Name:

Total Marks:

Pure Mathematics 2

Advanced Level

Practice Paper M15

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE A Level Specifications
- There are 12 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

Use proof by contradiction to show that there is no greatest positive rational number. **(4)**

(Total for question = 4 marks)

Question 2

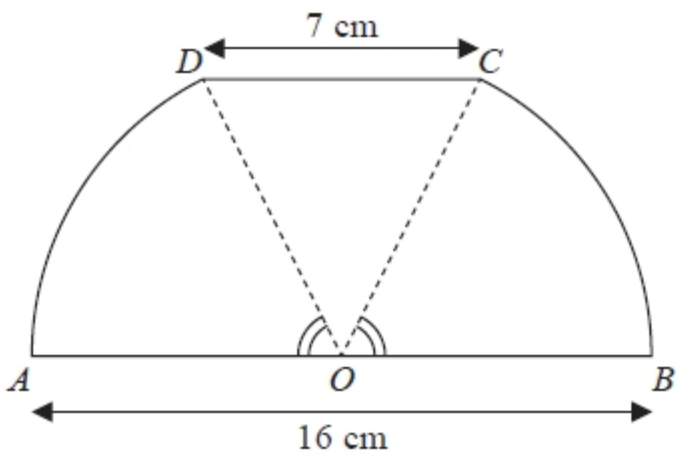

Figure 1

Figure 1 shows a sketch of a design for a scraper blade. The blade *AOBCDA* consists of an isosceles triangle *COD* joined along its equal sides to sectors *OBC* and *ODA* of a circle with centre *O* and radius 8 cm. Angles *AOD* and *BOC* are equal. *AOB* is a straight line and is parallel to the line *DC*. *DC* has length 7 cm.

(a) Show that the angle *COD* is 0.906 radians, correct to 3 significant figures. **(2)**

(b) Find the perimeter of *AOBCDA*, giving your answer to 3 significant figures. **(3)**

(c) Find the area of *AOBCDA*, giving your answer to 3 significant figures. **(3)**

(Total for question = 8 marks)

Question 3

Given that k is a **negative** constant and that the function f(x) is defined by

$$f(x) = 2 - \frac{(x-5k)(x-k)}{x^2 - 3kx + 2k^2}, \quad x \geqslant 0$$

(a) show that $f(x) = \dfrac{x+k}{x-2k}$ **(3)**

(b) Hence find f' (x) , giving your answer in its simplest form. **(3)**

(c) State, with a reason, whether f(x) is an increasing or a decreasing function.
Justify your answer. **(2)**

(Total for question = 9 marks)

Question 4

(a) Find the binomial expansion of

$$(4+5x)^{\frac{1}{2}}, \quad |x| < \frac{4}{5}$$

in ascending powers of x, up to and including the term in x^2.
Give each coefficient in its simplest form. **(5)**

(b) Find the exact value of $(4 + 5x)^{½}$ when $x = \frac{1}{10}$

Give your answer in the form $k\sqrt{2}$, where k is a constant to be determined. **(1)**

(c) Substitute $x = \frac{1}{10}$ into your binomial expansion from part (a) and hence find an approximate value for $\sqrt{2}$

Give your answer in the form $\frac{p}{q}$ where p and q are integers. **(2)**

(Total for question = 8 marks)

