

- 1** Express each of the following in the form $\log_a b = c$.
- a** $10^3 = 1000$ **b** $3^4 = 81$ **c** $256 = 2^8$ **d** $7^0 = 1$
- e** $3^{-3} = \frac{1}{27}$ **f** $32^{-\frac{1}{5}} = \frac{1}{2}$ **g** $19^1 = 19$ **h** $216 = 36^{\frac{3}{2}}$
- 2** Express each of the following using index notation.
- a** $\log_5 125 = 3$ **b** $\log_2 16 = 4$ **c** $5 = \log_{10} 100\,000$ **d** $\log_{23} 1 = 0$
- e** $\frac{1}{2} = \log_9 3$ **f** $\lg 0.01 = -2$ **g** $\log_2 \frac{1}{8} = -3$ **h** $\log_6 6 = 1$
- 3** Without using a calculator, find the exact value of
- a** $\log_7 49$ **b** $\log_4 64$ **c** $\log_2 128$ **d** $\log_3 27$
- e** $\log_5 625$ **f** $\log_8 8$ **g** $\log_7 1$ **h** $\log_{15} \frac{1}{15}$
- i** $\log_3 \frac{1}{9}$ **j** $\lg 0.001$ **k** $\log_{16} 2$ **l** $\log_4 8$
- m** $\log_9 243$ **n** $\log_{100} 0.001$ **o** $\log_{25} 125$ **p** $\log_{27} \frac{1}{9}$
- 4** Without using a calculator, find the exact value of x in each case.
- a** $\log_5 25 = x$ **b** $\log_2 x = 6$ **c** $\log_x 64 = 3$ **d** $\lg x = -3$
- e** $\log_x 16 = \frac{2}{3}$ **f** $\log_5 1 = x$ **g** $\log_x 9 = 1$ **h** $\lg 10^{12} = x$
- i** $2 \log_x 7 = 1$ **j** $\log_4 x = 1.5$ **k** $\log_x 0.1 = -\frac{1}{3}$ **l** $3 \log_8 x + 1 = 0$
- 5** Express in the form $\log_a n$
- a** $\log_a 4 + \log_a 7$ **b** $\log_a 10 - \log_a 5$ **c** $2 \log_a 6$
- d** $\log_a 9 - \log_a \frac{1}{3}$ **e** $\frac{1}{2} \log_a 25 + 2 \log_a 3$ **f** $\log_a 48 - 3 \log_a 2 - \frac{1}{2} \log_a 9$
- 6** Express in the form $p \log_q x$
- a** $\log_q x^5$ **b** $\frac{1}{2} \log_q x^{15}$ **c** $\log_q \frac{1}{x}$ **d** $\log_q \sqrt[3]{x}$
- e** $4 \log_q \frac{1}{\sqrt{x}}$ **f** $\log_q x^2 + \log_q x^5$ **g** $\log_q \frac{1}{x^2} + \log_q \frac{1}{x^3}$ **h** $3 \log_q x^2 - \frac{1}{2} \log_q x^4$
- 7** Express in the form $\lg n$
- a** $\lg 5 + \lg 4$ **b** $\lg 12 - \lg 6$ **c** $3 \lg 2$ **d** $4 \lg 3 - \lg 9$
- e** $\frac{1}{2} \lg 16 - \frac{1}{5} \lg 32$ **f** $1 + \lg 11$ **g** $\lg \frac{1}{50} + 2$ **h** $3 - \lg 40$
- 8** Without using a calculator, evaluate
- a** $\log_3 54 - \log_3 2$ **b** $\log_5 20 + \log_5 1.25$ **c** $\log_2 16 + \log_3 27$
- d** $\log_6 24 + \log_6 9$ **e** $\log_3 12 - \log_3 4$ **f** $\log_4 18 - \log_4 9$
- g** $\log_9 4 + \log_9 0.25$ **h** $2 \lg 2 + \lg 25$ **i** $\frac{1}{3} \log_3 8 - \log_3 18$
- j** $\frac{1}{3} \log_4 64 + 2 \log_5 25$ **k** $\frac{1}{2} \log_5 (1\frac{9}{16}) + 2 \log_5 10$ **l** $\log_3 5 - 2 \log_3 6 - \log_3 (3\frac{3}{4})$

