Name:

Total Marks:

Pure Mathematics 2

Advanced Level

Practice Paper M11

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE A Level Specifications
- There are 11 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

Prove by contradiction that $\sqrt{3}$ is irrational. **(Total 4 marks)**

Question 2

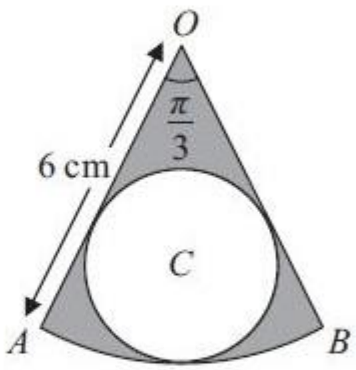

Figure 1

The shape shown in Figure 1 is a pattern for a pendant. It consists of a sector *OAB* of a circle centre *O*, of radius 6 cm, and angle $AOB = \frac{\pi}{3}$. The circle *C*, inside the sector, touches the two straight edges, *OA* and *OB*, and the arc *AB* as shown.

Find

(a) the area of the sector *OAB*, **(2)**

(b) the radius of the circle *C*. **(3)**

The region outside the circle *C* and inside the sector *OAB* is shown shaded in Figure 1.

(c) Find the area of the shaded region. **(2)**

(Total 7 marks)

Question 3

The second and third terms of a geometric series are 192 and 144 respectively.

For this series, find

(a) the common ratio, **(2)**

(b) the first term, **(2)**

(c) the sum to infinity, **(2)**

(d) the smallest value of n for which the sum of the first n terms of the series exceeds 1000. **(4)**

(Total 10 marks)

Question 4

$$\mathrm{f}(x) = \frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{x^2-9}, \qquad x \neq \pm 3,\ x \neq -\frac{1}{2}$$

(a) Show that

$$\mathrm{f}(x) = \frac{5}{(2x+1)(x+3)}$$

(5)

The curve C has equation $y = \mathrm{f}(x)$. The point $P\left(-1, -\frac{5}{2}\right)$ lies on C.

(b) Find an equation of the normal to C at P. **(8)**

(Total 13 marks)

Question 5

Find the gradient of the curve with equation

$$\ln y = 2x \ln x, \quad x > 0, \ y > 0$$

at the point on the curve where $x = 2$. Give your answer as an exact value. **(7)**

(Total 7 marks)

Question 6

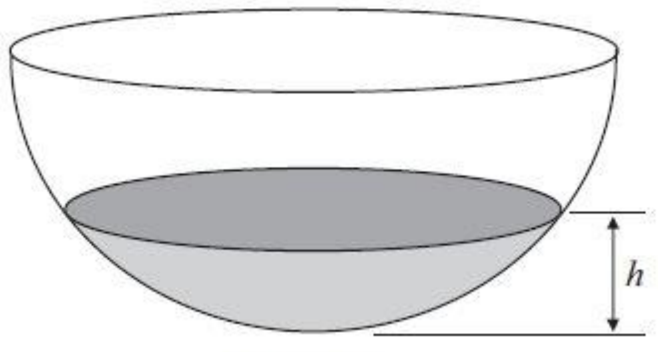

Figure 1

A hollow hemispherical bowl is shown in Figure 1. Water is flowing into the bowl. When the depth of the water is h m, the volume V m^3 is given by

$$V = \frac{1}{12}\pi h^2 (3 - 4h), \qquad 0 \leqslant h \leqslant 0.25$$

(a) Find, in terms of π, $\frac{dV}{dh}$ when $h = 0.1$ **(4)**

Water flows into the bowl at a rate of $\frac{\pi}{800}$ $m^3 s^{-1}$.

(b) Find the rate of change of h, in m s^{-1}, when $h = 0.1$ **(2)**

(Total 6 marks)