Question 5

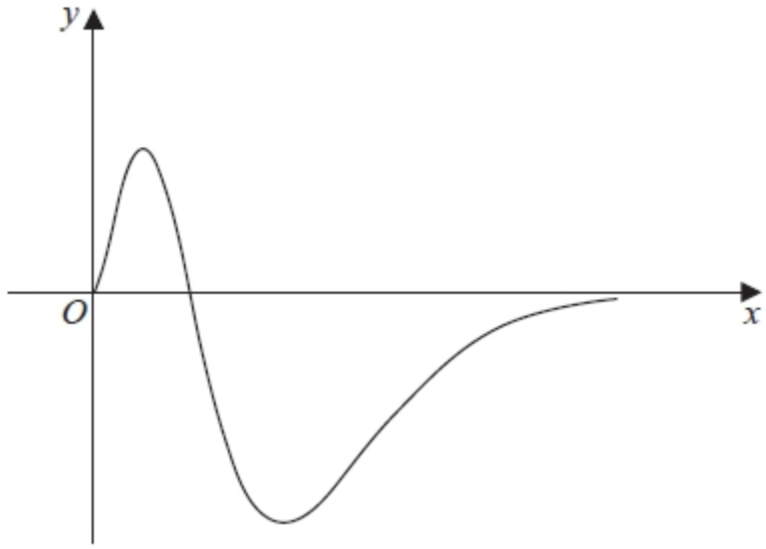

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$g(x) = x^2(1 - x)e^{-2x}, \quad x \geqslant 0$$

(a) Show that g' (x) = f(x)e^{-2x}, where f(x) is a cubic function to be found. **(3)**

(b) Hence find the range of g. **(6)**

(c) State a reason why the function g$^{-1}(x)$ does not exist. **(1)**

(Total for question = 10 marks)

Question 6

Given that

$$f(x) = 2e^x - 5, \quad x \in \mathbb{R}$$

(a) sketch, on separate diagrams, the curve with equation

(i) $y = f(x)$

(ii) $y = |f(x)|$

On each diagram, show the coordinates of each point at which the curve meets or cuts the axes.

On each diagram state the equation of the asymptote. **(6)**

(b) Deduce the set of values of *x* for which $f(x) = |f(x)|$ **(1)**

(c) Find the exact solutions of the equation $|f(x)| = 2$ **(3)**

(Total for question = 10 marks)

Question 7

The point *P* lies on the curve with equation

$$x = (4y - \sin 2y)^2$$

Given that *P* has (*x*, *y*) coordinates $\left(p, \frac{\pi}{2}\right)$, where *p* is a constant,

(a) find the exact value of *p*. **(1)**

The tangent to the curve at *P* cuts the *y*-axis at the point *A*.

(b) Use calculus to find the coordinates of *A*. **(6)**

(Total for question = 7 marks)

Question 8

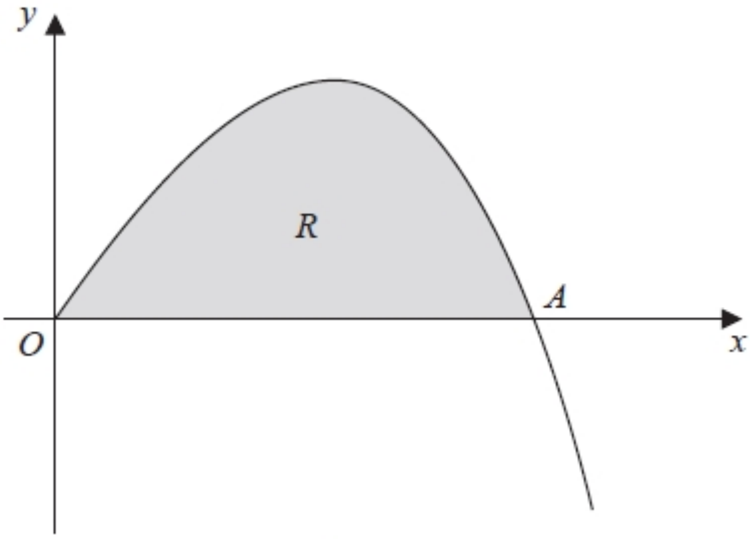

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = 4x - xe^{\frac{1}{2}x},\ x \geqslant 0$

The curve meets the *x*-axis at the origin *O* and cuts the *x*-axis at the point *A*.

