Name:

Total Marks:

Pure Mathematics 2

Advanced Level

Practice Paper M14

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE A Level Specifications
- There are 13 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

The curve C has equation $y = 2x^3 + 12x^2 - 24x - 3$

a Show that C is concave on the interval [–5, –3]. (3)

b Find the coordinates of the point of inflection. (3)

(Total 6 marks)

Question 2

The first term of a geometric series is 20 and the common ratio is $\frac{7}{8}$

The sum to infinity of the series is S_∞

(a) Find the value of S_∞ (2)

The sum to N terms of the series is S_N

(b) Find, to 1 decimal place, the value of S_{12} (2)

(c) Find the smallest value of N, for which

$$S_\infty - S_N < 0.5$$ (4)

(Total 8 marks)

Question 3

$$g(x) = \frac{x}{x+3} + \frac{3(2x+1)}{x^2+x-6}, \quad x > 3$$

(a) Show that $g(x) = \frac{x+1}{x-2}, \quad x > 3$ (4)

(b) Find the range of g. (2)

(c) Find the exact value of a for which $g(a) = g^{-1}(a)$. (4)

(Total 10 marks)

Question 4

Given that the binomial expansion of $(1 + kx)^{-4}$, $|kx| < 1$, is

$$1 - 6x + Ax^2 + \ldots$$

(a) find the value of the constant k,

(2)

(b) find the value of the constant A, giving your answer in its simplest form.

(3)

(Total 5 marks)

Question 5

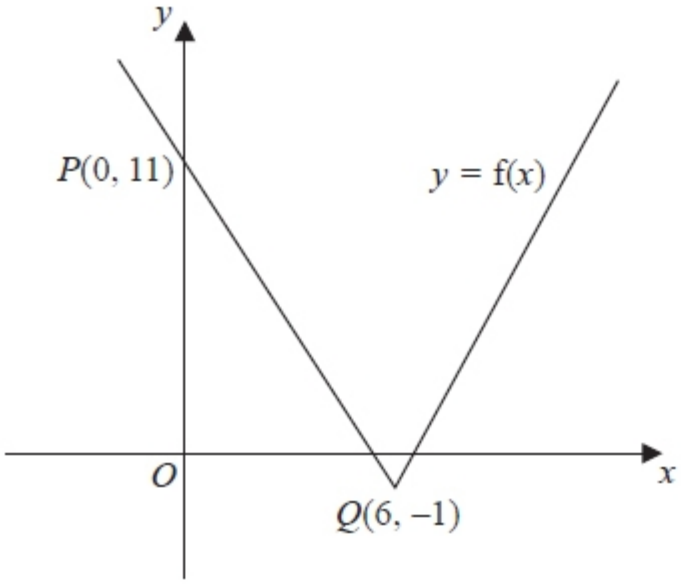

Figure 1

Figure 1 shows part of the graph with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $Q(6, -1)$.

The graph crosses the y-axis at the point $P(0, 11)$.

Sketch, on separate diagrams, the graphs of

(a) $y = |\mathrm{f}(x)|$ (2)

(b) $y = 2\mathrm{f}(-x) + 3$ (3)

On each diagram, show the coordinates of the points corresponding to P and Q.

Given that $\mathrm{f}(x) = a|x - b| - 1$, where a and b are constants,

(c) state the value of a and the value of b. (2)

(Total 7 marks)

Question 6

The curve C has equation $y = f(x)$ where

$$f(x) = \frac{4x+1}{x-2}, \quad x > 2$$

(a) Show that

$$f'(x) = \frac{-9}{(x-2)^2}$$ **(3)**

Given that P is a point on C such that $f'(x) = -1$,

(b) find the coordinates of P. **(2)**

(Total 5 marks)

Question 7

The curve C has equation $x = 8y \tan 2y$

The point P has coordinates $\left(\pi, \frac{\pi}{8}\right)$

(a) Verify that P lies on C. **(1)**

(b) Find the equation of the tangent to C at P in the form $ay = x + b$, where the constants a and b are to be found in terms of π. **(6)**

(Total 7 marks)

Question 8

(a) Show that

$$\text{cosec } 2x + \cot 2x = \cot x, \quad x \neq 90n°, \quad n \in \mathbb{R}.$$ **(5)**

(b) Hence, or otherwise, solve, for $0 \leq \theta < 180°$,

$$\text{cosec } (4\theta + 10°) + \cot (4\theta + 10°) = \sqrt{3}$$

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*) **(5)**

(Total 10 marks)

Question 9

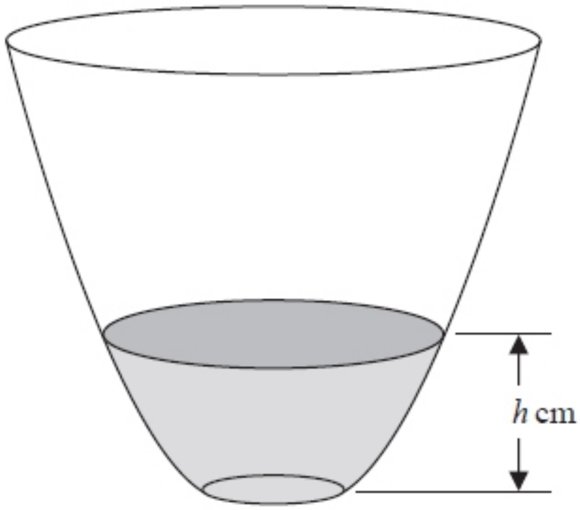

Figure 2

A vase with a circular cross-section is shown in Figure 2. Water is flowing into the vase.