Question 7

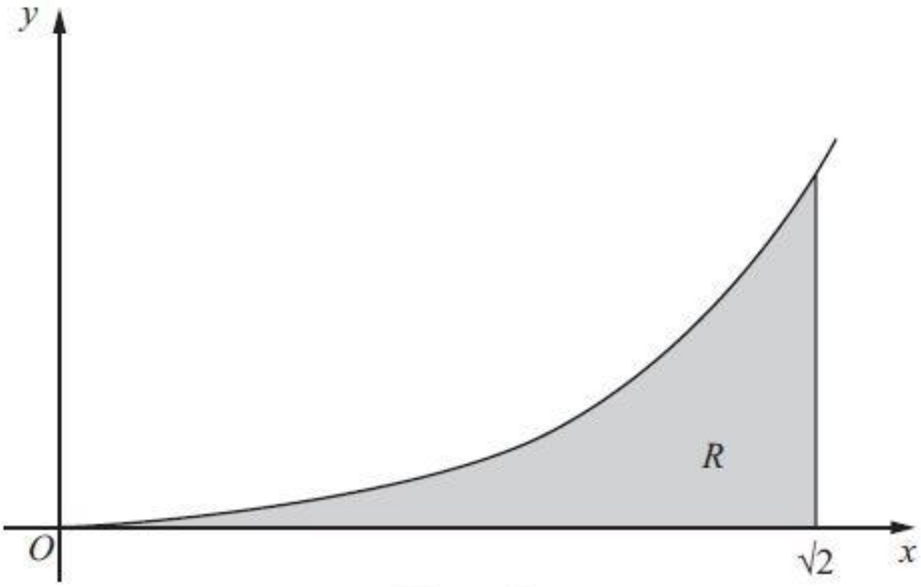

Figure 2

(a) Use the substitution $u = x^2 + 2$ to show that the area of R is

$$\frac{1}{2}\int_{2}^{4}(u-2)\ln u \, \mathrm{d}u$$

(4)

(b) Hence, or otherwise, find the exact area of R. **(6)**

(Total 10 marks)

Question 8

(a) Prove that

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \tan\theta, \quad \theta \neq 90n°, \ n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise,

(i) show that $\tan 15° = 2 - \sqrt{3}$,

(3)

(ii) solve, for $0 < x < 360°$,

$$\operatorname{cosec} 4x - \cot 4x = 1$$

(5)

(Total 12 marks)

Question 9

(a) Express 2cos 3*x* − 3sin 3*x* in the form *R* cos (3*x* + α), where *R* and α are constants, *R* > 0 and $0 < \alpha < \frac{\pi}{2}$. Give your answers to 3 significant figures. **(4)**

$$f(x) = e^{2x} \cos 3x$$

(b) Show that f '(*x*) can be written in the form

$$f'(x) = R e^{2x} \cos(3x + \alpha)$$

where *R* and α are the constants found in part (a). **(5)**

(c) Hence, or otherwise, find the smallest positive value of *x* for which the curve with equation *y* = f(*x*) has a turning point. **(3)**

(Total 12 marks)

Question 10

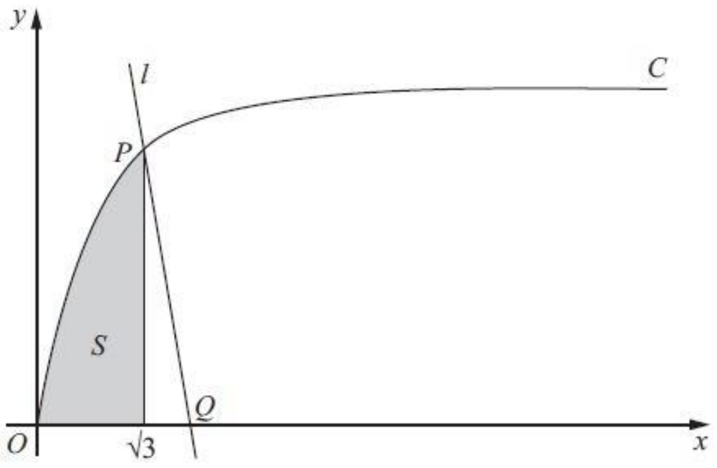

Figure 3

Figure 3 shows part of the curve *C* with parametric equations

$$x = \tan\theta, \qquad y = \sin\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point *P* lies on *C* and has coordinates $(\sqrt{3}, \frac{1}{2}\sqrt{3})$

(a) Find the value of θ at the point *P*. **(2)**

The line *l* is a normal to *C* at *P*. The normal cuts the *x*-axis at the point *Q*.

(b) Show that *Q* has coordinates ($k\sqrt{3}$, 0), giving the value of the constant *k*. **(6)**

(Total 8 marks)

Question 11

The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$$m = pe^{-kt}$$

where k and p are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(a) Write down the value of p. **(1)**

(b) Show that $k = \frac{1}{4}\ln 3$. **(4)**

(c) Find the value of t when $\frac{dm}{dt} = -0.6\ln 3$. **(6)**

(Total 11 marks)

TOTAL FOR PAPER IS 100 MARKS