1 (5)
	(c) Finding gradient of normal $\left(m=\frac{1}{13}\right)$		M1
	$y--1=\frac{1}{13}(x-1)$		M1 A1ft
	$\underline{x-13y-14=0}$	o.e.	A1cso (4)
			13

(a)	M1 for attempting PQ or PQ^2 using their P and their Q. Usual rules about quoting formulae. We must see attempt at $1^2+(y_P-y_Q)^2$ for M1. $PQ^2=\sqrt{...}$ etc could be M1A0. A1cso for proceeding to the correct answer with no incorrect working seen.
(b)	1st M1 for multiplying by x^2, the x^3 or $-6x^2$ must be correct. 2nd M1 for some correct differentiation, at least one term must be correct as printed. 1st A1 for a fully correct derivative. **These 3 marks can be awarded anywhere when first seen.** 3rd M1 for attempting to substitute $x=1$ or $x=2$ in their derivative. Substituting in y is M0. 2nd A1 for -13 from both substitutions <u>and</u> a brief comment. The -13 must come from their derivative.
(c)	1st M1 for use of the perpendicular gradient rule. Follow through their -13. 2nd M1 for full method to find the equation of the normal or tangent at P. If formula is quoted allow slips in substitution, otherwise a correct substitution is required. 1st A1ft for a correct expression. Follow through their -1 and their changed gradient. 2nd A1cso for a correct equation with $=0$ and integer coefficients. This mark is dependent upon the -13 coming from their derivative in (b) hence cso. Tangent can get M0M1A0A0, changed gradient can get M0M1A1A0orM1M1A1A0.
MR	Condone confusion over terminology of tangent and normal, mark gradient and equation. Allow for $-\frac{4}{x}$ or $(x+6)$ but not omitting $4x^{-1}$ or treating it as $4x$.

Q7.

Question number	Scheme	Marks
	(a) $4x^2+6xy=600$	M1 A1
	$V=2x^2y=2x^2\left(\frac{600-4x^2}{6x}\right)$ $\quad V=200x-\frac{4x^3}{3}$ (*)	M1 A1cso (4)
	(b) $\frac{dV}{dx}=200-4x^2$	B1
	Equate their $\frac{dV}{dx}$ to 0 and solve for x^2 or x : $x^2=50$ or $x=\sqrt{50}$ (7.07…)	M1 A1
	Evaluate V: $V=200(\sqrt{50})-\frac{4}{3}(50\sqrt{50})=943\ \text{cm}^3$ Allow awrt	M1 A1 (5)
	(c) $\frac{d^2V}{dx^2}=-8x$ Negative, $\therefore$ Maximum	M1, A1ft (2)
		11

	(a) 1st M: Attempting an expression in terms of x and y for the total surface area (the expression should be dimensionally correct). 1st A: Correct expression (not necessarily simplified), equated to 600. 2nd M: Substituting their y into $2x^2y$ to form an expression in terms of x only. (Or substituting y from $2x^2y$ into their area equation). (b) 1st A: Ignore $x = -\sqrt{50}$, if seen. The 2nd M mark (for substituting their x value into the given expression for V) is dependent on the 1st M. Final A: Allow also exact value $\frac{400\sqrt{50}}{3}$ or $\frac{2000\sqrt{2}}{3}$ or equiv. <u>single term</u>. (c) Allow marks if the work for (c) is seen in (b) (or vice-versa). M: Find second derivative <u>and consider its sign</u>. A: Second derivative following through correctly from their $\frac{dV}{dx}$, and correct reason/conclusion (it must be a maximum, not a minimum). An actual value of x does not have to be used… this mark can still be awarded if no x value has been found or if a wrong x value is used. <u>Alternative</u>: M: Find <u>value</u> of $\frac{dV}{dx}$ on each side of "$x = \sqrt{50}$" and consider sign. A: Indicate sign change of positive to negative for $\frac{dV}{dx}$, and conclude max. <u>Alternative</u>: M: Find <u>value</u> of V on each side of "$x = \sqrt{50}$" and compare with "943". A: Indicate that both values are less than 943, and conclude max.	

Q8.

Question number	Scheme	Marks
	(a) $\left(\frac{dy}{dx}\right) = 6x^1 + \frac{4}{2}x^{-\frac{1}{2}}$ or $\left(6x + 2x^{-\frac{1}{2}}\right)$	M1 A1 (2)
	(b) $6 + -x^{-\frac{3}{2}}$ or $6 + -1 \times x^{-\frac{3}{2}}$	M1 A1ft (2)
	(c) $x^3 + \frac{8}{3}x^{\frac{3}{2}} + C$ A1: $\frac{3}{3}x^3$ or $\frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}$ A1: both, simplified and $+C$	M1 A1 A1 (3)
		7

(a) M1 for some attempt to differentiate: $x^n \to x^{n-1}$
Condone missing $\frac{dy}{dx}$ or $y = \ldots$

A1 for both terms correct, as written or better. No $+C$ here. Of course $\frac{2}{\sqrt{x}}$ is acceptable.

