Name:

Total Marks:

Pure Mathematics 2

Advanced Level

Practice Paper J9

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE A Level Specifications
- There are 10 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

$$f(x) = 3\sqrt{x} + \frac{18}{\sqrt{x}} - 20$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [1.1, 1.2]. (2)

(b) Find $f'(x)$. (3)

(c) Using $x_0 = 1.1$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 significant figures. (4)

(Total 9 marks)

Question 2

$$f(x) = \frac{27x^2 + 32x + 16}{(3x+2)^2(1-x)}, \quad |x| < \frac{2}{3}$$

Given that f (x) can be expressed in the form

$$f(x) = \frac{A}{(3x+2)} + \frac{B}{(3x+2)^2} + \frac{C}{(1-x)},$$

(a) find the values of B and C and show that $A = 0$. (4)

(b) Hence, or otherwise, find the series expansion of f (x), in ascending powers of x, up to and including the term in x^2. Simplify each term. (6)

(c) Find the percentage error made in using the series expansion in part (b) to estimate the value of f (0.2). Give your answer to 2 significant figures. (4)

(Total 14 marks)

Question 3

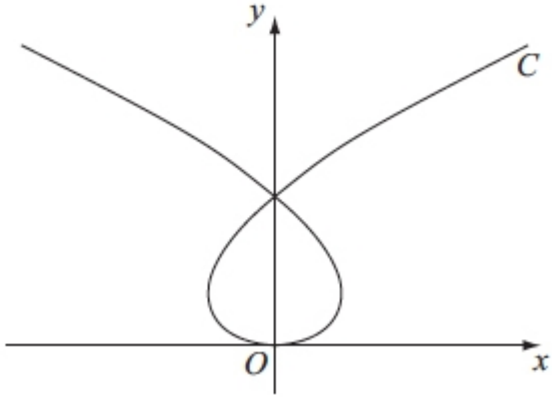

Figure 3

The curve C shown in Figure 3 has parametric equations

$$x = t^3 - 8t, \; y = t^2$$

where t is a parameter. Given that the point A has parameter $t = -1$,

(a) find the coordinates of A. (1)

The line l is the tangent to C at A.

(b) Show that an equation for l is $2x - 5y - 9 = 0$. (5)

The line l also intersects the curve at the point B.

(c) Find the coordinates of B. (6)

(Total 12 marks)

Question 4

(a) Find the value of $\frac{dy}{dx}$ at the point where $x = 2$ on the curve with equation

$$y = x^2 \sqrt{(5x - 1)}.$$ (6)

(b) Differentiate $\frac{\sin 2x}{x^2}$ with respect to x. (4)

(Total 10 marks)

Question 5

Find the equation of the tangent to the curve $x = \cos(2y + \pi)$ at $\left(0, \frac{\pi}{4}\right)$.

Give your answer in the form $y = ax + b$, where a and b are constants to be found. (6)

(Total 6 marks)

Question 6

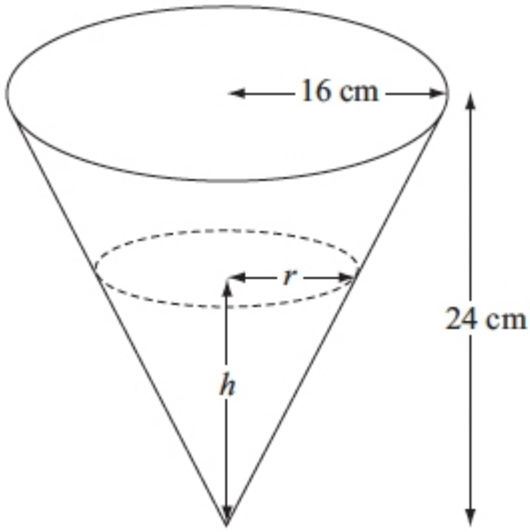

Figure 2

A container is made in the shape of a hollow inverted right circular cone. The height of the container is 24 cm and the radius is 16 cm, as shown in Figure 2. Water is flowing into the container. When the height of water is h cm, the surface of the water has radius r cm and the volume of water is V cm^3.

(a) Show that $V = \frac{4\pi h^3}{27}$. (2)

[The volume V of a right circular cone with vertical height h and base radius r is given by the formula $V = \frac{1}{3}\pi r^2 h$.*]*

Water flows into the container at a rate of 8 cm^3 s^{-1}.

(b) Find, in terms of π, the rate of change of h when $h = 12$. (5)

(Total 7 marks)

Question 7

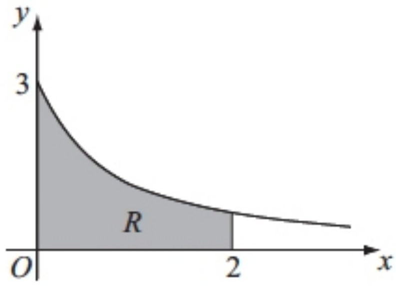

Figure 1

Figure 1 shows part of the curve $y = \frac{3}{\sqrt{(1+4x)}}$. The region R is bounded by the curve, the x-axis, and the lines $x = 0$ and $x = 2$, as shown shaded in Figure 1.

(a) Use integration to find the area of R. (4)

(Total 4 marks)

Question 8

(a) Find $\int \tan^2 x \; dx$. (2)

(b) Use integration by parts to find $\int \ln x \; dx$. (4)

(c) Use the substitution $u = 1 + e^x$ to show that

$$\int \frac{e^{3x}}{1+e^x} dx = \frac{1}{2} e^{2x} - e^x + \ln(1 + e^x) + k,$$

where k is a constant. (7)

(Total 13 marks)

Question 9

(a) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3\theta. \quad (4)$$

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8 \sin^3\theta - 6 \sin \theta + 1 = 0.$$

(i) Give your answers in terms of π. (5)

(b) Using $\sin(\theta - \alpha) = \sin \theta \cos\alpha - \cos\theta \sin\alpha$, or otherwise, show that

$$\sin 15° = \frac{1}{4} (\sqrt{6} - \sqrt{2}). \quad (4)$$

(Total 13 marks)

Question 10

(a) Express $3 \cos \theta + 4 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$. **(4)**

(b) Hence find the maximum value of $3 \cos \theta + 4 \sin \theta$ and the smallest positive value of θ for which this maximum occurs. **(3)**

The temperature, f (t), of a warehouse is modelled using the equation

$$f(t) = 10 + 3 \cos(15t)° + 4 \sin(15t)°,$$

where t is the time in hours from midday and $0 \leqslant t < 24$.

(c) Calculate the minimum temperature of the warehouse as given by this model. **(2)**

(d) Find the value of t when this minimum temperature occurs. **(3)**

(Total 12 marks)

TOTAL FOR PAPER IS 100 MARKS