Pure Mathematics 1

Advanced Subsidiary

Practice Paper M11

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE AS Level Specifications
- There are 11 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

$$f(x) = x^2 + (k+3)x + k$$

where *k* is a real constant.

(a) Find the discriminant of f(*x*) in terms of *k*.   **(2)**

(b) Show that the discriminant of f(*x*) can be expressed in the form $(k + a)^2 + b$, where *a* and *b* are integers to be found.   **(2)**

(c) Show that, for all values of *k*, the equation   f(*x*) = 0   has real roots.   **(2)**

(Total 6 marks)

Question 2

$$f(x) = 2x^3 - 7x^2 - 5x + 4$$

(a) Find the remainder when f(*x*) is divided by (*x* −1).   **(2)**

(b) Use the factor theorem to show that (*x*+1) is a factor of f(*x*).   **(2)**

(c) Factorise f(*x*) completely.   **(4)**

(Total 8 marks)

Question 3

The circle *C* has equation

$$x^2 + y^2 + 4x - 2y - 11 = 0$$

Find

(a) the coordinates of the centre of *C*,   **(2)**

(b) the radius of *C*,   **(2)**

(c) the coordinates of the points where *C* crosses the *y*-axis,
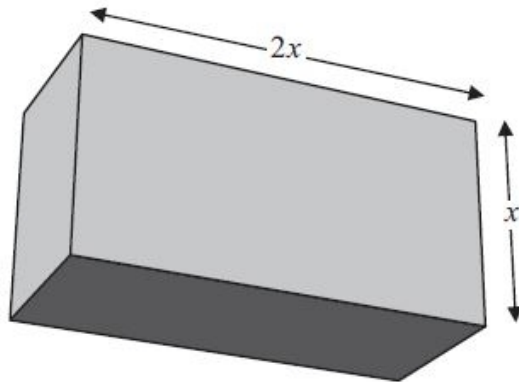giving your answers as simplified surds.   **(4)**

(Total 8 marks)

Figure 2

A cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cm, as shown in Figure 2.
The volume of the cuboid is 81 cubic centimetres.

(a) Show that the total length, L cm, of the twelve edges of the cuboid is given by

$$L = 12x + \frac{162}{x^2}$$

(3)

(b) Use calculus to find the minimum value of L. (6)

(c) Justify, by further differentiation, that the value of L that you have found is a minimum. (2)

(Total 11 marks)

Question 5

The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$$m = pe^{-kt}$$

where k and p are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(a) Write down the value of p. (1)

(b) Show that $k = \frac{1}{4}\ln 3$. (4)

(c) Find the value of t when $\frac{dm}{dt} = -0.6\ln 3$. (6)

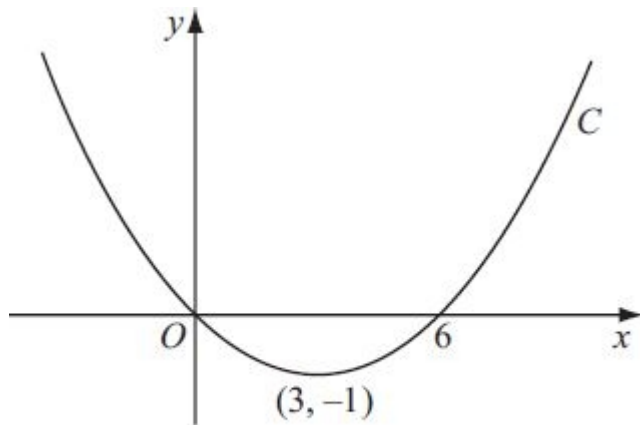**(Total 11 marks)**

Question 6

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$.
The curve C passes through the origin and through $(6, 0)$.
The curve C has a minimum at the point $(3, -1)$.

On separate diagrams, sketch the curve with equation

(a) $y = f(2x)$,   **(3)**

(b) $y = -f(x)$,   **(3)**

(c) $y = f(x + p)$, where p is a constant and $0 < p < 3$.   **(4)**

(Total 10 marks)

Question 7

The curve *C* has equation

$$y = (x + 1)(x + 3)^2$$

(a) Sketch *C*, showing the coordinates of the points at which *C* meets the axes. **(4)**

(b) Show that $\dfrac{dy}{dx} = 3x^2 + 14x + 15.$ **(3)**

The point *A*, with *x*-coordinate −5, lies on *C*.

(c) Find the equation of the tangent to *C* at *A*, giving your answer in the form *y* = *mx* + *c*, where *m* and *c* are constants. **(4)**

Another point *B* also lies on *C*. The tangents to *C* at *A* and *B* are parallel.

