Integration by Substitution - Edexcel Past Exam Questions MARK SCHEME

Question 1: June 05 Q4

Question Number	Scheme	Marks
	$\int \frac{1}{(1-x^2)^{\frac{3}{2}}}\,dx = \int \frac{1}{(1-\sin^2\theta)^{\frac{3}{2}}}\cos\theta\,d\theta$ Use of $x = \sin\theta$ and $\frac{dx}{d\theta} = \cos\theta$	M1
	$= \int \frac{1}{\cos^2\theta}\,d\theta$	M1 A1
	$= \int \sec^2\theta\,d\theta = \tan\theta$	M1 A1
	Using the limits 0 and $\frac{\pi}{6}$ to evaluate integral	M1
	$[\tan\theta]_0^{\frac{\pi}{6}} = \frac{1}{\sqrt{3}}$ $\left(= \frac{\sqrt{3}}{3}\right)$ cao	A1
		[7]
	Alternative for final M1 A1	
	Returning to the variable x and using the limits 0 and $\frac{1}{2}$ to evaluate integral	M1
	$\left[\frac{x}{\sqrt{(1-x^2)}}\right]_0^{\frac{1}{2}} = \frac{1}{\sqrt{3}}$ $\left(= \frac{\sqrt{3}}{3}\right)$ cao	A1

Question 2: Jan 06 Q3

Question Number	Scheme	Marks
	Uses substitution to obtain $x = f(u) \left[\frac{u^2+1}{2}\right]$,	M1
	and to obtain $u\frac{du}{dx}$ = const. or equiv.	M1
	Reaches $\int \frac{3(u^2+1)}{2u} u du$ or equivalent	A1
	Simplifies integrand to $\int \left(3u^2 + \frac{3}{2}\right) du$ or equiv.	M1
	Integrates to $\frac{1}{2}u^3 + \frac{3}{2}u$	M1 A1√
	A1√ dependent on all previous Ms	
	Uses new limits 3 and 1 substituting and subtracting (or returning to function of x with old limits)	M1
	To give 16 cso	A1 [8]
	"By Parts" Attempt at " right direction" by parts — M1 $[\ 3x\left(2x-1\right)^{\frac{1}{2}}\ ] - \{\int 3(2x-1)^{\frac{1}{2}}\,dx\}]$ — M1{M1A1} .... $-\ (2x-1)^{\frac{3}{2}}$ — M1A1√ Uses limits 5 and 1 correctly; [42 – 26] 16 — M1A1	

Question 3: Jan 07 Q8

Question Number	Scheme		Marks
	$t=(3x+1)^{\frac{1}{2}} \Rightarrow \frac{dt}{dx}=\frac{1}{2}.3.(3x+1)^{-\frac{1}{2}}$	$A(3x+1)^{-\frac{1}{2}}$ or $t\frac{dt}{dx}=A$	M1
	... or $t^2=3x+1 \Rightarrow 2t\frac{dt}{dx}=3$	$\frac{3}{2}(3x+1)^{-\frac{1}{2}}$ or $2t\frac{dt}{dx}=3$	A1
	so $\frac{dt}{dx}=\frac{3}{2.(3x+1)^{\frac{1}{2}}}=\frac{3}{2t} \Rightarrow \frac{dx}{dt}=\frac{2t}{3}$	Candidate obtains either $\frac{dt}{dx}$ or $\frac{dx}{dt}$ in terms of t ...	
	$\therefore I=\int e^{\sqrt{(3x+1)}}\,dx=\int e^t\frac{dx}{dt}.dt=\int e^t.\frac{2t}{3}.dt$	... and moves on to substitute this into I to convert an integral wrt x to an integral wrt t.	dM1
	$\therefore I=\int \frac{2}{3}te^t\,dt$	$\int \frac{2}{3}te^t$	A1
	change limits: when x = 0, t = 1 & when x = 5, t = 4	changes limits $x \to t$ so that $0 \to 1$ and $5 \to 4$	B1
	Hence $I=\int_1^4 \frac{2}{3}te^t\,dt$; where $a=1$, $b=4$, $k=\frac{2}{3}$		
			[5]

Question 4: June 07 Q2

Question Number	Scheme		Marks
	$\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx$, with substitution $u=2^x$		
	$\frac{du}{dx}=2^x.\ln 2 \Rightarrow \frac{dx}{du}=\frac{1}{2^x.\ln 2}$	$\frac{du}{dx}=2^x.\ln 2$ or $\frac{du}{dx}=u.\ln 2$ or $\left(\frac{1}{u}\right)\frac{du}{dx}=\ln 2$	B1
	$\int \frac{2^x}{(2^x+1)^2}\,dx=\left(\frac{1}{\ln 2}\right)\int \frac{1}{(u+1)^2}\,du$	$k\int \frac{1}{(u+1)^2}\,du$ where k is constant	M1*