(a) Find, in terms of ln2, the *x* coordinate of the point *A*. **(2)**

(b) Find

$$\int xe^{\frac{1}{2}x}\,dx$$

(3)

The finite region *R*, shown shaded in Figure 1, is bounded by the *x*-axis and the curve with equation

$$y = 4x - xe^{\frac{1}{2}x},\ x \geqslant 0$$

(c) Find, by integration, the exact value for the area of *R*.
Give your answer in terms of ln2 **(3)**

(Total for question = 8 marks)

Question 9

A curve *C* has parametric equations

$$x = 4t + 3, \quad y = 4t + 8 + \frac{5}{2t}, \quad t \neq 0$$

(a) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point on *C* where $t = 2$, giving your answer as a fraction in its simplest form. **(3)**

(b) Show that the cartesian equation of the curve *C* can be written in the form

$$y = \frac{x^2 + ax + b}{x - 3}, \quad x \neq 3$$

where *a* and *b* are integers to be determined. **(3)**

(Total for question = 6 marks)

Question 10

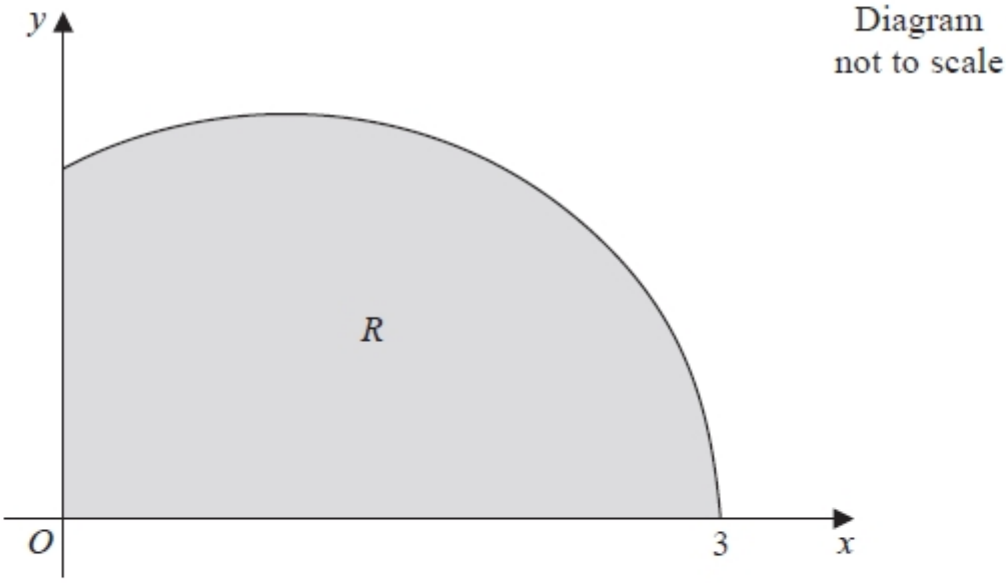

Figure 2

Figure 2 shows a sketch of the curve with equation $y = \sqrt{(3-x)(x+1)}, \quad 0 \leqslant x \leqslant 3$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis, and the y-axis.

(a) Use the substitution $x = 1 + 2\sin\theta$ to show that

$$\int_0^3 \sqrt{(3-x)(x+1)}\,\mathrm{d}x = k\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\,\mathrm{d}\theta$$

where k is a constant to be determined. **(5)**

(b) Hence find, by integration, the exact area of R. **(3)**

(Total for question = 8 marks)

Question 11

(a) Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A}, \qquad A \neq \frac{(2n+1)\pi}{4},\ n \in \mathbb{Z}$$

(5)

(b) Hence solve, for $0 \leqslant \theta < 2\pi$,

$$\sec 2\theta + \tan 2\theta = \frac{1}{2}$$

Give your answers to 3 decimal places. **(4)**

(Total for question = 9 marks)

Question 12

(a) Express $\dfrac{2}{P(P-2)}$ in partial fractions. **(3)**

A team of biologists is studying a population of a particular species of animal.

The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{2}P(P-2)\cos 2t,\ \ t \geqslant 0$$

where *P* is the population in thousands, and *t* is the time measured in years since the start of the study.

Given that *P* = 3 when *t* = 0,

(b) solve this differential equation to show that

$$P = \frac{6}{3 - e^{\frac{1}{2}\sin 2t}}$$

(7)

(c) find the time taken for the population to reach 4000 for the first time.

Give your answer in years to 3 significant figures. **(3)**

(Total for question = 13 marks)

TOTAL FOR PAPER IS 100 MARKS