Pure Mathematics 2 Practice Paper J12 MARK SCHEME

Question 1

Question	Scheme	Marks
(a)	$S_{10} = \frac{10}{2}[2P + 9\times 2T]$ or $\frac{10}{2}(P + [P + 18T])$	M1
	e.g. $5[2P+18T]$ = (£) $(10P + 90T)$ or (£) $10P + 90T$ (*)	A1cso (2)
(b)	Scheme 2: $S_{10} = \frac{10}{2}[2(P+1800)+9T] = \{10P + 18000 + 45T\}$	M1A1
	$10P + 90T = 10P + 18000 + 45T$	M1
	$90T = 18000 + 45T$	
	$T = 400$ (only)	A1 (4)
(c)	Scheme 2, Year 10 salary: $[a+(n-1)d=](P+1800)+9T$	B1ft
	$P + 1800 +$ "3600" $= 29850$	M1
	$P =$ (£) 24450	A1 (3)
		9 marks
	Notes	
(a)	M1 for identifying $a = P$ or $d = 2T$ and attempt at S_{10}. Using $n = 10$ and one of a or d correct. Must see evidence for M mark, at least one line before the answer.	
	A1cso for simplifying to given answer. No incorrect working seen. Do not penalise missing end bracket in working eg $5(2P + 18T$	
List	M1A1 for a full list seen (with + signs or written in columns) and no incorrect working seen. Any missing terms is M0A0	
(b)	1st M1 for attempting S_{10} for scheme 2 (allow missing (…) brackets e.g. $2P + 1800 + 9T$) Using $n = 10$ and at least one of a or d correct.	
	1st A1 for a correct expression for S_{10} using scheme 2 (needn't be multiplied out)	
List	Allow M1A1 if they reach $10P + 18000 + 45T$ with no incorrect working seen. $10P + 18000 + 45T$ with no working is M1A1	
	2nd M1 for forming an equation using the two sums that would enable P to be eliminated. Follow through their expressions provided P would disappear.	
	2nd A1 for $T = 400$ Answer only (4/4)	
(c)	B1 for using u_{10} for scheme 2 . Can be $9T$ or follow through their value of T	
	M1 for forming an equation based on u_{10} for scheme 2 and using 29850 and their value of T	
	A1 for 24450 seen Answer only (3/3)	
MR	If they misread scheme 2 as scheme 1 in part (c) apply MR rule and award B0M1A0 max for an equation based on u_{10} for scheme 1 and using 29850 and their value of T	