(b) M1 for some attempt to differentiate again. Follow through their $\frac{dy}{dx}$, at least one term correct or correct follow through.

A1f.t. as written or better, follow through must have 2 distinct terms and simplified e.g. $\frac{4}{4} = 1$.

(c) M1 for some attempt to integrate: $x^n \to x^{n+1}$. Condone misreading $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$ for y.
($+C$ alone is not sufficient)

1st A1 for either $\frac{3}{3}x^3$ or $\frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}$ (or better) $\frac{2}{3} \times 4x^{\frac{3}{2}}$ is OK here too but not for 2nd A1.

2nd A1 for both x^3 and $\frac{8}{3}x^{\frac{3}{2}}$ or $\frac{8}{3}x\sqrt{x}$ i.e. simplified terms and $+C$ all on one line.
$2\frac{2}{3}$ instead of $\frac{8}{3}$ is OK

Q9.

Question number	Scheme	Marks
	(a) $x = \dfrac{\log 0.8}{\log 8}$ or $\log_8 0.8$, $= -0.107$ Allow awrt	M1, A1 (2)
	(b) $2\log x = \log x^2$	B1
	$\log x^2 - \log 7x = \log \dfrac{x^2}{7x}$	M1
	"Remove logs" to form equation in x, using the base correctly: $\dfrac{x^2}{7x} = 3$	M1
	$x = 21$ (Ignore $x = 0$, if seen)	A1cso (4) **6**
	(a) Allow also the 'implicit' answer $8^{-0.107}$ (M1 A1). Answer only: -0.107 or awrt: Full marks. Answer only: -0.11 or awrt (insufficient accuracy): M1 A0 Trial and improvement: Award marks as for "answer only". (b) <u>Alternative:</u> $2\log x = \log x^2$ B1 $\log 7x + 1 = \log 7x + \log 3 = \log 21x$ M1 "Remove logs" to form equation in x: $x^2 = 21x$ M1 $x = 21$ (Ignore $x = 0$, if seen) A1 <u>Alternative:</u> $\log 7x = \log 7 + \log x$ B1 $2\log x - (\log 7 + \log x) = 1$ $\log_3 x = 1 + \log_3 7$ M1 $x = 3^{(1+\log_3 7)}$ $\left(= 3^{2.771..}\right)$ or $\log_3 x = \log_3 3 + \log_3 7$ M1 $x = 21$ A1 Attempts using change of base will usually require the same steps as in the main scheme or alternatives, so can be marked equivalently. <u>A common mistake:</u> $\log x^2 - \log 7x = \dfrac{\log x^2}{\log 7x}$ B1 M0 $\dfrac{x^2}{7x} = 3$ $x = 21$ M1('Recovery'), but A0	

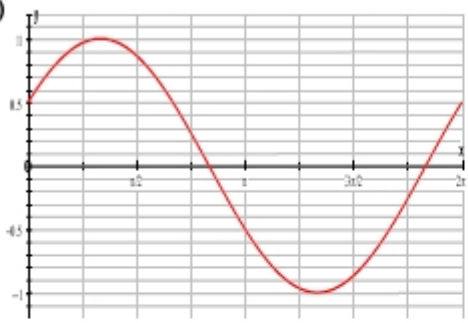
Q10.

