Differentiation, Tangents & Normal - Edexcel Past Exam Questions

1. Given that $y = 5x^3 + 7x + 3$, find

(*a*) $\frac{\mathrm{d}y}{\mathrm{d}x}$, **(3)**

(*b*) $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$. **(1)**

Jan 05 Q2

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2. The curve C has equation $y = 4x^2 + \frac{5-x}{x}$, $x \neq 0$. The point P on C has x-coordinate 1.

(*a*) Show that the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at P is 3. **(5)**

(*b*) Find an equation of the tangent to C at P. **(3)**

This tangent meets the x-axis at the point $(k, 0)$.

(*c*) Find the value of k. **(2)**

Jan 05 Q7

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3. Given that $y = 6x - \frac{4}{x^2}$, $x \neq 0$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ **(2)**

June 05 Q2

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4. The curve C has equation $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$.

The point P has coordinates $(3, 0)$.

(*a*) Show that P lies on C. **(1)**

(*b*) Find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants. **(5)**

Another point Q also lies on C. The tangent to C at Q is parallel to the tangent to C at P.

(*c*) Find the coordinates of Q. **(5)**

June 05 Q10

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5. Given that $y = 2x^2 - \frac{6}{x^3}$, $x \neq 0$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ **(2)**

Jan 06 Q4

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6. Differentiate with respect to x

(a) $x^4 + 6\sqrt{x}$, **(3)**

(b) $\dfrac{(x+4)^2}{x}$. **(4)**

June 06 Q5

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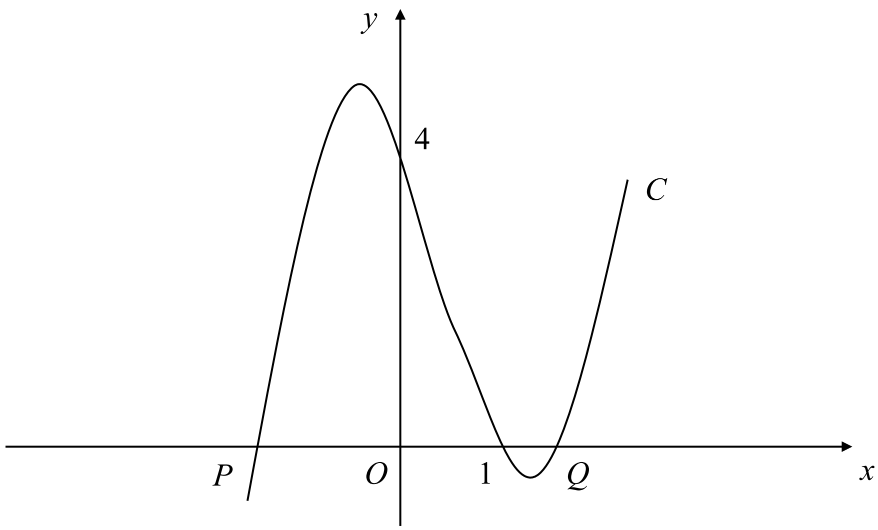

Figure 2 shows part of the curve C with equation

$$y = (x - 1)(x^2 - 4).$$

The curve cuts the x-axis at the points P, $(1, 0)$ and Q, as shown in Figure 2.

(a) Write down the x-coordinate of P and the x-coordinate of Q. **(2)**

(b) Show that $\dfrac{dy}{dx} = 3x^2 - 2x - 4$. **(3)**

(c) Show that $y = x + 7$ is an equation of the tangent to C at the point $(-1, 6)$. **(2)**

The tangent to C at the point R is parallel to the tangent at the point $(-1, 6)$.

(d) Find the exact coordinates of R. **(5)**

Jan 06 Q9

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8. Given that $y = 4x^3 - 1 + 2x^{\frac{1}{2}}$, $x > 0$, find $\dfrac{dy}{dx}$. **(4)**

Jan 07 Q1

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9. The curve C has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2$, $x > 0$.

(a) Find an expression for $\dfrac{dy}{dx}$. **(3)**

(b) Show that the point $P(4, 8)$ lies on C. **(1)**

(c) Show that an equation of the normal to C at the point P is

$$3y = x + 20.$$ **(4)**

The normal to C at P cuts the x-axis at the point Q.

(d) Find the length PQ, giving your answer in a simplified surd form. **(3)**

Jan 07 Q8

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10. Given that $y = 3x^2 + 4\sqrt{x}$, $x > 0$, find

(a) $\dfrac{dy}{dx}$, **(2)**

(b) $\dfrac{d^2y}{dx^2}$, **(2)**

June 07 Q3

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11. The curve C has equation $y = x^2(x - 6) + \dfrac{4}{x}$, $x > 0$.

The points P and Q lie on C and have x-coordinates 1 and 2 respectively.