Question 2

Question Number	Scheme		Marks
(a)	$\dfrac{1}{(2-5x)^2} = (2-5x)^{-2} = \underline{(2)^{-2}}\left(1-\dfrac{5x}{2}\right)^{-2} = \underline{\dfrac{1}{4}}\left(1-\dfrac{5x}{2}\right)^{-2}$	$\underline{(2)^{-2}}$ or $\underline{\dfrac{1}{4}}$	B1
	$= \left\{\dfrac{1}{4}\right\}\left[1+(-2)(**x)+\dfrac{(-2)(-3)}{2!}(**x)^2+\ldots\right]$	see notes	M1 A1ft
	$= \left\{\dfrac{1}{4}\right\}\left[\underline{1+(-2)\left(-\dfrac{5x}{2}\right)+\dfrac{(-2)(-3)}{2!}\left(-\dfrac{5x}{2}\right)^2}+\ldots\right]$		
	$= \dfrac{1}{4}\left[1+5x;+\dfrac{75}{4}x^2+\ldots\right]$	See notes below!	
	$= \dfrac{1}{4}+\dfrac{5}{4}x;+\dfrac{75}{16}x^2+\ldots$		A1; A1
			[5]
(b)	$\left\{\dfrac{2+kx}{(2-5x)^2}\right\} = (2+kx)\left(\dfrac{1}{4}+\dfrac{5}{4}x+\left\{\dfrac{75}{16}x^2+\ldots\right\}\right)$	*Can be implied by later work even in part (c).*	M1
	x terms: $\dfrac{2(5x)}{4}+\dfrac{kx}{4}=\dfrac{7x}{4}$		
	giving, $10+k=7 \Rightarrow \underline{k=-3}$	$\underline{k=-3}$	A1
			[2]
(c)	x^2 terms: $\dfrac{150x^2}{16}+\dfrac{5kx^2}{4}$		M1
	So, $A = \dfrac{75}{8}+\dfrac{5(-3)}{4} = \dfrac{75}{8}-\dfrac{15}{4} = \underline{\dfrac{45}{8}}$	$\underline{\dfrac{45}{8}}$ or $\underline{5\dfrac{5}{8}}$ or $\underline{5.625}$	A1
			[2] **9**
(a)	**B1**: $(2)^{-2}$ or $\dfrac{1}{4}$ outside brackets or $\dfrac{1}{4}$ as candidate's constant term in their binomial expansion. **M1:** Expands to give a simplified or an un-simplified, $1+(-2)(**x)$ or $(-2)(**x)+\dfrac{(-2)(-3)}{2!}(**x)^2$ or $1+\ldots\ldots+\dfrac{(-2)(-3)}{2!}(**x)^2$, **where** $** \neq 1$. **A1:** A correct simplified or an un-simplified $1+(-2)(**x)+\dfrac{(-2)(-3)}{2!}(**x)^2$ expansion with candidate's follow through $(**x)$. Note that $(**x)$ must be consistent. You would award B1M1A0 for $=\dfrac{1}{4}\left[\underline{1+(-2)\left(-\dfrac{5x}{2}\right)+\dfrac{(-2)(-3)}{2!}(-5x)^2}+\ldots\right]$ because ** is not consistent. ***Invisible brackets*** $\left\{\dfrac{1}{4}\right\}\left[\underline{1+(-2)\left(-\dfrac{5x}{2}\right)+\dfrac{(-2)(-3)}{2!}\left(-\dfrac{5x^2}{2}\right)}+\ldots\right]$ is M1A0 unless recovered. **A1:** For $\dfrac{1}{4}+\dfrac{5}{4}x$ **(simplified fractions)** or Also allow $0.25+1.25x$ or $\dfrac{1}{4}+1\dfrac{1}{4}x$. **Allow Special Case A1 for either SC:** $\dfrac{1}{4}[1+5x;\ldots]$ or SC: $K\left[1+5x+\dfrac{75}{4}x^2+\ldots\right]$. **A1:** Accept only $\dfrac{75}{16}x^2$ or $4\dfrac{11}{16}x^2$ or $4.6875x^2$ **<u>Alternative method:</u>** Candidates can apply an alternative form of the binomial expansion. (See next page).		

(b)	**M1:** Candidate writes down $(2 + kx)$(their part (a) answer, at least up to the term in x.) $(2+kx)\left(\frac{1}{4}+\frac{5}{4}x+...\right)$ or $(2+kx)\left(\frac{1}{4}+\frac{5}{4}x+\frac{75}{16}x^2+...\right)$ are fine. This mark can also be implied by candidate multiplying out to find two terms (or coefficients) in x. **A1:** $k = -3$
(c)	**M1:** Multiplies out their $(2+kx)\left(\frac{1}{4}+\frac{5}{4}x+\frac{75}{16}x^2+...\right)$ to give **exactly** two terms (or coefficients) in x^2 and attempts to find A using a numerical value of k. **A1:** Either $\frac{45}{8}$ or $5\frac{5}{8}$ or 5.625 **Note:** $\frac{45}{8}x^2$ is A0. *Alternative method for part (a)* $(2-5x)^{-2} = (2)^{-2} + (-2)(2)^{-3}(-5x); + \frac{(-2)(-3)}{2!}(2)^{-4}(-5x)^2$ **B1:** $\frac{1}{4}$ or $(2)^{-2}$, **M1:** Any two of three (un-simplified) terms correct. **A1:** All three (un-simplified) terms correct. **A1:** $\frac{1}{4}+\frac{5}{4}x$ **A1:** $\frac{75}{16}x^2$ **Note:** The terms in C need to be evaluated, so ${}^{-2}C_0(2)^{-2} + {}^{-2}C_1(2)^{-3}(-5x); + {}^{-2}C_2(2)^{-4}(-5x)^2$ without further working is B0M0A0.
	Alternative method for parts (b) and (c) $(2+kx) = (2-5x)^2\left(\frac{1}{2}+\frac{7}{4}x+Ax^2+...\right)$ $(2+kx) = (4-20x+25x^2)\left(\frac{1}{2}+\frac{7}{4}x+Ax^2+...\right)$ $(2+kx) = 2 + (7x-10x) + \left(4Ax^2 - 35x^2 + \frac{25}{2}x^2\right)$ Equate x terms: $\underline{k = -3}$ Equate x^2 terms: $0 = 4A - 35 + \frac{25}{2} \Rightarrow 4A = \frac{45}{2} \Rightarrow \underline{A = \frac{45}{8}}$
(b)	**M1:** For $(2+kx) = (4 \pm \lambda x + 25x^2)\left(\frac{1}{2}+\frac{7}{4}x+Ax^2+...\right)$, where $\lambda \neq 0$ **A1:** $k = -3$
(c)	**M1:** Multiplies out to obtain three x^2 terms/coefficients, equates to 0 and attempts to find A. **A1:** Either $\frac{45}{8}$ or $5\frac{5}{8}$ or 5.625 **Note:** $\frac{45}{8}x^2$ is A0.

