

1 Factorise

a  $x^2 + 4x + 3$

b  $x^2 + 7x + 10$

c  $y^2 - 3y + 2$

d  $x^2 - 6x + 9$

e  $y^2 - y - 2$

f  $a^2 + 2a - 8$

g  $x^2 - 1$

h  $p^2 + 9p + 14$

i  $x^2 - 2x - 15$

j  $16 - 10m + m^2$

k  $t^2 + 3t - 18$

l  $y^2 - 13y + 40$

m  $r^2 - 16$

n  $y^2 - 2y - 63$

o  $121 + 22a + a^2$

p  $x^2 + 6x - 72$

q  $26 - 15x + x^2$

r  $s^2 + 23s + 120$

s  $p^2 + 14p - 51$

t  $m^2 - m - 90$

2 Factorise

a  $2x^2 + 3x + 1$

b  $2 + 7p + 3p^2$

c  $2y^2 - 5y + 3$

d  $2 - m - m^2$

e  $3r^2 - 2r - 1$

f  $5 - 19y - 4y^2$

g  $4 - 13a + 3a^2$

h  $5x^2 - 8x - 4$

i  $4x^2 + 8x + 3$

j  $9s^2 - 6s + 1$

k  $4m^2 - 25$

l  $2 - y - 6y^2$

m  $4u^2 + 17u + 4$

n  $6p^2 + 5p - 4$

o  $8x^2 + 19x + 6$

p  $12r^2 + 8r - 15$

3 Using factorisation, solve each equation.

a  $x^2 - 4x + 3 = 0$

b  $x^2 + 6x + 8 = 0$

c  $x^2 + 4x - 5 = 0$

d  $x^2 - 7x = 8$

e  $x^2 - 25 = 0$

f  $x(x - 1) = 42$

g  $x^2 = 3x$

h  $27 + 12x + x^2 = 0$

i  $60 - 4x - x^2 = 0$

j  $5x + 14 = x^2$

k  $2x^2 - 3x + 1 = 0$

l  $x(x - 1) = 6(x - 2)$

m  $3x^2 + 11x = 4$

n  $x(2x - 3) = 5$

o  $6 + 23x - 4x^2 = 0$

p  $6x^2 + 10 = 19x$

q  $4x^2 + 4x + 1 = 0$

r  $3(x^2 + 4) = 13x$

s  $(2x + 5)^2 = 5 - x$

t  $3x(2x - 7) = 2(7x + 3)$

4 Factorise fully

a  $2y^2 - 10y + 12$

b  $x^3 + x^2 - 2x$

c  $p^3 - 4p$

d  $3m^3 + 21m^2 + 18m$

e  $a^4 + 4a^2 + 3$

f  $t^4 + 3t^2 - 10$

g  $12 + 20x - 8x^2$

h  $6r^2 - 9r - 42$

i  $6x^3 - 26x^2 + 8x$

j  $y^4 + 3y^3 - 18y^2$

k  $m^4 - 1$

l  $p^5 - 4p^3 + 4p$

5 Sketch each curve showing the coordinates of any points of intersection with the coordinate axes.

a  $y = x^2 - 3x + 2$

b  $y = x^2 + 5x + 6$

c  $y = x^2 - 9$

d  $y = x^2 - 2x$

e  $y = x^2 - 10x + 25$

f  $y = 2x^2 - 14x + 20$

g  $y = -x^2 + 5x - 4$

h  $y = 2 + x - x^2$

i  $y = 2x^2 - 3x + 1$

j  $y = 2x^2 + 13x + 6$

k  $y = 3 - 8x + 4x^2$

l  $y = 2 + 7x - 4x^2$

m  $y = 5x^2 - 17x + 6$

n  $y = -6x^2 + 7x - 2$

o  $y = 6x^2 + x - 5$

6 Solve each of the following equations.

a  $x - 5 + \frac{4}{x} = 0$

b  $x - \frac{10}{x} = 3$

c  $2x^3 - x^2 - 3x = 0$

d  $x^2(10 - x^2) = 9$

e  $\frac{5}{x^2} + \frac{4}{x} - 1 = 0$

f  $\frac{x-6}{x-4} = x$

g  $x + 5 = \frac{3}{x+3}$

h  $x^2 - \frac{4}{x^2} = 3$

i  $4x^4 + 7x^2 = 2$

j  $\frac{2x}{3-x} = \frac{1}{x+2}$

k  $\frac{2x+1}{x+3} = \frac{2}{x}$

l  $\frac{7}{x+2} - 3x = 2$

- 1 Express in the form  $(x + a)^2 + b$
- |                            |                                  |                                   |   |
|----------------------------|----------------------------------|-----------------------------------|---|
| <b>a</b> $x^2 + 2x + 4$    | <b>b</b> $x^2 - 2x + 4$          | <b>c</b> $x^2 - 4x + 1$           | <b>d</b> $x^2 + 6x$                         |
| <b>e</b> $x^2 + 4x + 8$    | <b>f</b> $x^2 - 8x - 5$          | <b>g</b> $x^2 + 12x + 30$         | <b>h</b> $x^2 - 10x + 25$                   |
| <b>i</b> $x^2 + 6x - 9$    | <b>j</b> $18 - 4x + x^2$         | <b>k</b> $x^2 + 3x + 3$           | <b>l</b> $x^2 + x - 1$                      |
| <b>m</b> $x^2 - 18x + 100$ | <b>n</b> $x^2 - x - \frac{1}{2}$ | <b>o</b> $20 + 9x + x^2$          | <b>p</b> $x^2 - 7x - 2$                     |
| <b>q</b> $5 - 3x + x^2$    | <b>r</b> $x^2 - 11x + 37$        | <b>s</b> $x^2 + \frac{2}{3}x + 1$ | <b>t</b> $x^2 - \frac{1}{2}x - \frac{1}{4}$ |
- 2 Express in the form  $a(x + b)^2 + c$
- |                            |                           |                            |  |
