Name:

Total Marks:

Pure

Mathematics 2

Advanced Level

Practice Paper J10

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE A Level Specifications
- There are 13 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

Show that, for a small angle θ, where θ is in radians,

$$1 + \cos Q - 3\cos^2 Q \approx \ -1 + \frac{5}{2}Q^2$$ **(3)**

(Total 3 marks)

Question 2

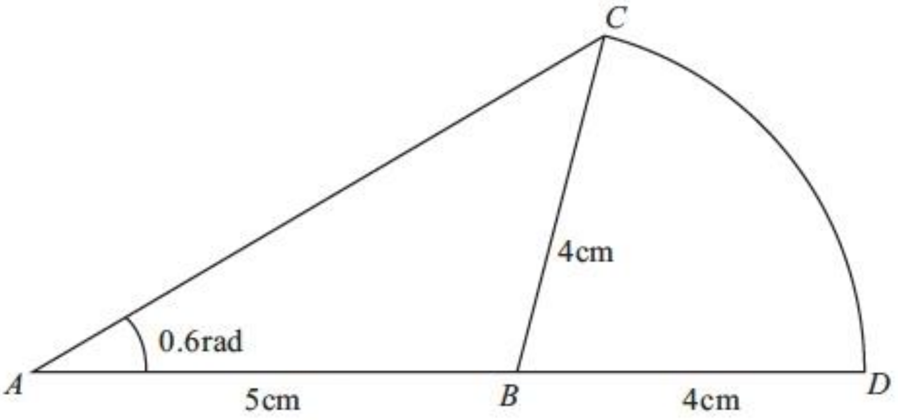

Figure 1

An emblem, as shown in Figure 1, consists of a triangle *ABC* joined to a sector *CBD* of a circle with radius 4 cm and centre *B*. The points *A*, *B* and *D* lie on a straight line with *AB* = 5 cm and *BD* = 4 cm. Angle *BAC* = 0.6 radians and *AC* is the longest side of the triangle *ABC*.

(a) Show that angle *ABC* = 1.76 radians, correct to 3 significant figures. **(4)**

(b) Find the area of the emblem. **(3)**

(Total 7 marks)

Question 3

A car was purchased for £18 000 on 1st January.
On 1st January each following year, the value of the car is 80% of its value on 1st January in the previous year.

(a) Show that the value of the car exactly 3 years after it was purchased is £9216. **(1)**

The value of the car falls below £1000 for the first time *n* years after it was purchased.

(b) Find the value of *n*. **(3)**

An insurance company has a scheme to cover the maintenance of the car.
The cost is £200 for the first year, and for every following year the cost increases by 12% so that for the 3rd year the cost of the scheme is £250.88

(c) Find the cost of the scheme for the 5th year, giving your answer to the nearest penny. **(2)**

(d) Find the total cost of the insurance scheme for the first 15 years. **(3)**

(Total 9 marks)

Question 4

(a) Find the binomial expansion of

$\sqrt{(1-8)}$, $|x| < \frac{1}{8}$,

in ascending powers of x up to and including the term in x^3, simplifying each term. **(4)**

(b) Show that, when $x = \frac{\sqrt{23}}{5}$, the exact value of $\sqrt{(1-8x)}$ is $\frac{\sqrt{23}}{5}$, **(2)**

(c) Substitute $\chi = \frac{\sqrt{23}}{5}$ into the binomial expansion in part (a) and hence obtain an approximation to $\sqrt{23}$. Give your answer to 5 decimal places. **(3)**

(Total 9 marks)

Question 5

The curve C has the equation

$$\cos 2x + \cos 3y = 1, \qquad -\frac{\pi}{4} \leqslant x \leqslant \frac{\pi}{4}, \quad 0 \leqslant y \leqslant \frac{\pi}{6}$$

(a) Find $\frac{dy}{dx}$ in terms of x and y. **(3)**

The point P lies on C where $x = \frac{\pi}{6}$.

(b) Find the value of y at P. **(3)**

(c) Find the equation of the tangent to C at P, giving your answer in the form $ax + by + c\pi = 0$, where a, b and c are integers. **(3)**

(Total 9 marks)

Question 6

(a) By writing $\sec x$ as $\frac{1}{\cos x}$, show that $\frac{d(\sec x)}{dx} = \sec x \tan x$. **(3)**

Given that $y = e^{2x} \sec 3x$,

(b) find $\frac{dy}{dx}$. **(4)**

The curve with equation $y = e^{2x} \sec 3x$, $-\frac{\pi}{6} < x < \frac{\pi}{6}$, has a minimum turning point at (a, b).