Question 3

Question No	Scheme	Marks
	(a)	
	$\frac{d}{dx}(\ln(3x)) \rightarrow \frac{B}{x}$ for any constant B	M1
	Applying vu'+uv' , $\ln(3x) \times 2x + x$	M1, A1 A1
		(4)
	(b)	
	Applying $\frac{vu' - uv\prime}{v^2}$	
	$\frac{x^3 \times 4cos(4x) - \sin(4x) \times 3x^2}{x^6}$	M1 A1+A1 A1
	$= \frac{4xcos(4x) - 3\sin(4x)}{x^4}$	A1
		(5)
		(9 MARKS)

Question 4

Question Number	Scheme	Marks
(a)	$\left\{\frac{\cancel{dy}}{\cancel{dx}}\times\right\}\ \underline{2+6y\frac{dy}{dx}}+\left(\underline{\underline{6xy+3x^2\frac{dy}{dx}}}\right)=\underline{8x}$	M1 <u>A1</u> <u>B1</u>
	$\left\{\frac{dy}{dx}=\frac{8x-2-6xy}{6y+3x^2}\right\}$ *not necessarily required.*	
	At $P(-1,1)$, $m(T)=\frac{dy}{dx}=\frac{8(-1)-2-6(-1)(1)}{6(1)+3(-1)^2}=-\frac{4}{9}$	dM1 A1 cso
		[5]
(b)	So, $m(N)=\frac{-1}{-\frac{4}{9}}\left\{=\frac{9}{4}\right\}$	M1
	N: $y-1=\frac{9}{4}(x+1)$	M1
	N: $9x-4y+13=0$	A1
		[3]
		8
(a)	**M1**: Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $3x^2\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx}=\right)$). **A1**: $(2x+3y^2)\to\left(2+6y\frac{dy}{dx}\right)$ and $(4x^2\to\underline{8x})$. **Note:** If an extra "sixth" term appears then award A0. **B1**: $6xy+3x^2\frac{dy}{dx}$. **dM1**: Substituting $x=-1$ and $y=1$ into an equation involving $\frac{dy}{dx}$. Allow this mark if either the numerator or denominator of $\frac{dy}{dx}=\frac{8x-2-6xy}{6y+3x^2}$ is substituted into or evaluated correctly. If it is clear, however, that the candidate is intending to substitute $x=1$ and $y=-1$, then award M0. Candidates who substitute $x=1$ and $y=-1$, will usually achieve $m(T)=-4$ Note that this mark is dependent on the previous method mark being awarded. **A1**: For $-\frac{4}{9}$ or $-\frac{8}{18}$ or $-0.\dot{4}$ or awrt -0.44 If the candidate's solution is not completely correct, then do not give this mark.	
(b)	**M1**: Applies $m(N)=-\frac{1}{\text{their } m(T)}$. **M1**: Uses $y-1=(m_N)(x--1)$ or finds c using $x=-1$ and $y=1$ and uses $y=(m_N)x+\text{"}c\text{"}$, Where $m_N=-\frac{1}{\text{their } m(T)}$ or $m_N=\frac{1}{\text{their } m(T)}$ or $m_N=-\text{their } m(T)$. **A1**: $9x-4y+13=0$ or $-9x+4y-13=0$ or $4y-9x-13=0$ or $18x-8y+26=0$ etc. Must be "$=0$". So do not allow $9x+13=4y$ etc. **Note**: $m_N=-\left(\frac{6y+3x^2}{8x-2-6xy}\right)$ is M0M0 unless a numerical value is then found for m_N.	
	<u>*Alternative method for part (a): Differentiating with respect to y*</u> $\left\{\frac{\cancel{dx}}{\cancel{dy}}\times\right\}\ \underline{2\frac{dx}{dy}+6y}+\left(\underline{\underline{6xy\frac{dx}{dy}+3x^2}}\right)=\underline{8x\frac{dx}{dy}}$ **M1**: Differentiates implicitly to include either $2\frac{dx}{dy}$ or $6xy\frac{dx}{dy}$ or $\pm kx\frac{dx}{dy}$. (Ignore $\left(\frac{dx}{dy}=\right)$). **A1**: $(2x+3y^2)\to\left(2\frac{dx}{dy}+6y\right)$ and $\left(4x^2\to 8x\frac{dx}{dy}\right)$. **Note:** If an extra "sixth" term appears then award A0. **B1**: $6xy+3x^2\frac{dy}{dx}$. **dM1**: Substituting $x=-1$ and $y=1$ into an equation involving $\frac{dx}{dy}$ or $\frac{dy}{dx}$. Allow this mark if either the numerator or denominator of $\frac{dx}{dy}=\frac{6y+3x^2}{8x-2-6xy}$ is substituted into or evaluated correctly. If it is clear, however, that the candidate is intending to substitute $x=1$ and $y=-1$, then award M0. Candidates who substitute $x=1$ and $y=-1$, will usually achieve $m(T)=-4$	

