Name:

Total Marks:

Pure Mathematics 2

Advanced Level

Practice Paper M13

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE A Level Specifications
- There are 10 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

A company, which is making 200 mobile phones each week, plans to increase its production.

The number of mobile phones produced is to be increased by 20 each week from 200 in week 1 to 220 in week 2, to 240 in week 3 and so on, until it is producing 600 in week N.

(a) Find the value of N. **(2)**

The company then plans to continue to make 600 mobile phones each week.

(b) Find the total number of mobile phones that will be made in the first 52 weeks starting from and including week 1. **(5)**

(Total 7 marks)

Question 2

(a) Use the binomial expansion to show that

$$\sqrt{\left(\frac{1+x}{1-x}\right)} \approx 1 + x + \frac{1}{2}x^2, \quad |x| < 1$$

(6)

(b) Substitute $x = \frac{1}{26}$ into

$$\sqrt{\left(\frac{1+x}{1-x}\right)} = 1 + x + \frac{1}{2}x^2$$

to obtain an approximation to $\sqrt{3}$

Give your answer in the form $\frac{a}{b}$ where a and b are integers. **(3)**

(Total 9 marks)

Question 3

The function f has domain $-2 \leq x \leq 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = \mathrm{f}(x)$ is shown in Figure 1.

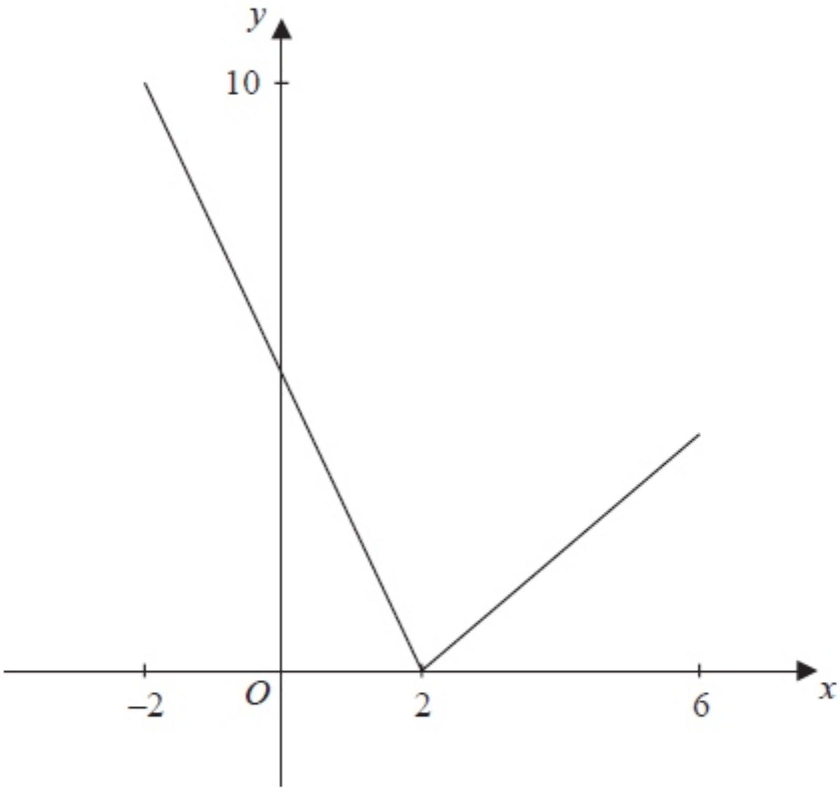

Figure 1

(a) Write down the range of f. **(1)**

(b) Find ff(0). **(2)**

The function g is defined by

$$\mathrm{g} : x \rightarrow \frac{4 + 3x}{5 - x}, \qquad x \in \mathbb{R}, \qquad x \neq 5$$

(c) Find $\mathrm{g}^{-1}(x)$ **(3)**

(d) Solve the equation $\mathrm{gf}(x) = 16$ **(5)**

(Total 11 marks)

Question 4

$$x^2 + 4xy + y^2 + 27 = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y. **(4)**

A point Q lies on the curve.

The tangent to the curve at Q is parallel to the y-axis.