1 Express in the form $p \log_{10} a + q \log_{10} b$

a $\log_{10} ab$

b $\log_{10} ab^7$

c $\log_{10} \frac{a^3}{b}$

d $\log_{10} a \sqrt{b}$

e $\log_{10} (ab)^2$

f $\log_{10} \frac{1}{ab}$

g $\log_{10} \sqrt{a^3 b^5}$

h $3 \log_{10} \frac{a^2}{\sqrt[3]{b}}$

2 Given that $y = \log_q 8$, express each of the following in terms of y .

a $\log_q 64$

b $\log_q 2$

c $\log_q \frac{16}{q}$

d $\log_q 4q^3$

3 Given that $a = \lg 2$ and $b = \lg 3$, express each of the following in terms of a and b .

a $\lg 18$

b $\lg 96$

c $\lg \frac{9}{16}$

d $\lg 6 - \lg 8$

e $\lg \sqrt{6}$

f $\frac{3}{2} \lg 16 + \frac{1}{2} \lg 81$

g $4 \lg 3 - 3 \lg 6$

h $\lg 60 + \lg 20 - 2$

4 Without using a calculator, evaluate

a $\frac{1}{3} \log_5 1000 - \frac{1}{2} \log_5 4$

b $2 \log_{12} 4 + \frac{1}{2} \log_{12} 81$

c $\log_4 12 + \log_4 \frac{2}{3}$

d $\frac{\log_7 81}{\log_7 3}$

e $3 \log_{27} 12 - 2 \log_{27} 72$

f $\frac{\log_{11} 25}{\log_{11} \frac{1}{5}}$

5 Solve each equation, giving your answers correct to 3 significant figures.

a $\log_3 x = 1.8$

b $\log_5 x = -0.3$

c $\log_8 (x - 3) = 2.1$

d $\log_4 (\frac{1}{2}x + 1) = 3.2$

e $15 - \log_2 3y = 9.7$

f $\log_6 (1 - 5t) + 4.2 = 3.6$

6 Express in the form $\log_2 [f(x)]$

a $5 \log_2 x$

b $\log_2 x + \log_2 (x + 4)$

c $2 \log_2 x + \frac{1}{5} \log_2 x^5$

d $3 \log_2 (x - 2) - 4 \log_2 x$

e $\log_2 (x^2 - 1) - \log_2 (x + 1)$

f $\log_2 x - \frac{1}{2} \log_2 x^4 + \frac{1}{3} \log_2 x^2$

7 Solve each of the following equations.

a $\log_3 x + \log_3 5 = \log_3 (2x + 3)$

b $\log_9 x + \log_9 10 = \frac{3}{2}$

c $\log_4 x - \log_4 (x - 1) = \log_4 3 + \frac{1}{2}$

d $\log_5 5x - \log_5 (x + 2) = \log_5 (x + 6) - \log_5 x$

e $2 \log_6 x = \log_6 (2x - 5) + \log_6 5$

f $\log_7 4x = \log_7 \frac{1}{x-6} + 1$

8 Solve each pair of simultaneous equations.

a $\log_x y = 2$

$xy = 27$

b $\log_5 x - 2 \log_5 y = \log_5 2$

$x + y^2 = 12$

c $\log_2 x = 3 - 2 \log_2 y$

d $\log_y x = \frac{3}{2}$

$\log_y 32 = -\frac{5}{2}$

$x^{\frac{1}{3}} + 3y^{\frac{1}{2}} = 20$

e $\log_a x + \log_a 3 = \frac{1}{2} \log_a y$

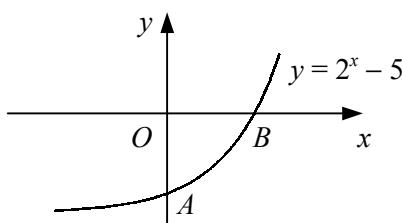
f $\log_{10} y + 2 \log_{10} x = 3$

$3x + y = 20$

$\log_2 y - \log_2 x = 3$

- 1 Find, to 3 significant figures, the value of
a $\log_{10} 60$ **b** $\log_{10} 6$ **c** $\log_{10} 253$ **d** $\log_{10} 0.4$
- 2 Solve each equation, giving your answers to 2 decimal places.
a $10^x = 14$ **b** $2(10^x) - 8 = 0$ **c** $10^{3x} = 49$
d $10^{x-4} = 23$ **e** $10^{2x+1} = 130$ **f** $100^x - 5 = 0$
- 3 Show that $\log_a b = \frac{\log_c b}{\log_c a}$, where a , b and c are positive constants.
- 4 Find, to 3 significant figures, the value of
a $\log_2 7$ **b** $\log_{20} 172$ **c** $\log_5 49$ **d** $\log_9 4$
- 5 Solve each equation, giving your answers to 3 significant figures.
a $3^x = 12$ **b** $2^x = 0.7$ **c** $8^{-y} = 3$ **d** $4^{\frac{1}{2}x} - 0.3 = 0$
e $5^{t+3} = 24$ **f** $16 - 3^{4+x} = 0$ **g** $7^{2x+4} = 12$ **h** $5(2^{3x+1}) = 62$
i $4^{2-3x} = 32.7$ **j** $5^x = 6^{x-1}$ **k** $7^{y+2} = 9^{y+1}$ **l** $4^{5-x} = 11^{2x-1}$
m $4^{\frac{1}{2}x+3} - 5^{1-2x} = 0$ **n** $2^{3y-2} = 3^{2y+5}$ **o** $7^{2x+5} = 7(11^{3x-4})$ **p** $3^{2x} = 3^{x-1} \times 2^{4+x}$
- 6 Solve the following equations, giving your answers to 2 decimal places where appropriate.
a $2^{2x} + 2^x - 6 = 0$ **b** $3^{2x} - 5(3^x) + 4 = 0$ **c** $5^{2x} + 12 = 8(5^x)$
d $2(4^x) + 3(4^{-x}) = 7$ **e** $2^{2y+1} + 7(2^y) - 15 = 0$ **f** $3^{2x+1} - 17(3^x) + 10 = 0$
g $25^t + 5^{t+1} - 24 = 0$ **h** $3^{2x+1} + 15 = 2(3^{x+2})$ **i** $3(16^x) - 4^{x+2} + 5 = 0$
- 7 Sketch each pair of curves on the same diagram, showing the coordinates of any points of intersection with the coordinate axes.
a $y = 2^x$ **b** $y = 3^x$ **c** $y = 4^x$ **d** $y = 2^x$
 $y = 5^x$ $y = (\frac{1}{3})^x$ $y = 4^x - 1$ $y = 2^{x+3}$
- 8 A curve has the equation $y = 2 + a^x$ where a is a constant and $a > 1$.
a Sketch the curve, showing the coordinates of any points of intersection with the coordinate axes and the equations of any asymptotes.
Given also that the curve passes through the point $(3, 29)$,
b find the value of a .

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The diagram shows the curve with equation $y = 2^x - 5$ which intersects the coordinate axes at the points A and B. Find the length AB correct to 3 significant figures.