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Question 5

Question Number	Scheme	Marks
(a)	$\int x\sin 3x\,\mathrm{d}x = -\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x\,\{\mathrm{d}x\}$	M1 A1
	$= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\,\{+c\}$	A1
		[3]
(b)	$\int x^2\cos 3x\,\mathrm{d}x = \frac{1}{3}x^2\sin 3x - \int \frac{2}{3}x\sin 3x\,\{\mathrm{d}x\}$	M1 A1
	$= \frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right)\ \{+c\}$	A1 isw
	$\left\{= \frac{1}{3}x^2\sin 3x + \frac{2}{9}x\cos 3x - \frac{2}{27}\sin 3x\ \{+c\}\right\}$ *Ignore subsequent working*	[3]
		6
(a)	**M1:** Use of 'integration by parts' formula $uv - \int vu'$ (whether stated or not stated) in the **correct direction**, where $u = x \to u' = 1$ and $v' = \sin 3x \to v = k\cos 3x$ (seen or implied), where k is a positive or negative constant. (Allow $k = 1$). This means that the candidate must achieve $x(k\cos 3x) - \int (k\cos 3x)$, where k is a consistent constant. If x^2 appears after the integral, this would imply that the candidate is applying integration by parts in the wrong direction, so M0. **A1:** $-\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x\,\{\mathrm{d}x\}$. Can be un-simplified. Ignore the $\{\mathrm{d}x\}$. **A1:** $-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x$ with/without $+c$. Can be un-simplified.	
(b)	**M1:** Use of 'integration by parts' formula $uv - \int vu'$ (whether stated or not stated) in the **correct direction**, where $u = x^2 \to u' = 2x$ or x and $v' = \cos 3x \to v = \lambda\sin 3x$ (seen or implied), where λ is a positive or negative constant. (Allow $\lambda = 1$). This means that the candidate must achieve $x^2(\lambda\sin 3x) - \int 2x(\lambda\sin 3x)$, where $u' = 2x$ or $x^2(\lambda\sin 3x) - \int x(\lambda\sin 3x)$, where $u' = x$. If x^3 appears after the integral, this would imply that the candidate is applying integration by parts in the wrong direction, so M0. **A1:** $\frac{1}{3}x^2\sin 3x - \int \frac{2}{3}x\sin 3x\,\{\mathrm{d}x\}$. Can be un-simplified. Ignore the $\{\mathrm{d}x\}$. **A1:** $\frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right)$ with/without $+c$, can be un-simplified. You can ignore subsequent working here. **Special Case:** If the candidate scores the first two marks of M1A1 in part (b), then you can award the final A1 as a follow through for $\frac{1}{3}x^2\sin 3x - \frac{2}{3}$(their follow through part(a) answer).	