(d) Find the *x*-coordinate of *B*. **(3)**
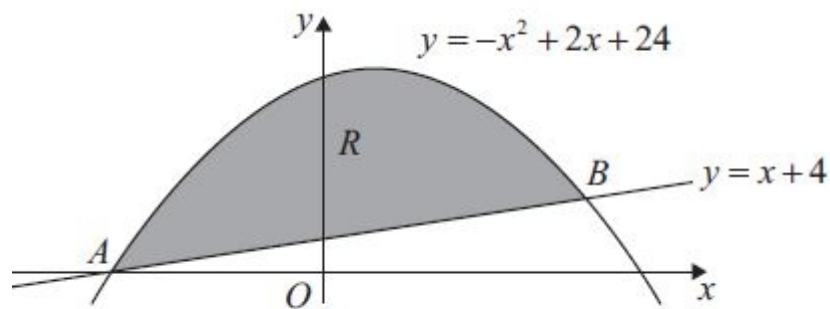
(Total 14 marks)

Question 8

Figure 3

The straight line with equation *y* = *x* + 4 cuts the curve with equation *y* = −*x*² + 2*x* + 24 at the points *A* and *B*, as shown in Figure 3.

(a) Use algebra to find the coordinates of the points *A* and *B*. **(4)**

The finite region *R* is bounded by the straight line and the curve and is shown shaded in Figure 3.

(b) Use calculus to find the exact area of *R*. **(7)**

(Total 11 marks)

Question 9

(a) Solve for 0 ≤ x < 360°, giving your answers in degrees to 1 decimal place,

$$3\sin(x + 45°) = 2$$ **(4)**

(b) Find, for $0 \leq x < 360^0$, all the solutions of

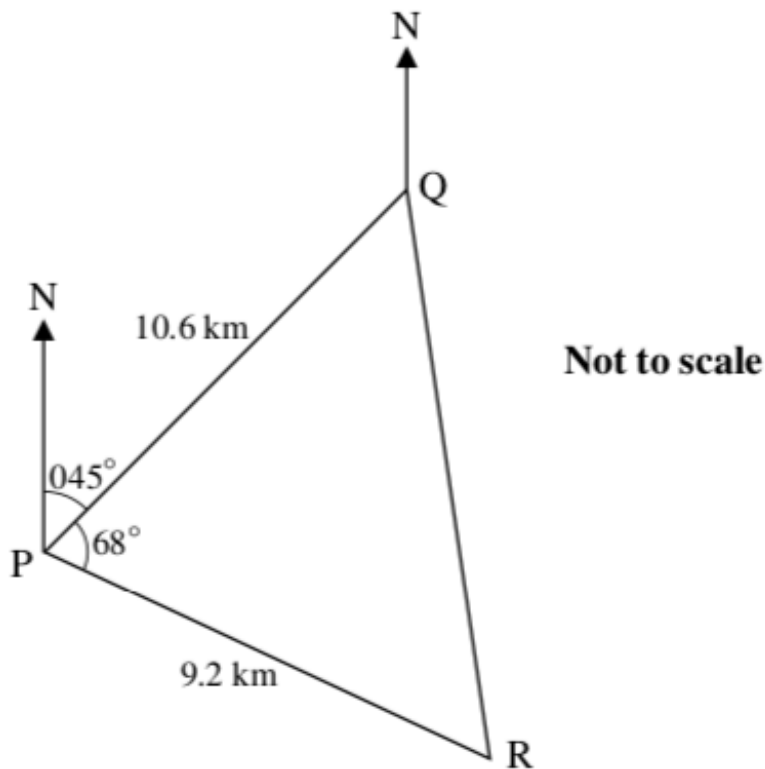$$2\sin^2 x + 2 = 7\cos x.$$

You must show clearly how you obtained your answers. **(6)**

(Total 10 marks)

Question 10

N

Q

N

10.6 km

Not to scale

045°

68°

P

9.2 km

R

A boat travels from P to Q and then to R. As shown in Fig. 4. Q is 10 km from P on a bearing of 045^0. R is 9.2 km from P on a bearing of 113^0, so that angle QPR is 68^0.

Calculate the distance and bearing of R from Q **(5)**

(Total 5 marks)

Question 11
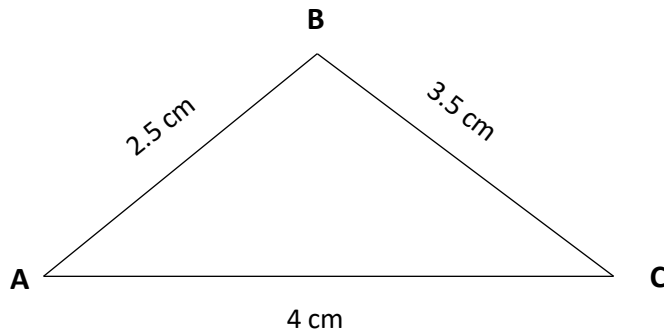
The diagram shows the triangle ABC with AB = 2.5 cm, BC = 3.5 cm and AC = 4 cm

(a) Show that $\cos B = \dfrac{1}{7}$ **(3)**

(b) Hence find the exact value of $\sin B$ **(3)**

(Total 6 marks)

TOTAL FOR PAPER IS 100 MARKS