Name:

Total Marks:

Pure Mathematics 2

Advanced Level

Practice Paper J12

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE A Level Specifications
- There are 11 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

A company offers two salary schemes for a 10-year period, Year 1 to Year 10 inclusive.

Scheme 1: Salary in Year 1 is £P.

Salary increases by £($2T$) each year, forming an arithmetic sequence.

Scheme 2: Salary in Year 1 is £(P + 1800).

Salary increases by £T each year, forming an arithmetic sequence.

(a) Show that the **total** earned under Salary Scheme 1 for the 10-year period is

£($10P + 90T$) **(2)**

For the 10-year period, the **total** earned is the same for both salary schemes.

(b) Find the value of T. **(4)**

For this value of T, the salary in Year 10 under Salary Scheme 2 is Â£29 850

(c) Find the value of P. **(3)**

(Total 9 marks)

Question 2

(a) Expand

$$\frac{1}{(2-5x)^2}, \quad |x| < \frac{2}{5}$$

in ascending powers of x, up to and including the term in x^2, giving each term as a simplified fraction. **(5)**

Given that the binomial expansion of

$$\frac{2+kx}{(2-5x)^2}, \quad |x| < \frac{2}{5} \quad \text{is}$$

$$\frac{1}{2} + \frac{7}{4}x + Ax^2 + \ldots$$

(b) find the value of the constant k, **(2)**

(c) find the value of the constant A. **(2)**

(Total 9 marks)

Question 3

Differentiate with respect to x, giving your answer in its simplest form,

(a) $x^2\ln(3x)$ (4)

(b) $\dfrac{\sin 4x}{x^3}$ (5)

(Total 9 marks)

Question 4

The curve C has the equation $2x + 3y^2 + 3x^2 y = 4x^2$.

The point P on the curve has coordinates $(-1, 1)$.

(a) Find the gradient of the curve at P. (5)

(b) Hence find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. (3)

(Total 8 marks)

Question 5

(a) Use integration by parts to find $\int x \sin 3x dx$. (3)

(b) Using your answer to part (a), find $\int x^2 \cos 3x \, dx$. (3)

(Total 6 marks)

Question 6

Solve, for $0 \leqslant \theta \leqslant 180°$,

$$2\cot^2 3\theta = 7 \operatorname{cosec} 3\theta - 5$$

Give your answers in degrees to 1 decimal place. (7)

(Total 7 marks)

Question 7

The point P is the point on the curve $x = 2\tan\left(y + \frac{\pi}{12}\right)$ with y-coordinate $\frac{\pi}{4}$.

Find an equation of the normal to the curve at P. (7)

(Total 7 marks)

Question 8

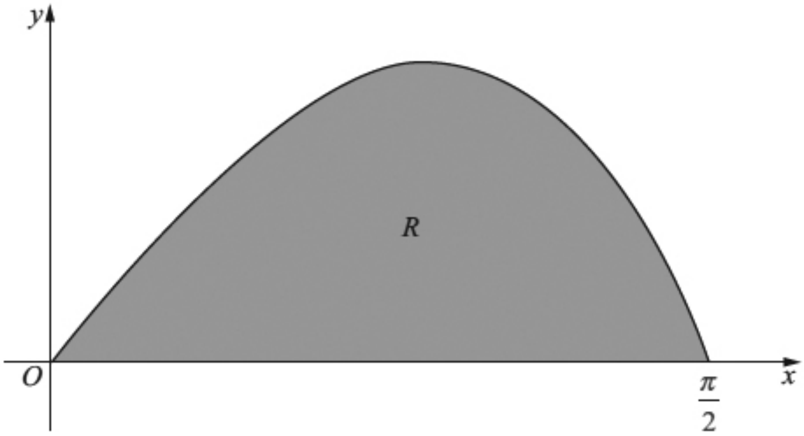

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \frac{2\sin 2x}{(1+\cos x)}$, $0 \leqslant x \leqslant \frac{\pi}{2}$.