|----------------------------|---------------------------|----------------------------|--|
| <b>a</b> $2x^2 + 4x + 3$   | <b>b</b> $2x^2 - 8x - 7$  | <b>c</b> $3 - 6x + 3x^2$   | <b>d</b> $4x^2 + 24x + 11$                             |
| <b>e</b> $-x^2 - 2x - 5$   | <b>f</b> $1 + 10x - x^2$  | <b>g</b> $2x^2 + 2x - 1$   | <b>h</b> $3x^2 - 9x + 5$                               |
| <b>i</b> $3x^2 - 24x + 48$ | <b>j</b> $3x^2 - 15x$     | <b>k</b> $70 + 40x + 5x^2$ | <b>l</b> $2x^2 + 5x + 2$                               |
| <b>m</b> $4x^2 + 6x - 7$   | <b>n</b> $-2x^2 + 4x - 1$ | <b>o</b> $4 - 2x - 3x^2$   | <b>p</b> $\frac{1}{3}x^2 + \frac{1}{2}x - \frac{1}{4}$ |
- 3 Solve each equation by completing the square, giving your answers as simply as possible in terms of surds where appropriate.
- |                             |                               |                             |                              |
|-----------------------------|-------------------------------|-----------------------------|------------------------------|
| <b>a</b> $y^2 - 4y + 2 = 0$ | <b>b</b> $p^2 + 2p - 2 = 0$   | <b>c</b> $x^2 - 6x + 4 = 0$ | <b>d</b> $7 + 10r + r^2 = 0$ |
| <b>e</b> $x^2 - 2x = 11$    | <b>f</b> $a^2 - 12a - 18 = 0$ | <b>g</b> $m^2 - 3m + 1 = 0$ | <b>h</b> $9 - 7t + t^2 = 0$  |
| <b>i</b> $u^2 + 7u = 44$    | <b>j</b> $2y^2 - 4y + 1 = 0$  | <b>k</b> $3p^2 + 18p = -23$ | <b>l</b> $2x^2 + 12x = 9$    |
| <b>m</b> $-m^2 + m + 1 = 0$ | <b>n</b> $4x^2 + 49 = 28x$    | <b>o</b> $1 - t - 3t^2 = 0$ | <b>p</b> $2a^2 - 7a + 4 = 0$ |
- 4 By completing the square, find the maximum or minimum value of  $y$  and the value of  $x$  for which this occurs. State whether your value of  $y$  is a maximum or a minimum in each case.
- |                               |                               |                               |
|-------------------------------|-------------------------------|-------------------------------|
| <b>a</b> $y = x^2 - 2x + 7$   | <b>b</b> $y = x^2 + 2x - 3$   | <b>c</b> $y = 1 - 6x + x^2$   |
| <b>d</b> $y = x^2 + 10x + 35$ | <b>e</b> $y = -x^2 + 4x + 4$  | <b>f</b> $y = x^2 + 3x - 2$   |
| <b>g</b> $y = 2x^2 + 8x + 5$  | <b>h</b> $y = -3x^2 + 6x$     | <b>i</b> $y = 7 - 5x - x^2$   |
| <b>j</b> $y = 4x^2 - 12x + 9$ | <b>k</b> $y = 4x^2 + 20x - 8$ | <b>l</b> $y = 17 - 2x - 2x^2$ |
- 5 Sketch each curve showing the exact coordinates of its turning point and the point where it crosses the  $y$ -axis.
- |                              |                                |                                |
|------------------------------|--------------------------------|--------------------------------|
| <b>a</b> $y = x^2 - 4x + 3$  | <b>b</b> $y = x^2 + 2x - 24$   | <b>c</b> $y = x^2 - 2x + 5$    |
| <b>d</b> $y = 30 + 8x + x^2$ | <b>e</b> $y = x^2 + 2x + 1$    | <b>f</b> $y = 8 + 2x - x^2$    |
| <b>g</b> $y = -x^2 + 8x - 7$ | <b>h</b> $y = -x^2 - 4x - 7$   | <b>i</b> $y = x^2 - 5x + 4$    |
| <b>j</b> $y = x^2 + 3x + 3$  | <b>k</b> $y = 3 + 8x + 4x^2$   | <b>l</b> $y = -2x^2 + 8x - 15$ |
| <b>m</b> $y = 1 - x - 2x^2$  | <b>n</b> $y = 25 - 20x + 4x^2$ | <b>o</b> $y = 3x^2 - 4x + 2$   |
- 6 **a** Express  $x^2 - 4\sqrt{2}x + 5$  in the form  $a(x + b)^2 + c$ .  
**b** Write down an equation of the line of symmetry of the curve  $y = x^2 + 4\sqrt{2}x + 5$ .
- 7  $f(x) \equiv x^2 + 2kx - 3$ .
- By completing the square, find the roots of the equation  $f(x) = 0$  in terms of the constant  $k$ .