Question 6

Question No	Scheme	Marks
	Uses the identity $cot^2(3\theta) = cosec^2(3\theta) - 1$ *in* $2cot^2(3\theta) = 7cosec(3\theta) - 5$	**M1**
	$2cosec^2(3\theta) - 7cosec(3\theta) + 3 = 0$	**A1**
	$(2cosec3\theta - 1)(cosec3\theta - 3) = 0$	**dM1**
	$cosec3\theta = 3$	**A1**
	$\theta = \frac{invsin(\frac{1}{3})}{3}, \ \frac{19.5°}{3} = awrt\ 6.5°$	**ddM1, A1**
	$\theta = \frac{180° - invsin(\frac{1}{3})}{3}, 53.5°$ Correct 2nd value	A1 **ddM1,A1**
	$\theta = \frac{360° + invsin(\frac{1}{3})}{3}$ Correct 3rd value	**ddM1**
	All 4 correct answers awrt 6.5°,53.5°,126.5 °or 173.5°	**A1** **(10 marks)**

Question 7

Question No	Scheme	Marks
	$\left(\frac{dx}{dy}\right) = 2sec^2\left(y + \frac{\pi}{12}\right)$	M1,A1
	substitute $y = \frac{\pi}{4}$ into their $\frac{dx}{dy} = 2sec^2\left(\frac{\pi}{4} + \frac{\pi}{12}\right) = 8$	M1, A1
	When $y = \frac{\pi}{4}$, $x = 2\sqrt{3}$ awrt 3.46	B1
	$\left(y - \frac{\pi}{4}\right) = their\ m(x - their\ 2\sqrt{3})$	M1
	$\left(y - \frac{\pi}{4}\right) = -8(x - 2\sqrt{3})$ oe	A1 (7 marks)

Question 8

Question Number	Scheme	Marks
(a)	0.73508	B1 **cao** **[1]**
(b)	Area $\approx \frac{1}{2} \times \frac{\pi}{8}$; $\times \left[0 + 2(\text{their } 0.73508 + 1.17157 + 1.02280) + 0\right]$	B1 M1
	$= \frac{\pi}{16} \times 5.8589... = 1.150392325... = 1.1504$ (4 dp) awrt 1.1504	A1 **[3]**
(c)	$\{u = 1 + \cos x\} \Rightarrow \frac{du}{dx} = -\sin x$	B1
	$\left\{\int \frac{2\sin 2x}{(1+\cos x)}\,dx =\right\} \int \frac{2(2\sin x \cos x)}{(1+\cos x)}\,dx$ $\sin 2x = 2\sin x \cos x$	B1
	$= \int \frac{4(u-1)}{u}.(-1)\,du \quad \left\{= 4\int \frac{(1-u)}{u}\,du\right\}$	M1
	$= 4\int \left(\frac{1}{u} - 1\right) du = 4(\ln u - u) + c$	dM1
	$= 4\ln(1+\cos x) - 4(1+\cos x) + c = 4\ln(1+\cos x) - 4\cos x + k$ **AG**	A1 **cso** **[5]**
(d)	$= \left[4\ln\left(1+\cos\frac{\pi}{2}\right) - 4\cos\frac{\pi}{2}\right] - \left[4\ln(1+\cos 0) - 4\cos 0\right]$ Applying limits $x = \frac{\pi}{2}$ and $x = 0$ either way round.	M1
	$= [4\ln 1 - 0] - [4\ln 2 - 4]$	
	$= 4 - 4\ln 2 \; \{= 1.227411278...\}$ $\pm 4(1 - \ln 2)$ or $\pm(4 - 4\ln 2)$ or awrt ± 1.2, *however* found.	A1
	Error $= \lvert(4 - 4\ln 2) - 1.1504...\rvert$ $= 0.0770112776... = 0.077$ (2sf) awrt ± 0.077 or awrt $\pm 6.3(\%)$	A1 **cso** **[3]**
		12