(c) Find the values of the constants a and b, giving your answers to 3 significant figures. **(4)**

(Total 11 marks)

Question 7

Solve $\operatorname{cosec}^2 2x - \cot 2x = 1$ for $0 \leqslant x \leqslant 180°$. **(7)**

(Total 7 marks)

Question 8

The area A of a circle is increasing at a constant rate of 1.5 cm^2 s^{-1}. Find, to 3 significant figures, the rate at which the radius r of the circle is increasing when the area of the circle is 2 cm^2. **(5)**

(Total 5 marks)

Question 9

(i) Given that $y = \dfrac{\ln(x^2+1)}{x}$, find $\dfrac{dy}{dx}$. **(4)**

(ii) Given that $x = \tan y$, show that $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$. **(5)**

(Total 9 marks)

Question 10

Using the substitution $x = 2\cos u$, or otherwise, find the exact value of

$$\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{(4-x^2)}}\,dx$$

(7)

(Total 7 marks)

Question 11

(a) Find $\int \dfrac{9x+6}{x}\,dx$, $x > 0$ **(2)**

(b) Given that $y = 8$ at $x = 1$, solve the differential equation

$$\frac{dy}{dx} = \frac{(9x+6)y^{\frac{1}{3}}}{x}$$

giving your answer in the form $y^2 = g(x)$. **(6)**

(Total 8 marks)

Question 12

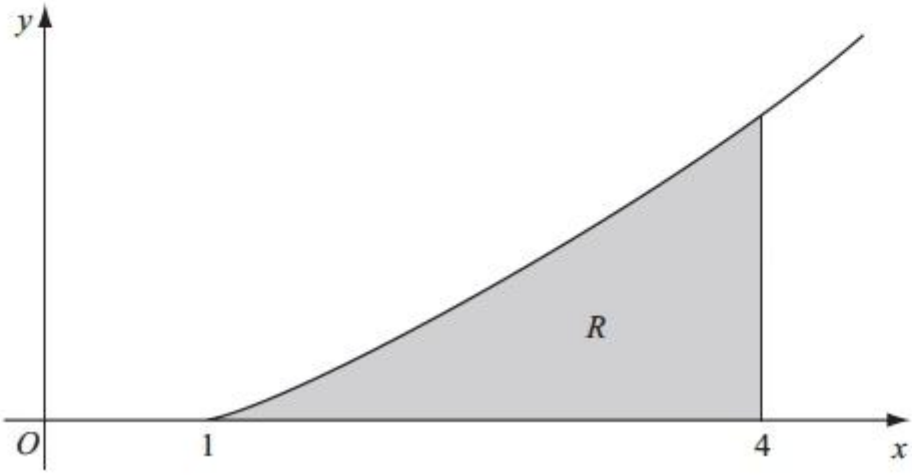

Figure 1

Figure 1 shows a sketch of the curve with equation $y = x \, 1\mathrm{n} \, x$, $x \geqslant 1$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 4$.

(a) Use integration by parts to find $\int x \ln x \, dx$.

(b) Hence find the exact area of R, giving your answer in the form $\frac{1}{4}$ $(a \ln 2 + b)$ where a and b are integers. **(7)**

(Total 7 marks)

Question 13

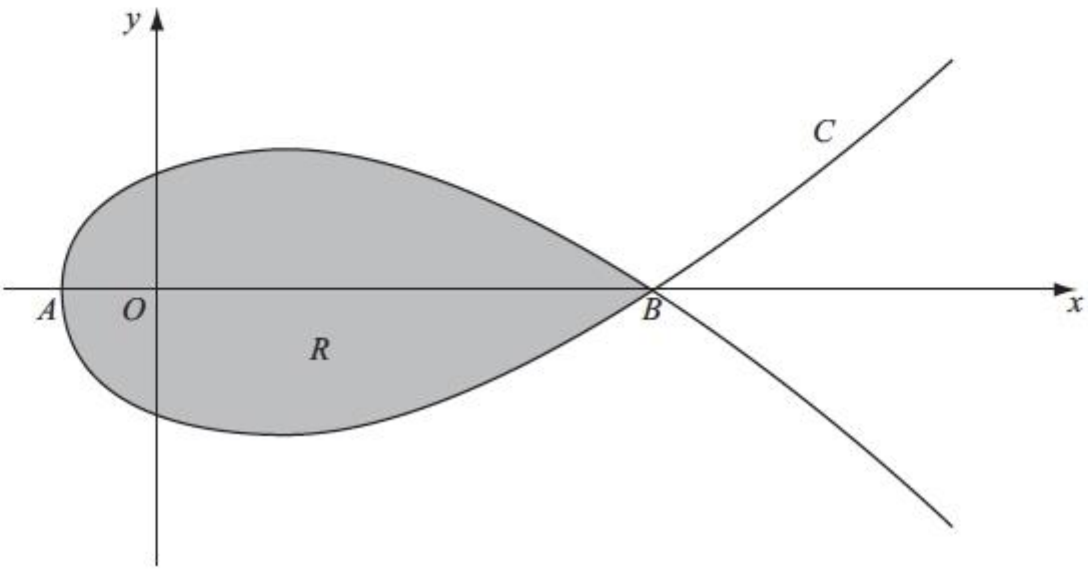

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 5t^2 - 4, \qquad y = t(9 - t^2)$$

The curve C cuts the x-axis at the points A and B.

(a) Find the x-coordinate at the point A and the x-coordinate at the point B. **(3)**

The region R, as shown shaded in Figure 2, is enclosed by the loop of the curve.

(b) Use integration to find the area of R. **(6)**

(Total 9 marks)

TOTAL FOR PAPER IS 100 MARKS