Question number	Scheme	Marks
	(a) $f(x)=\frac{6x^3}{3}-\frac{10x^2}{2}-12x\ (+C)$	M1 A1
	$x=5$: $250-125-60+C=65$ $C=0$	M1 A1 (4)
	(b) $x(2x^2-5x-12)$ or $(2x^2+3x)(x-4)$ or $(2x+3)(x^2-4x)$	M1
	$=x(2x+3)(x-4)$ (*)	A1cso (2)
	(c) Shape	B1
	Through origin	B1
	$\left(-\frac{3}{2},0\right)$ and (4,0)	B1 (3)
		9
(a)	1st M1 for attempting to integrate, $x^n \to x^{n+1}$ 1st A1 for all x terms correct, need not be simplified. Ignore + C here. 2nd M1 for some use of $x=5$ and f(5)=65 to form an equation in C based on their integration. There must be some visible attempt to use $x=5$ and f(5)=65. No $+C$ is M0. 2nd A1 for $C=0$. This mark cannot be scored unless a suitable equation is seen.	
(b)	M1 for attempting to take out a correct factor or to verify. Allow usual errors on signs. They must get to the equivalent of one of the given partially factorised expressions or, if verifying, $x(2x^2+3x-8x-12)$ i.e. with no errors in signs. A1cso for proceeding to printed answer with no incorrect working seen. Comment <u>not</u> required. This mark is <u>dependent upon a fully correct solution to part (a)</u> so M1A1M0A0M1A0 for (a) & (b). Will be common or M1A1M1A0M1A0. To score 2 in (b) they must score 4 in (a).	
(c)	1st B1 for positive x^3 shaped curve (with a max and a min) positioned anywhere. 2nd B1 for any curve that passes through the origin (B0 if it only touches at the origin) 3rd B1 for the two points <u>clearly</u> given as coords or values marked in appropriate places on x axis. Ignore any extra crossing points (they should have lost first B1). Condone (1.5, 0) if clearly marked on –ve x-axis. Condone (0, 4) etc if marked on +ve x axis. Curve can <u>stop</u> (i.e. not pass through) at (-1.5, 0) and (4, 0). A point on the graph overrides coordinates given elsewhere.	

Q11.

Question number	Scheme	Marks
	(a) $4^2 = 5^2 + 6^2 - (2 \times 5 \times 6 \cos\theta)$	M1
	$\cos\theta = \dfrac{5^2 + 6^2 - 4^2}{2 \times 5 \times 6}$	A1
	$\left(= \dfrac{45}{60}\right) = \dfrac{3}{4}$ (*)	A1cso (3)
	(b) $\sin^2 A + \left(\dfrac{3}{4}\right)^2 = 1$ (or equiv. Pythag. method)	M1
	$\left(\sin^2 A = \dfrac{7}{16}\right)$ $\sin A = \dfrac{1}{4}\sqrt{7}$ or equivalent exact form, e.g. $\sqrt{\dfrac{7}{16}}$, $\sqrt{0.4375}$	A1 (2)
		5
	(a) M: Is also scored for $5^2 = 4^2 + 6^2 - (2 \times 4 \times 6 \cos\theta)$ or $6^2 = 5^2 + 4^2 - (2 \times 5 \times 4 \cos\theta)$ or $\cos\theta = \dfrac{4^2 + 6^2 - 5^2}{2 \times 4 \times 6}$ or $\cos\theta = \dfrac{5^2 + 4^2 - 6^2}{2 \times 5 \times 4}$. 1st A: Rearranged correctly and numerically correct (possibly unsimplified), in the form $\cos\theta = \ldots$ or $60\cos\theta = 45$ (or equiv. in the form $p\cos\theta = q$). <u>Alternative</u> (verification): $4^2 = 5^2 + 6^2 - \left(2 \times 5 \times 6 \times \dfrac{3}{4}\right)$ [M1] Evaluate correctly, at least to $16 = 25 + 36 - 45$ [A1] Conclusion (perhaps as simple as a tick). [A1cso] (Just achieving $16 = 16$ is insufficient without at least a tick). (b) M: Using a correct method to find an equation in $\sin^2 A$ or $\sin A$ which would give an exact value. <u>Correct answer without working</u> (or with unclear working or decimals): Still scores both marks.	

Q12.

Question number	Scheme	Marks
	(a)  Sine wave (anywhere) with at least 2 turning points.	M1
	Starting on positive y-axis, going up to a max., then min. below x-axis, no further turning points in range, finishing above x-axis at $x = 2\pi$ or $360°$. There must be <u>some</u> indication of scale on the y-axis… (e.g. 1, −1 or 0.5) Ignore parts of graph outside 0 to 2π.	A1 (2)
	n.b. Give credit if necessary for what is seen on an initial sketch (before any transformation has been performed).	
	(b) $(0, \frac{1}{2})$, $(150^0, 0)$, $(330^0, 0)$	B1, B1, B1 (3)
	(c) $180^0 - 40.54^0 = 139.46^0$	B1
	Subtract 30^0 from all values	M1
	10.54^0 , 109.45^0	M1
		A1, A1 (5)
		10

Q13.

Question number	Scheme	Marks
(a)	$\frac{dT}{dt} = -7.5e^{\frac{-t}{6}}$	M1
	Sub t =15 $\frac{dT}{dt} = -7.5e^{\frac{-15}{6}} = 24.38$ mins	A1
(b)	As $\rightarrow \infty$, T $\rightarrow$ 25 Hence T will never falls below 25^0	A1