Name:

Total Marks:

Pure Mathematics 2

Advanced Level

Practice Paper M10

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE A Level Specifications
- There are 11 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

The adult population of a town is 25 000 at the end of Year 1.

A model predicts that the adult population of the town will increase by 3% each year, forming a geometric sequence.

(a) Show that the predicted adult population at the end of Year 2 is 25 750. **(1)**

(b) Write down the common ratio of the geometric sequence. **(1)**

The model predicts that Year *N* will be the first year in which the adult population of the town exceeds 40 000.

(c) Show that

$$(N-1)\log 1.03 > \log 1.6$$

(3)

(d) Find the value of *N*. **(2)**

At the end of each year, each member of the adult population of the town will give £1 to a charity fund.

Assuming the population model,

(e) find the total amount that will be given to the charity fund for the 10 years from the end of Year 1 to the end of Year 10, giving your answer to the nearest £1000. **(3)**

(Total 10 marks)

Question 2

$$\frac{2x^2+5x-10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}$$

(a) Find the values of the constants *A*, *B* and *C*. (3)

(b) Hence, or otherwise, expand $\frac{2x^2+5x-10}{(x-1)(x+2)}$ in ascending powers of *x*, as far as the term in x^2.
Give each coefficient as a simplified fraction. (6)

(Total 9 marks)

Question 3

A curve *C* has equation

$$y = \frac{3}{(5-3x)^2}, \quad x \neq \frac{5}{3}$$

The point *P* on *C* has *x*-coordinate 2. Find an equation of the normal to *C* at *P* in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers. (6)

(Total 6 marks)

Question 4

A curve *C* has equation

$$2^x + y^2 = 2xy$$

Find the exact value of $\frac{dy}{dx}$ at the point on *C* with coordinates (3, 2). (7)

(Total 7 marks)

Question 5

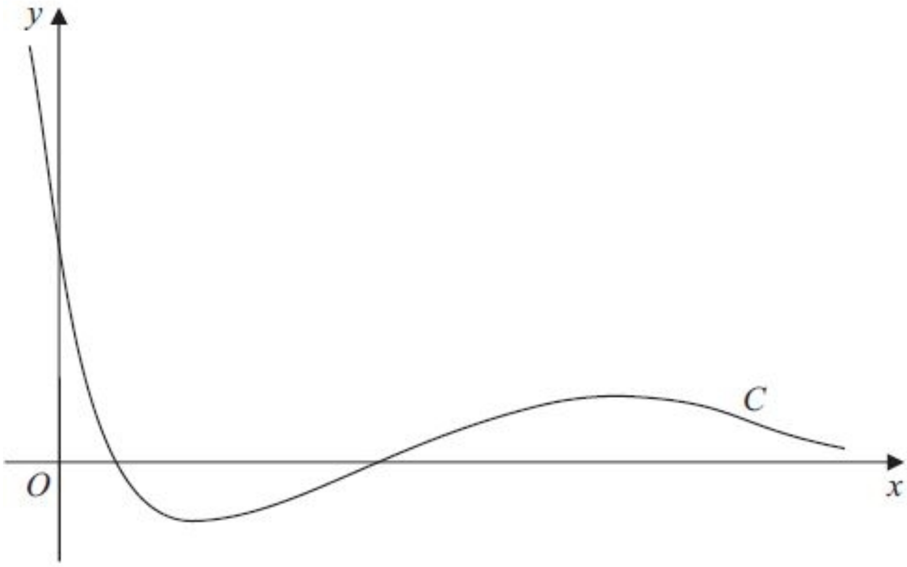

Figure 1

Figure 1 shows a sketch of the curve C with the equation $y = (2x^2 - 5x + 2)e^{-x}$.

(a) Find the coordinates of the point where C crosses the y-axis. (1)

(b) Show that C crosses the x-axis at $x = 2$ and find the x-coordinate of the other point where C crosses the x-axis. (3)

(c) Find $\frac{dy}{dx}$. (3)

(d) Hence find the exact coordinates of the turning points of C. (5)

(Total 12 marks)

Question 6

$$f(\theta) = 4\cos^2\theta - 3\sin^2\theta$$

(a) Show that $f(\theta) = \frac{1}{2} + \frac{7}{2}\cos 2\theta$. (3)

(b) Hence, using calculus, find the exact value of $\int_0^{\frac{\pi}{2}} \theta f(\theta)\, d\theta$. (7)

(Total 10 marks)

Question 7

(a) Show that

$$\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$$

(2)

(b) Hence find, for −180° ≤ θ < 180°, all the solutions of

$$\frac{2\sin 2\theta}{1+\cos 2\theta} = 1$$

Give your answers to 1 decimal place.

(3)

(Total 5 marks)

Question 8

Using the substitution $u = \cos x + 1$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{\cos x + 1} \sin x \, dx = e(e-1)$$

(6)

(Total 6 marks)

Question 9

A curve C has parametric equations

$$x = \sin^2 t, \quad y = 2\tan t, \quad 0 \leqslant t < \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of t. (4)

The tangent to C at the point where $t = \frac{\pi}{3}$ cuts the x-axis at the point P.

(b) Find the x-coordinate of P. (6)

(Total 10 marks)

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Question 10

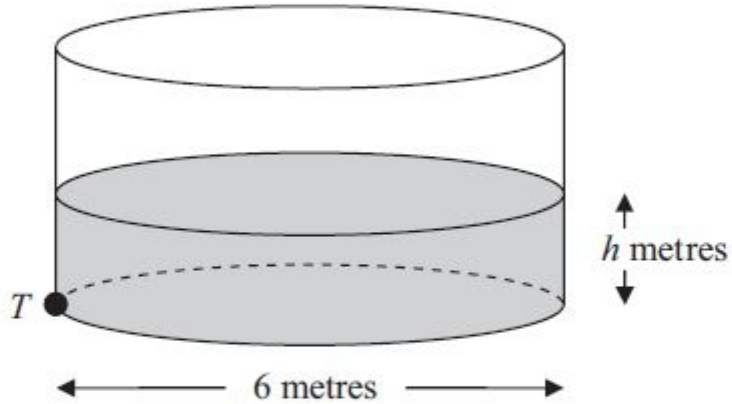

Figure 2

Figure 2 shows a cylindrical water tank. The diameter of a circular cross-section of the tank is 6 m. Water is flowing into the tank at a constant rate of 0.48π m^3 min^{-1}. At time t minutes, the depth of the water in the tank is h metres. There is a tap at a point T at the bottom of the tank. When the tap is open, water leaves the tank at a rate of $0.6\pi h$ m^3 min^{-1}.

(a) Show that t minutes after the tap has been opened

$$75\frac{\mathrm{d}h}{\mathrm{d}t} = (4 - 5h)$$

(5)

When $t = 0$, $h = 0.2$

(b) Find the value of t when $h = 0.5$ **(6)**

(Total 11 marks)

Question 11

Express $2\sin\theta - 1.5\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(a) Give the value of α to 4 decimal places. **(3)**

(b) (i) Find the maximum value of $2 \sin \theta - 1.5 \cos \theta$.

(ii) Find the value of θ, for $0 \le \theta < \pi$, at which this maximum occurs. **(3)**

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right), \quad 0 \leqslant t < 12,$$

where t hours is the number of hours after midday.

(c) Calculate the maximum value of H predicted by this model and the value of t, to 2 decimal places, when this maximum occurs. **(3)**

(d) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres. **(6)**

(Total 15 marks)

TOTAL FOR PAPER IS 100 MARKS