The finite region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

The table below shows corresponding values of x and y for $y = \frac{2\sin 2x}{(1+\cos x)}$

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

(a) Complete the table above giving the missing value of y to 5 decimal places. (1)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving
your answer to 4 decimal places. (3)

(c) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2\sin 2x}{(1+\cos x)}\,dx = 4\ln(1+\cos x) - 4\cos x + k$$

where k is a constant. (5)

(d) Hence calculate the error of the estimate in part (b), giving your answer to 2 significant figures. (3)

(Total 12 marks)

Question 9

(a) Starting from the formulae for sin (*A* + *B*) and cos (*A* + *B*), prove that

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

(4)

(b) Deduce that

$$\tan\left(\theta+\frac{\pi}{6}\right)=\frac{1+\sqrt{3}\tan\theta}{\sqrt{3}-\tan\theta}$$

(3)

(c) Hence, or otherwise, solve, for $0\leqslant\theta\leqslant\pi$,

$$1+\sqrt{3}\tan\theta=(\sqrt{3}-\tan\theta)\tan(\pi-\theta)$$

Give your answers as multiples of π. (6)

(Total 13 marks)

Question 10

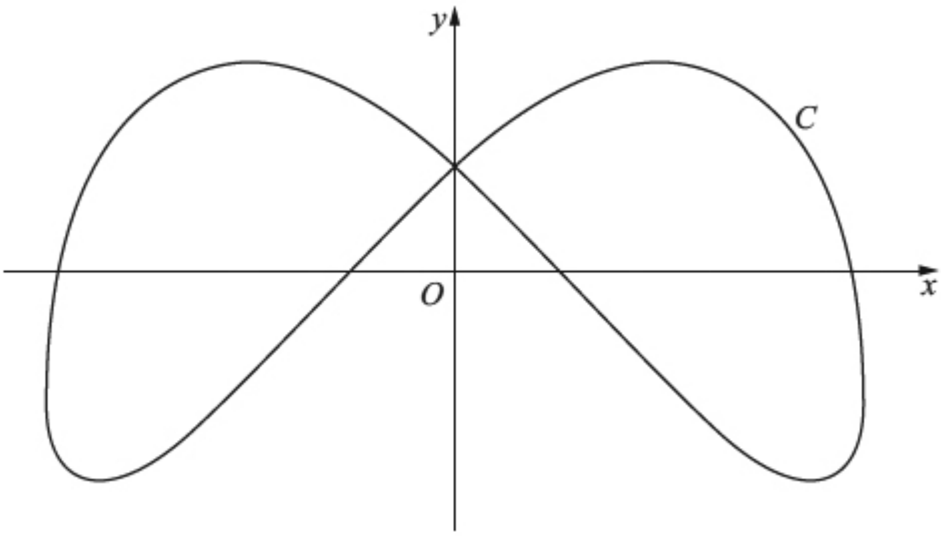

Figure 2

Figure 2 shows a sketch of the curve *C* with parametric equations

$$x=4\sin\left(t+\frac{\pi}{6}\right),\qquad y=3\cos 2t,\qquad 0\leqslant t<2\pi$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of *t*. (3)

Find the coordinates of all the points on *C* where $\frac{dy}{dx}=0$ (5)

(Total 8 marks)

Question 11

(*a*) Express $\dfrac{1}{P(5-P)}$ in partial fractions. **(3)**

A team of conservationists is studying the population of meerkats on a nature reserve. The population is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{15}P(5-P), \qquad t \geq 0,$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that when $t = 0$, $P = 1$,

(*b*) solve the differential equation, giving your answer in the form,

$$P = \frac{a}{b + c\mathrm{e}^{-\frac{1}{3}t}}$$

where a, b and c are integers. **(8)**

(*c*) Hence show that the population cannot exceed 5000. **(1)**

TOTAL FOR PAPER IS 100 MARKS