(a)	**B1:** 0.73508 correct answer only. Look for this on the table or in the candidate's working.
(b)	**B1:** Outside brackets $\frac{1}{2} \times \frac{\pi}{8}$ or $\frac{\pi}{16}$ or awrt 0.196 **M1:** For structure of trapezium rule $[\ ....\ ]$; (0 can be implied). **A1:** anything that rounds to 1.1504 Bracketing mistake: **Unless the final answer implies that the calculation has been done correctly** Award B1M0A0 for $\frac{1}{2} \times \frac{\pi}{8} + 2(\text{their } 0.73508 + 1.17157 + 1.02280)$ (nb: answer of 6.0552). Award B1M0A0 for $\frac{1}{2} \times \frac{\pi}{8}(0 + 0) + 2(\text{their } 0.73508 + 1.17157 + 1.02280)$ (nb: answer of 5.8589). *Alternative method for part (b): Adding individual trapezia* Area $\approx \frac{\pi}{8} \times \left[\frac{0 + 0.73508}{2} + \frac{0.73508 + 1.17157}{2} + \frac{1.17157 + 1.02280}{2} + \frac{1.02280 + 0}{2}\right] = 1.150392325...$ B1: $\frac{\pi}{8}$ and a divisor of 2 on all terms inside brackets. M1: One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2. A1: anything that rounds to 1.1504

Question 9

Question No	Scheme	Marks
	(a) $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB}$	M1A1
	$= \frac{\frac{sinA}{cosA}+\frac{sinB}{cosB}}{1-\frac{sinAsinB}{cosAcosB}}$ $(\div cosAcosB)$	M1
	$= \frac{tanA + tanB}{1 - tanAtanB}$	A1 *
		(4)
	(b) $\tan\left(\theta+\frac{\pi}{6}\right) = \frac{tan\theta+tan\frac{\pi}{6}}{1-tan\theta tan\frac{\pi}{6}}$	M1
	$= \frac{tan\theta+\frac{1}{\sqrt{3}}}{1-tan\theta\frac{1}{\sqrt{3}}}$	M1
	$= \frac{\sqrt{3}tan\theta+1}{\sqrt{3}-tan\theta}$	A1 *
		(3)
	(c) $\tan\left(\theta+\frac{\pi}{6}\right) = \tan(\pi-\theta).$	M1
	$\left(\theta+\frac{\pi}{6}\right) = (\pi-\theta)$	dM1
	$\theta = \frac{5}{12}\pi$	ddM1 A1
	$\tan\left(\theta+\frac{\pi}{6}\right) = \tan(2\pi-\theta)$	dddM1
	$\theta = \frac{11}{12}\pi$	A1
		(6)
		(13 MARKS)

Question 10

Question Number	Scheme	Marks
	$x = 4\sin\left(t+\frac{\pi}{6}\right),\quad y = 3\cos 2t,\quad 0 \leqslant t < 2\pi$	
(a)	$\frac{dx}{dt} = 4\cos\left(t+\frac{\pi}{6}\right),\quad \frac{dy}{dt} = -6\sin 2t$	B1 B1
	So, $\frac{dy}{dx} = \frac{-6\sin 2t}{4\cos\left(t+\frac{\pi}{6}\right)}$	B1$\sqrt{\ }$ oe
		[3]
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}\ -6\sin 2t = 0$	M1 oe
	@ $t = 0,\quad x = 4\sin\left(\frac{\pi}{6}\right) = 2,\quad y = 3\cos 0 = 3 \quad \rightarrow (2, 3)$	M1
	@ $t = \frac{\pi}{2},\quad x = 4\sin\left(\frac{2\pi}{3}\right) = \frac{4\sqrt{3}}{2},\quad y = 3\cos\pi = -3 \rightarrow (2\sqrt{3}, -3)$	
	@ $t = \pi,\quad x = 4\sin\left(\frac{7\pi}{6}\right) = -2,\quad y = 3\cos 2\pi = 3 \rightarrow (-2, 3)$	
	@ $t = \frac{3\pi}{2}, x = 4\sin\left(\frac{5\pi}{3}\right) = \frac{4(-\sqrt{3})}{2},\quad y = 3\cos 3\pi = -3 \rightarrow (-2\sqrt{3}, -3)$	A1A1A1
		[5] 8

(a)	**B1:** Either one of $\frac{dx}{dt} = 4\cos\left(t+\frac{\pi}{6}\right)$ or $\frac{dy}{dt} = -6\sin 2t$. They do not have to be simplified. **B1:** Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ correct. They do not have to be simplified. Any or both of the first two marks can be implied. Don't worry too much about their notation for the first two B1 marks. **B1:** Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or their $\frac{dy}{dt} \times \frac{1}{\text{their}\left(\frac{dx}{dt}\right)}$. **Note:** This is a follow through mark. <u>*Alternative differentiation in part (a)*</u> $x = 2\sqrt{3}\sin t + 2\cos t \Rightarrow \frac{dx}{dt} = 2\sqrt{3}\cos t - 2\sin t$ $y = 3(2\cos^2 t - 1) \Rightarrow \frac{dy}{dt} = 3(-4\cos t\sin t)$ or $y = 3\cos^2 t - 3\sin^2 t \Rightarrow \frac{dy}{dt} = -6\cos t\sin t - 6\sin t\cos t$ or $y = 3(1 - 2\sin^2 t) \Rightarrow \frac{dy}{dt} = 3(-4\cos t\sin t)$