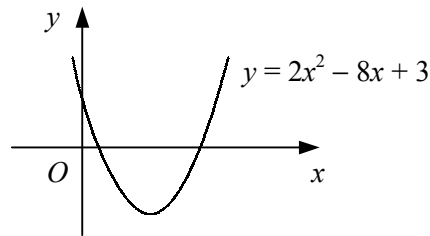
- 1 By completing the square, show that the roots of the equation  $ax^2 + bx + c = 0$  are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

- 2 Use the quadratic formula to solve each equation, giving your answers as simply as possible in terms of surds where appropriate.

|                                |                             |  |                                |
|--------------------------------|-----------------------------|--|--------------------------------|
| <b>a</b> $x^2 + 4x + 1 = 0$    | <b>b</b> $4 + 8t - t^2 = 0$ | <b>c</b> $y^2 - 20y + 91 = 0$                | <b>d</b> $r^2 + 2r - 7 = 0$    |
| <b>e</b> $6 + 18a + a^2 = 0$   | <b>f</b> $m(m - 5) = 5$     | <b>g</b> $x^2 + 11x + 27 = 0$                | <b>h</b> $2u^2 + 6u + 3 = 0$   |
| <b>i</b> $5 - y - y^2 = 0$     | <b>j</b> $2x^2 - 3x = 2$    | <b>k</b> $3p^2 + 7p + 1 = 0$                 | <b>l</b> $t^2 - 14t = 14$      |
| <b>m</b> $0.1r^2 + 1.4r = 0.9$ | <b>n</b> $6u^2 + 4u = 1$    | <b>o</b> $\frac{1}{2}y^2 - 3y = \frac{2}{3}$ | <b>p</b> $4x(x - 3) = 11 - 4x$ |

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The diagram shows the curve with equation  $y = 2x^2 - 8x + 3$ .

Find and simplify the exact coordinates of the points where the curve crosses the  $x$ -axis.

- 4 State the condition for which the roots of the equation  $ax^2 + bx + c = 0$  are

**a** real and distinct                      **b** real and equal                      **c** not real

- 5 Sketch the curve  $y = ax^2 + bx + c$  and the  $x$ -axis in the cases where

|                                      |                                      |
|--------------------------------------|--------------------------------------|
| <b>a</b> $a > 0$ and $b^2 - 4ac > 0$ | <b>b</b> $a < 0$ and $b^2 - 4ac < 0$ |
| <b>c</b> $a > 0$ and $b^2 - 4ac = 0$ | <b>d</b> $a < 0$ and $b^2 - 4ac > 0$ |

- 6 By evaluating the discriminant, determine whether the roots of each equation are real and distinct, real and equal or not real.

|                               |   |   |  |
|-------------------------------|---|---|--|
| <b>a</b> $x^2 + 2x - 7 = 0$   | <b>b</b> $x^2 + x + 3 = 0$                  | <b>c</b> $x^2 - 4x + 5 = 0$                     | <b>d</b> $x^2 - 6x + 3 = 0$                                |
| <b>e</b> $x^2 + 14x + 49 = 0$ | <b>f</b> $x^2 - 9x + 17 = 0$                | <b>g</b> $x^2 + 3x = 11$                        | <b>h</b> $2 + 3x + 2x^2 = 0$                               |
| <b>i</b> $5x^2 + 8x + 3 = 0$  | <b>j</b> $3x^2 - 7x + 5 = 0$                | <b>k</b> $9x^2 - 12x + 4 = 0$                   | <b>l</b> $13x^2 + 19x + 7 = 0$                             |
| <b>m</b> $4 - 11x + 8x^2 = 0$ | <b>n</b> $x^2 + \frac{2}{3}x = \frac{1}{4}$ | <b>o</b> $x^2 - \frac{3}{4}x + \frac{1}{8} = 0$ | <b>p</b> $\frac{2}{5}x^2 + \frac{3}{5}x + \frac{1}{3} = 0$ |

- 7 Find the value of the constant  $p$  such that the equation  $x^2 + x + p = 0$  has equal roots.

- 8 Given that  $q \neq 0$ , find the value of the constant  $q$  such that the equation  $x^2 + 2qx - q = 0$  has a repeated root.

- 9 Given that the  $x$ -axis is a tangent to the curve with the equation

$$y = x^2 + rx - 2x + 4,$$

find the two possible values of the constant  $r$ .