Given that the x coordinate of Q is negative,

(b) use your answer to part (a) to find the coordinates of Q. **(7)**

(Total 11 marks)

Question 5

Given that

$$2\cos(x + 50)° = \sin(x + 40)°$$

(a) Show, without using a calculator, that

$$\tan x° = \frac{1}{3}\tan 40°$$

(4)

(b) Hence solve, for $0 \leq \theta < 360$,

$$2\cos(2\theta + 50)° = \sin(2\theta + 40)°$$

giving your answers to 1 decimal place. **(4)**

(Total 8 marks)

Question 6

A curve C has parametric equations

$$x = 2\sin t, \quad y = 1 - \cos 2t, \quad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ at the point where $t = \frac{\pi}{6}$ **(4)**

(b) Find a cartesian equation for C in the form

$$y = f(x), \; -k \leq x \leq k,$$

stating the value of the constant k. **(3)**

(c) Write down the range of $f(x)$. **(2)**

(Total 9 marks)

Question 7

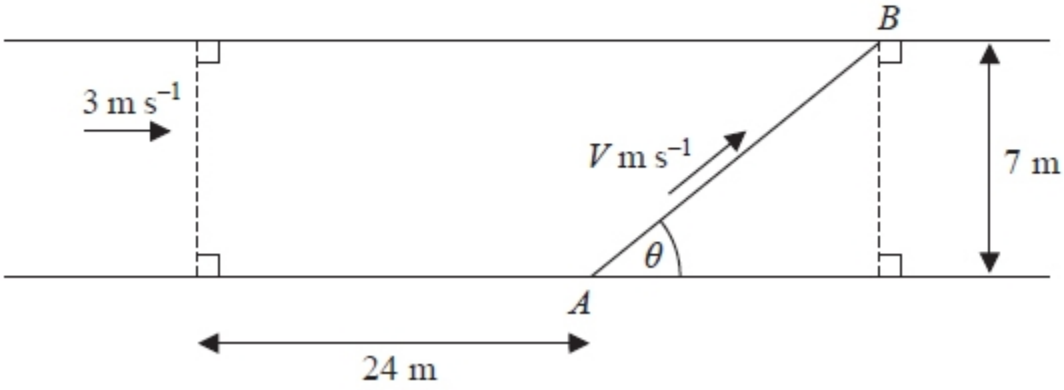

Figure 2

Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 m s^{-1}.

Kate is 24 m ahead of John when she starts to cross the road from the fixed point *A*.

John passes her as she reaches the other side of the road at a variable point *B*, as shown in Figure 2.

Kate's speed is *V*m s^{-1} and she moves in a straight line, which makes an angle θ, $0 < \theta < 150°$, with the edge of the road, as shown in Figure 2.

You may assume that *V* is given by the formula

$$V = \frac{21}{24\sin\theta + 7\cos\theta}, \qquad 0 < \theta < 150°$$

(a) Express $24\sin\theta + 7\cos\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants and where $R > 0$ and $0 < \alpha$ α decimal places. **(3)**

Given that θ varies,

(b) find the minimum value of *V*. **(2)**

Given that Kate's speed has the value found in part (b),

(c) find the distance *AB*. **(3)**

Given instead that Kate's speed is 1.68 m s^{-1},

(d) find the two possible values of the angle θ, given that $0 < \theta < 150°$. **(6)**

(Total 14 marks)

Question 8

(a) Use the substitution $x = u^2$, $u > 0$, to show that

$$\int \frac{1}{x(2\sqrt{x} - 1)}\,\mathrm{d}x = \int \frac{2}{u(2u - 1)}\,\mathrm{d}u$$

(3)

(b) Hence show that

$$\int_1^9 \frac{1}{x(2\sqrt{x} - 1)}\,\mathrm{d}x = 2\ln\left(\frac{a}{b}\right)$$

where *a* and *b* are integers to be determined. (7)

(Total 10 marks)

Question 9

Given that

$$x = \sec^2 3y, \qquad 0 < y < \frac{\pi}{6}$$

(a) find $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of *y*. (2)

(b) Hence show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$

(4)

(c) Find an expression for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ in terms of *x*. Give your answer in its simplest form. (4)

(Total 10 marks)

Question 10

Water is being heated in a kettle. At time t seconds, the temperature of the water is θ °C.
The rate of increase of the temperature of the water at any time t is modelled by the differential equation

$$\frac{d\theta}{dt} = \lambda(120 - \theta), \qquad \theta \leqslant 100$$

where λ is a positive constant.

Given that $\theta = 20$ when $t = 0$,

(a) solve this differential equation to show that

$$\theta = 120 - 100e^{-\lambda t}$$ **(8)**

When the temperature of the water reaches 100 °C, the kettle switches off.

(b) Given that $\lambda = 0.01$, find the time, to the nearest second, when the kettle switches off. **(3)**

(Total 11 marks)

TOTAL FOR PAPER IS 100 MARKS