Question 11

Question Number	Scheme		Marks
(a)	$1 = A(5-P) + BP$	Can be implied.	M1
	$A = \frac{1}{5},\ B = \frac{1}{5}$	Either one.	A1
	giving $\frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}$	See notes.	A1 **cao, aef**
			[3]
(b)	$\int \frac{1}{P(5-P)}\,dP = \int \frac{1}{15}\,dt$		B1
	$\frac{1}{5}\ln P - \frac{1}{5}\ln(5-P) = \frac{1}{15}t \quad (+c)$		M1* A1ft
	$\{t=0, P=1 \Rightarrow\}\quad \frac{1}{5}\ln 1 - \frac{1}{5}\ln(4) = 0 + c \quad \left\{\Rightarrow\ c = -\frac{1}{5}\ln 4\right\}$		dM1*
	eg: $\frac{1}{5}\ln\left(\frac{P}{5-P}\right) = \frac{1}{15}t - \frac{1}{5}\ln 4$	Using any of the subtraction (or addition) laws for logarithms CORRECTLY	dM1*
	$\ln\left(\frac{4P}{5-P}\right) = \frac{1}{3}t$		
	eg: $\frac{4P}{5-P} = e^{\frac{1}{3}t}$ or eg: $\frac{5-P}{4P} = e^{-\frac{1}{3}t}$	Eliminate ln's correctly.	dM1*
	gives $4P = 5e^{\frac{1}{3}t} - Pe^{\frac{1}{3}t} \Rightarrow P(4 + e^{\frac{1}{3}t}) = 5e^{\frac{1}{3}t}$		
	$P = \frac{5e^{\frac{1}{3}t}}{(4 + e^{\frac{1}{3}t})} \quad \left\{\frac{(\div e^{\frac{1}{3}t})}{(\div e^{\frac{1}{3}t})}\right\}$	Make P the subject.	dM1*
	$P = \frac{5}{(1 + 4e^{-\frac{1}{3}t})}$ or $P = \frac{25}{(5 + 20e^{-\frac{1}{3}t})}$ etc.		A1
			[8]
(c)	$1 + 4e^{-\frac{1}{3}t} > 1 \Rightarrow P < 5$. So population cannot exceed 5000.		B1
			[1] **12**

(a)	**M1:** Forming a correct identity. For example, $1 = A(5-P) + BP$. **Note** A and B not referred to in question.
	A1: Either one of $A = \frac{1}{5}$ or $B = \frac{1}{5}$.
	A1: $\frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}$ or any equivalent form, eg: $\frac{1}{5P} + \frac{1}{25-5P}$, etc. Ignore subsequent working. This answer must be stated in part (a) only. A1 can also be given for a candidate who finds both $A = \frac{1}{5}$ and $B = \frac{1}{5}$ and $\frac{A}{P} + \frac{B}{5-P}$ is seen in their working.
	Candidate can use 'cover-up' rule to write down $\frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}$, as so gain all three marks.
	Candidate cannot gain the marks for part (a) in part (b).

(b)

B1: Separates variables as shown. $\mathrm{d}P$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.

M1*: Both $\pm\lambda\ln P$ and $\pm\mu\ln(\pm 5 \pm P)$, where λ and μ are constants.

Or $\pm\lambda\ln mP$ and $\pm\mu\ln(n(\pm 5 \pm P))$, where λ, μ, m and n are constants.

A1ft: Correct follow through integration of both sides from their $\int \frac{\lambda}{P} + \frac{\mu}{(5-P)}\,\mathrm{d}P = \int K\,\mathrm{d}t$

with or without $+c$

dM1*: Use of $t = 0$ and $P = 1$ in an integrated equation containing c

dM1*: Using ANY of the subtraction (or addition) laws for logarithms CORRECTLY.

dM1*: Apply logarithms (or take exponentials) to eliminate ln's CORRECTLY from their equation.

dM1*: A full ACCEPTABLE method of rearranging to make P the subject. **(See below for examples!)**

A1: $P = \dfrac{5}{(1+4\mathrm{e}^{-\frac{1}{3}t})}$ $\{\text{where } a = 5, b = 1, c = 4\}$.