(a) Show that the length of PQ is $\sqrt{170}$. **(4)**

(b) Show that the tangents to C at P and Q are parallel. **(5)**

(c) Find an equation for the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

June 07 Q10

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12. (a) Write $\dfrac{2\sqrt{x}+3}{x}$ in the form $2x^p + 3x^q$, where p and q are constants. **(2)**

Given that $y = 5x - 7 + \dfrac{2\sqrt{x}+3}{x}$, $x > 0$,

(*b*) find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, simplifying the coefficient of each term. **(4)**

Jan 08 Q5

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13. The curve C has equation

$$y = (x + 3)(x - 1)^2.$$

(*a*) Sketch C, showing clearly the coordinates of the points where the curve meets the coordinate axes.

(4)

(*b*) Show that the equation of C can be written in the form

$$y = x^3 + x^2 - 5x + k,$$

where k is a positive integer, and state the value of k. **(2)**

There are two points on C where the gradient of the tangent to C is equal to 3.

(*c*) Find the x-coordinates of these two points. **(6)**

Jan 08 Q10

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14. $\mathrm{f}(x) = 3x + x^3$, $\quad x > 0$.

(*a*) Differentiate to find $\mathrm{f}'(x)$. **(2)**

Given that $\mathrm{f}'(x) = 15$,

(*b*) find the value of x. **(3)**

June 08 Q4

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15. The curve C has equation $y = kx^3 - x^2 + x - 5$, where k is a constant.

(a) Find $\dfrac{dy}{dx}$. **(2)**

The point A with x-coordinate $-\frac{1}{2}$ lies on C. The tangent to C at A is parallel to the line with equation $2y - 7x + 1 = 0$.

Find

(b) the value of k, **(4)**

(c) the value of the y-coordinate of A. **(2)**

June 08 Q9

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16. Given that $\dfrac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $2x^p - x^q$,

(a) write down the value of p and the value of q. **(2)**

Given that $y = 5x^4 - 3 + \dfrac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$,

(b) find $\dfrac{dy}{dx}$, simplifying the coefficient of each term. **(4)**

Jan 09 Q6

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17. The curve C has equation

$$y = 9 - 4x - \frac{8}{x}, \quad x > 0.$$

The point P on C has x-coordinate equal to 2.

(a) Show that the equation of the tangent to C at the point P is $y = 1 - 2x$.

(6)

(b) Find an equation of the normal to C at the point P.

(3)

The tangent at P meets the x-axis at A and the normal at P meets the x-axis at B.

(c) Find the area of the triangle APB. **(4)**

Jan 09 Q11

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18. Given that $y = 2x^3 + \dfrac{3}{x^2}$, $x \neq 0$, find $\dfrac{dy}{dx}$ **(3)**

June 09 Q3

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19. $f(x) = \dfrac{(3 - 4\sqrt{x})^2}{\sqrt{x}}$, $x > 0$.

(a) Show that $f(x) = 9x^{-\frac{1}{2}} + Ax^{\frac{1}{2}} + B$, where A and B are constants to be found. **(3)**

(b) Find $f'(x)$. **(3)**

(c) Evaluate $f'(9)$. **(2)**

June 09 Q9

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20. The curve C has equation

$$y = x^3 - 2x^2 - x + 9, \quad x > 0.$$

The point P has coordinates (2, 7).

(a) Show that P lies on C. **(1)**

(b) Find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants. **(5)**

The point Q also lies on C.

Given that the tangent to C at Q is perpendicular to the tangent to C at P,

(c) show that the x-coordinate of Q is $\frac{1}{3}(2 + \sqrt{6})$. **(5)**

June 09 Q11

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21. Given that $y = x^4 + x^{\frac{1}{3}} + 3$, find $\dfrac{dy}{dx}$. **(3)**

Jan 10 Q1

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22. The curve C has equation

$$y = \frac{(x+3)(x-8)}{x}, \quad x > 0.$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in its simplest form. **(4)**

(b) Find an equation of the tangent to C at the point where $x = 2$. **(4)**

Jan 10 Q6

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23. Given that

$$y = 8x^3 - 4\sqrt{x} + \frac{3x^2 + 2}{x}, \qquad x > 0,$$

find $\frac{\mathrm{d}y}{\mathrm{d}x}$. **(6)**

June 10 Q7

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24. The curve C has equation

$$y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30, \qquad x > 0.$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. **(4)**

(b) Show that the point $P(4, -8)$ lies on C. **(2)**

(c) Find an equation of the normal to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(6)**

Jan 11 Q11

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25. Given that $y = 2x^5 + 7 + \frac{1}{x^3}$, $x \neq 0$, find, in their simplest form, $\frac{\mathrm{d}y}{\mathrm{d}x}$ **(3)**

June 11 Q2

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26. The curve C has equation

$$y = (x + 1)(x + 3)^2.$$

(a) Sketch C, showing the coordinates of the points at which C meets the axes. **(4)**

(b) Show that $\dfrac{dy}{dx} = 3x^2 + 14x + 15$. **(3)**

The point A, with x-coordinate -5, lies on C.

(c) Find the equation of the tangent to C at A, giving your answer in the form $y = mx + c$, where m and c are constants. **(4)**

Another point B also lies on C. The tangents to C at A and B are parallel.

(d) Find the x-coordinate of B. **(3)**

June 11 Q2