Name:...

Total Marks:...

GCSE (9-1) Grade 7 Solving Equations using Iteration

Instructions

Use **black** ink or ball-point pen.
Fill in the boxes at the top of this page with your name.
Answer **all** questions.
Answer the questions in the spaces provided
– there may be more space than you need.
Show all your working out

Information

The total mark for this paper is 55.
The marks for **each** question are shown in brackets.
Use this as a guide as to how much time to spend on each question.
Questions labelled with an **asterisk** (*) are ones where the quality of your written communication will be assessed

Advice

Read each question carefully before you start to answer it
Attempt every question
Check your answers if you have time at the end

1. (a) Show that the equation $x^2 - x - 7 = 0$ can be rearranged to give $x = \sqrt{x+7}$

……………………… **(1)**

This information can be used to obtain the iterative formula $x = \sqrt{x+7}$

(b) Starting with $x_0 = 4$ calculate the values of x_1, x_2 and x_3, giving all the figures on your calculator display.

……………………… **(3)**

(c) Find one solution of $x^2 - x - 7 = 0$ correct to 3 decimal places.

……………………… **(1)**

2. (a) Show that the equation $x^2 + x - 3 = 0$ can be rearranged to give $x = \dfrac{3}{1+x}$

.... **(1)**

(b) Use the iteration $x_{n+1} = \dfrac{3}{1+x_n}$, with $x_0 = 1$, to find the values of x_1, x_2 and x_3.

.... **(3)**

3. (a) Show that the equation $x^3 + 4x = 1$ has a solution between $x = 0$ and $x = 1$

……………………… **(2)**

(b) Show that the equation $x^3 + 4x = 1$ can be arranged to give $x = \frac{1}{4} - \frac{x^3}{4}$

……………………… **(1)**

(c) Starting with $x_0 = 0$ use the iteration formula $x_{n+1} = \frac{1}{4} - \frac{{x_n}^3}{4}$ twice, to find an estimate for the solution of $x^3 + 4x = 1$

……………………… **(3)**

4. Show that the equation $x^3 - 5x + 7 = 0$ can be rearranged to form the following iterative formulae.

(a) (i) $x = \frac{1}{5}\left(x^3 + 7\right)$

.... **(1)**

(ii) $x = \sqrt[3]{5x - 7}$

.... **(1)**

Use the following iterations where $x_0 = -2$ in each case to find the values of x_1, x_2 and x_3.

(b) (i) $x_{n+1} = \frac{1}{5}\left(x_n^{\ 3} + 7\right)$

.... **(3)**

(ii) $x_{n+1} = \sqrt[3]{5x_n - 7}$

.... **(3)**

5. (a) Show that the equation $x^3 - 10x = 30$ has a solution between x = 4 and x = 5

……………………. **(2)**

(b) Show that the equation $x^3 - 10x = 30$ can be arranged to give $x = \sqrt[3]{30 + 10x}$

……………………. **(1)**

(c) Starting with $x_0 = 4.5$ use the iteration formula $x_{n+1} = \sqrt[3]{30 + 10x_n}$ to find an estimate for the solution of $x^3 - 10x = 30$ to 2 decimal places.

……………………. **(3)**

6. (a) Complete the table for $y = x^3 - 5x + 4$

x	0	1	2	3	4
y		−8			40

(2)

(b) Between which two consecutive integers is there a solution to the equation $x^3 - 5x + 4 = 0$? Give a reason for your answer.

…………………….. **(2)**

(c) Show that the equation $x^3 - 5x + 4 = 0$ can be arranged to give $x = \sqrt[3]{5x+4}$

…………………….. **(1)**

(d) Starting with $x_0 = 2.5$ use the iteration formula $x_{n+1} = \sqrt[3]{5x_n + 4}$ to find an estimate for the solution of $x^3 - 5x + 4 = 0$ to 2 decimal places.

…………………….. **(3)**

7. (a) show that $x = 1 + \dfrac{11}{x - 3}$ is a rearrangement of the equation $x^2 - 4x - 8 = 0$

…………………….. **(2)**

(b) Use the iterative formula $x_{n+1} = 1 + \dfrac{11}{x_n - 3}$ together with a starting value of $x_0 = -2$

to obtain a solution of the equation $x^2 - 4x - 8 = 0$ correct to 1 decimal place.

…………………….. **(3)**

8. A computer uses the iteration $x_{n+1} = \dfrac{2}{x_n + 3}$ to find one solution for a quadratic equation.

(a) What quadratic equation is being solved?

.... **(2)**

(b) Find the positive solution of this equation.

.... **(3)**

9. (a) Show that $x = \dfrac{5}{x-1}$ can be rearranged to give $x^2 - x - 5 = 0$

……………………… **(1)**

(b) Use the iterative formula $x_{n+1} = \dfrac{5}{x_n - 1}$ together with a starting value of $x_0 = -2$

to obtain a solution of the equation $x^2 - x - 5 = 0$

……………………… **(3)**

(c) The other solution is approximately 3.

What happens if we use 3 as the starting value in the iteration $x_{n+1} = \dfrac{5}{x_n - 1}$?

……………………… **(2)**

(d) Show that $x = \dfrac{x^2 + 5}{2x - 1}$ can be rearranged to give $x^2 - x - 5 = 0$

……………………… **(1)**

(e) Use the iterative formula $x_{n+1} = \dfrac{{x_n}^2 + 5}{2x_n - 1}$ together with a starting value of $x_0 = -2$

to obtain the other solution of the equation $x^2 - x - 5 = 0$

……………………… **(3)**

TOTAL FOR PAPER: 55 MARKS