Also allow any "integer" multiples of this expression. For example: $P = \dfrac{25}{(5+20\mathrm{e}^{-\frac{1}{3}t})}$

Note: If the first method mark (M1*) is not awarded then the candidate cannot gain any of the six remaining marks for this part of the question.

Note: $\int \frac{1}{P(5-P)}\,\mathrm{d}P = \int 15\,\mathrm{d}t \Rightarrow \int \frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}\,\mathrm{d}P = \int 15\,\mathrm{d}t \Rightarrow \ln P - \ln(5-P) = 15t$ is B0M1A1ft.

<u>dM1* for making P the subject</u>

Note there are three type of manipulations here which are considered acceptable to make *P* the subject.

(1) M1 for $\dfrac{P}{5-P} = \mathrm{e}^{\frac{1}{3}t} \Rightarrow P = 5\mathrm{e}^{\frac{1}{3}t} - P\mathrm{e}^{\frac{1}{3}t} \Rightarrow P(1+\mathrm{e}^{\frac{1}{3}t}) = 5\mathrm{e}^{\frac{1}{3}t} \Rightarrow P = \dfrac{5}{(1+\mathrm{e}^{-\frac{1}{3}t})}$

(2) M1 for $\dfrac{P}{5-P} = \mathrm{e}^{\frac{1}{3}t} \Rightarrow \dfrac{5-P}{P} = \mathrm{e}^{\frac{1}{3}t} \Rightarrow \dfrac{5}{P} - 1 = \mathrm{e}^{\frac{1}{3}t} \Rightarrow \dfrac{5}{P} = \mathrm{e}^{\frac{1}{3}t} + 1 \Rightarrow P = \dfrac{5}{(1+\mathrm{e}^{\frac{1}{3}t})}$

(3) M1 for $P(5-P) = 4\mathrm{e}^{\frac{1}{3}t} \Rightarrow P^2 - 5P = -4\mathrm{e}^{\frac{1}{3}t} \Rightarrow \left(P - \dfrac{5}{2}\right)^2 - \dfrac{25}{4} = -4\mathrm{e}^{\frac{1}{3}t}$ leading to $P = \ldots$

Note: The incorrect manipulation of $\dfrac{P}{5-P} = \dfrac{P}{5} - 1$ or equivalent is awarded this dM0*.

(c)

B1: $1 + 4\mathrm{e}^{-\frac{1}{3}t} > 1$ **and** $P < 5$ **and** a conclusion relating population (or even P) or meerkats to 5000.

For $P = \dfrac{25}{(5+20\mathrm{e}^{-\frac{1}{3}t})}$, B1 can be awarded for $5 + 20\mathrm{e}^{-\frac{1}{3}t} > 5$ **and** $P < 5$ **and** a conclusion relating population (or even P) or meerkats to 5000.

B1 can only be obtained if candidates have correct values of a and b in their $P = \dfrac{a}{(b + c\mathrm{e}^{-\frac{1}{3}t})}$.

Award B0 for: As $t \to \infty$, $\mathrm{e}^{-\frac{1}{3}t} \to 0$. So $P \to \dfrac{5}{(1+0)} = 5$, so population cannot exceed 5000,

unless the candidate also proves that $P = \dfrac{5}{(1+4\mathrm{e}^{-\frac{1}{3}t})}$ oe. is an increasing function.

If unsure here, then send to review!

	Alternative method for part (b) **B1M1*A1:** as before for $\frac{1}{5}\ln P - \frac{1}{5}\ln(5 - P) = \frac{1}{15}t \ \ (+c)$ Award 3rd M1 for $\ln\left(\frac{P}{5-P}\right) = \frac{1}{3}t + c$ Award 4th M1 for $\frac{P}{5-P} = Ae^{\frac{1}{3}t}$ Award 2nd M1 for $t = 0,\ P = 1 \Rightarrow \frac{1}{5-1} = Ae^{0}\ \left\{\Rightarrow A = \frac{1}{4}\right\}$ $\frac{P}{5-P} = \frac{1}{4}e^{\frac{1}{3}t}$ then award the final M1A1 in the same way.