	$= \left(\frac{1}{\ln 2}\right)\left(\frac{-1}{(u+1)}\right) + c$	$(u+1)^{-2} \to a(u+1)^{-1}$ $(u+1)^{-2} \to -1.(u+1)^{-1}$	M1 A1
	change limits: when $x = 0$ & $x = 1$ then $u = 1$ & $u = 2$		
	$\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx = \frac{1}{\ln 2}\left[\frac{-1}{(u+1)}\right]_1^2$		
	$= \frac{1}{\ln 2}\left[\left(-\frac{1}{3}\right) - \left(-\frac{1}{2}\right)\right]$	Correct use of limits $u = 1$ and $u = 2$	depM1 *
	$= \frac{1}{6\ln 2}$	$\frac{1}{6\ln 2}$ or $\frac{1}{\ln 4} - \frac{1}{\ln 8}$ or $\frac{1}{2\ln 2} - \frac{1}{3\ln 2}$ **Exact value only!**	A1 aef [6]
	Alternatively candidate can revert back to x …		
	$\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx = \frac{1}{\ln 2}\left[\frac{-1}{(2^x+1)}\right]_0^1$		
	$= \frac{1}{\ln 2}\left[\left(-\frac{1}{3}\right) - \left(-\frac{1}{2}\right)\right]$	Correct use of limits $x = 0$ and $x = 1$	depM1 *
	$= \frac{1}{6\ln 2}$	$\frac{1}{6\ln 2}$ or $\frac{1}{\ln 4} - \frac{1}{\ln 8}$ or $\frac{1}{2\ln 2} - \frac{1}{3\ln 2}$ **Exact value only!**	A1 aef
			6 marks

If you see this **integration** applied anywhere in a candidate's working then you can award M1, A1

There are other acceptable answers for A1, eg: $\frac{1}{2\ln 8}$ or $\frac{1}{\ln 64}$

NB: Use your calculator to check eg. 0.240449…

Question 5: Jan 10 Q8

Question Number	Scheme		Marks	
	$\frac{dx}{du} = -2\sin u$		B1	
	$\int \frac{1}{x^2\sqrt{4-x^2}}dx = \int \frac{1}{(2\cos u)^2\sqrt{4-(2\cos u)^2}} \times -2\sin u\, du$		M1	
	$= \int \frac{-2\sin u}{4\cos^2 u\sqrt{4\sin^2 u}}du$	Use of $1-\cos^2 u = \sin^2 u$	M1	
	$= -\frac{1}{4}\int \frac{1}{\cos^2 u}du$	$\pm k\int \frac{1}{\cos^2 u}du$	M1	
	$= -\frac{1}{4}\tan u \ (+C)$	$\pm k\tan u$	M1	
	$x = \sqrt{2} \Rightarrow \sqrt{2} = 2\cos u \Rightarrow u = \frac{\pi}{4}$			
	$x = 1 \Rightarrow 1 = 2\cos u \Rightarrow u = \frac{\pi}{3}$		M1	
	$\left[-\frac{1}{4}\tan u\right]_{\frac{\pi}{3}}^{\frac{\pi}{4}} = -\frac{1}{4}\left(\tan\frac{\pi}{4} - \tan\frac{\pi}{3}\right)$			
	$= -\frac{1}{4}\left(1-\sqrt{3}\right) \quad \left(=\frac{\sqrt{3}-1}{4}\right)$		A1	(7)

Question 6: June 10 Q2

Question Number	Scheme		Marks	
	$\frac{du}{dx} = -\sin x$		B1	
	$\int \sin x\, e^{\cos x+1}dx = -\int e^u du$		M1 A1	
	$= -e^u$	ft sign error	A1ft	
	$= -e^{\cos x+1}$			
	$\left[-e^{\cos x+1}\right]_0^{\frac{\pi}{2}} = -e^1 - \left(-e^2\right)$	or equivalent with u	M1	
	$= e(e-1)$ ✱	cso	A1	(6)
				[6]

Question 7: Jan 11 Q7

Question Number	Scheme	Marks
	$\frac{dx}{du} = 2(u-4)$	B1
	$\int \frac{1}{4+\sqrt{(x-1)}} dx = \int \frac{1}{u} \times 2(u-4) du$	M1
	$= \int \left(2 - \frac{8}{u}\right) du$	A1
	$= 2u - 8\ln u$	M1 A1
	$x = 2 \Rightarrow u = 5, \quad x = 5 \Rightarrow u = 6$	B1
	$[2u - 8\ln u]_5^6 = (12 - 8\ln 6) - (10 - 8\ln 5)$	M1
	$= 2 + 8\ln\left(\frac{5}{6}\right)$	A1
		(8)

Question 8: June 11 Q4

Question Number	Scheme	Marks	
(a)	$u = x^2 + 2 \Rightarrow \dfrac{du}{dx} = 2x$	B1	
	$\text{Area}(R) = \int_0^{\sqrt{2}} x^3 \ln(x^2+2)\,dx$	B1	
	$\int x^3 \ln(x^2+2)\,dx = \int x^2 \ln(x^2+2)\,x\,dx = \int (u-2)(\ln u)\tfrac{1}{2}\,du$	M1	
	Hence $\text{Area}(R) = \frac{1}{2}\int_2^4 (u-2)\ln u\,du$ * cso	A1	(4)
	$\int (u-2)\ln u\,du = \left(\frac{u^2}{2} - 2u\right)\ln u - \int \left(\frac{u^2}{2} - 2u\right)\frac{1}{u}\,du$	M1 A1	
	$= \left(\frac{u^2}{2} - 2u\right)\ln u - \int \left(\frac{u}{2} - 2\right)du$		
	$= \left(\frac{u^2}{2} - 2u\right)\ln u - \left(\frac{u^2}{4} - 2u\right) \quad (+C)$	M1 A1	
(b)	$\text{Area}(R) = \frac{1}{2}\left[\left(\frac{u^2}{2} - 2u\right)\ln u - \left(\frac{u^2}{4} - 2u\right)\right]_2^4$		
	$= \tfrac{1}{2}\left[(8-8)\ln 4 - 4 + 8 - ((2-4)\ln 2 - 1 + 4)\right]$	M1	
	$= \tfrac{1}{2}(2\ln 2 + 1)$ $\quad \ln 2 + \tfrac{1}{2}$	A1	(6)