When the depth of the water is h cm, the volume of water V cm^3 is given by

$$V = 4\pi h(h + 4), \qquad 0 \leq h \leq 25$$

Water flows into the vase at a constant rate of 80π cm^{3}s^{-1}

Find the rate of change of the depth of the water, in cm s^{-1}, when $h = 6$ **(5)**

(Total 5 marks)

Question 10

A rare species of primrose is being studied. The population, P, of primroses at time t years after the study started is modelled by the equation

$$P = \frac{800e^{0.1t}}{1 + 3e^{0.1t}}, \qquad t \geq 0, t \in \mathbb{R}.$$

(a) Calculate the number of primroses at the start of the study. **(2)**

(b) Find the exact value of t when $P = 250$, giving your answer in the form $a \ln(b)$ where a and b are integers. **(4)**

(c) Find the exact value of ${}^{dP}\!/_{dt}$ when $t = 10$. Give your answer in its simplest form. **(4)**

(d) Explain why the population of primroses can never be 270 **(1)**

(Total 11 marks)

Question 11

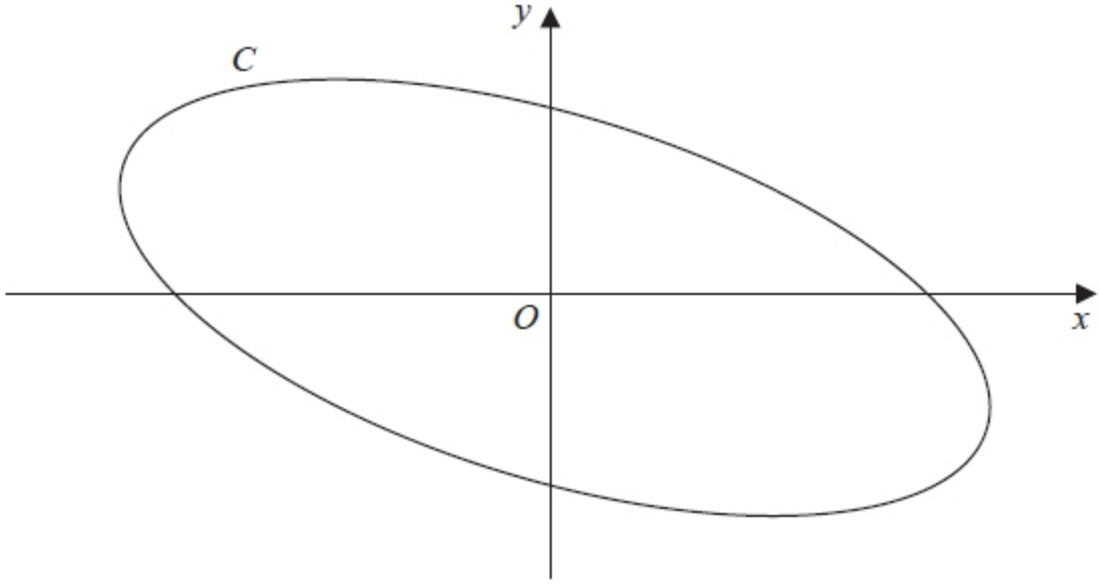

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right), \qquad y = 2\sin t, \qquad 0 \leq t < 2\pi$$

(a) Show that

$$x + y = \sqrt{3}\cos t$$ **(3)**

(b) Show that a cartesian equation of C is

$$(x + y)^2 + ay^2 = b$$

where a and b are integers to be determined. **(2)**

(Total 5 marks)

Question 12

(a) Express 2 sin θ − 4 cos θ in the form R sin($\theta - \alpha$), where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the value of α to 3 decimal places.

(3)

$$H(\theta) = 4 + 5(2\sin 3\theta - 4\cos 3\theta)^2$$

Find

(b) (i) the maximum value of H(θ),

(ii) the smallest value of θ, for $0 \le \theta < \pi$, at which this maximum value occurs.

(3)

Find

(c) (i) the minimum value of H(θ),

(ii) the largest value of θ, for $0 \le \theta < \pi$, at which this minimum value occurs.

(3)

(Total 9 marks)

Question 13

(i) Find

$$\int x\, e^{4x}\, dx$$

(3)

(ii) Find

$$\int \frac{8}{(2x-1)^3}\, dx, \qquad x > \tfrac{1}{2}$$

(2)

Given that $y = \frac{\pi}{6}$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y$$

(7)

(Total 12 marks)

TOTAL FOR PAPER IS 100 MARKS