C1 COORDINATE GEOMETRY — Worksheet A

1 Find the gradient of the line segment joining each pair of points.

a (3, 1) and (5, 5) **b** (4, 7) and (10, 9) **c** (6, 1) and (2, 5) **d** (–2, 2) and (2, 8)

e (1, 3) and (7, –1) **f** (4, 5) and (–5, –7) **g** (–2, 0) and (0, –8) **h** (8, 6) and (–7, –2)

2 Write down the gradient and y-intercept of each line.

a $y = 4x - 1$ **b** $y = \frac{1}{3}x + 3$ **c** $y = 6 - x$ **d** $y = -2x - \frac{3}{5}$

3 Find the gradient and y-intercept of each line.

a $x + y + 3 = 0$ **b** $x - 2y - 6 = 0$ **c** $3x + 3y - 2 = 0$ **d** $4x - 5y + 1 = 0$

4 Write down, in the form $y - y_1 = m(x - x_1)$, the equation of the straight line with the given gradient which passes through the given point.

a gradient 2, point (4, 1) **b** gradient 5, point (2, –5)

c gradient –3, point (–1, 1) **d** gradient $\frac{1}{2}$, point (1, 6)

e gradient –2, point ($\frac{3}{4}$, $-\frac{1}{4}$) **f** gradient $-\frac{1}{5}$, point (–3, –7)

5 Find, in the form $y = mx + c$, the equation of the straight line with the given gradient which passes through the given point.

a gradient 3, point (1, 2) **b** gradient –1, point (5, 3)

c gradient 4, point (–2, –3) **d** gradient –2, point (–4, 1)

e gradient $\frac{1}{3}$, point (–3, 1) **f** gradient $-\frac{5}{6}$, point (9, –2)

6 Find, in each case, the equation of the straight line with gradient m which passes through the point P. Give your answers in the form $ax + by + c = 0$, where a, b and c are integers.

a $m = 1$, P (2, –4) **b** $m = \frac{1}{2}$, P (6, 1) **c** $m = -4$, P (–1, 8)

d $m = \frac{2}{5}$, P (–3, 5) **e** $m = -3$, P ($\frac{3}{2}$, $-\frac{1}{8}$) **f** $m = -\frac{3}{4}$, P ($\frac{2}{3}$, –7)

7 Find, in the form $y = mx + c$, the equation of the straight line passing through each pair of points.

a (0, 1) and (4, 13) **b** (2, 9) and (7, –1) **c** (–4, 3) and (2, 7)

d ($-\frac{1}{2}$, –2) and (2, 8) **e** (3, –2) and (18, –5) **f** (–3.2, 4) and (–2, 0.4)

8 Find, in the form $ax + by + c = 0$, where a, b and c are integers, the equation of the straight line which passes through each pair of points.

a (3, 0) and (5, 2) **b** (–1, 8) and (5, –4) **c** (–5, 3) and (7, 5)

d (–4, –1) and (8, –17) **e** (2, –1.5) and (7, 0) **f** ($-\frac{3}{5}$, $\frac{1}{10}$) and (3, 1)

9 The straight line l passes through the points A (–6, 8) and B (3, 2).

a Find an equation of the line l.

b Show that the point C (9, –2) lies on l.

10 The point M (k, $2k$) lies on the line with equation $x - 3y + 15 = 0$.

Find the value of the constant k.

11 The point with coordinates $(4p, p^2)$ lies on the line with equation $2x - 4y + 5 = 0$.

Find the two possible values of the constant p.

12 Find the coordinates of the points at which each straight line crosses the coordinate axes.

a $y = 2x + 5$ **b** $x - 3y + 6 = 0$ **c** $2x + 4y - 3 = 0$ **d** $5x - 3y = 10$

13 The line l has the equation $5x - 18y - 30 = 0$.

a Find the coordinates of the points A and B where the line l crosses the coordinate axes.

b Find the area of triangle OAB where O is the origin.

14 Find the exact length of the line segment joining each pair of points, giving your answers in terms of surds where appropriate.

a (1, 1) and (4, 5) **b** (0, 0) and (3, 1) **c** (1, −4) and (9, 11)

d (7, −8) and (−9, 4) **e** (3, 12) and (1, 7) **f** (−6, −3) and (2, −7)

15 The points P (22, 15), Q (−13, c) and R (k, 24) all lie on a circle, centre (2, 0).

Find the radius of the circle and the possible values of the constants c and k.

16 The points A (−2, 7) and B (6, −3) lie at either end of the diameter of a circle.

Find the area of the circle, giving your answer as an exact multiple of π.

17 The corners of a triangle are the points P (4, 7), Q (−2, 5) and R (3, −10).

a Find the length of each side of triangle PQR, giving your answers in terms of surds.

b Hence, verify that triangle PQR contains a right-angle.

c Find the area of triangle PQR.

18 Find the coordinates of the mid-point of the line segment joining each pair of points.

a (0, 2) and (8, 4) **b** (1, 9) and (7, 5) **c** (−5, 1) and (3, −7)

d (−5, −7) and (7, −5) **e** (1, 0) and (2, 9) **f** (−1, −2) and (4, −5)

g (2.4, 3.1) and (0.6, 4.5) **h** $(0, 3)$ and $(\frac{1}{2}, \frac{3}{2})$ **i** $(-\frac{5}{4}, 2)$ and $(-1, -\frac{3}{5})$

19 The straight line l_1 passes through the points P (−2, 1) and Q (4, −1).

a Find the equation of l_1 in the form $ax + by + c = 0$, where a, b and c are integers.

The straight line l_2 passes through the point R (2, 4) and through the mid-point of PQ.

b Find the equation of l_2 in the form $y = mx + c$.

20 Find the coordinates of the point of intersection of each pair of straight lines.

a $y = 2x + 1$, $y = 3x - 1$ **b** $y = x + 7$, $y = 4 - 2x$ **c** $y = 5x - 4$, $y = 3x - 1$

d $x + 2y - 4 = 0$, $3x - 2y + 4 = 0$ **e** $2x + y - 2 = 0$, $x + 3y + 9 = 0$ **f** $3x + 2y = 0$, $x + 4y - 2 = 0$

21 The line l with equation $x - 2y + 2 = 0$ crosses the y-axis at the point P. The line m with equation $3x + y - 15 = 0$ crosses the y-axis at the point Q and intersects l at the point R.

Find the area of triangle PQR.

1 The straight line l has gradient -3 and passes through the point with coordinates $(3, -5)$.

a Find an equation of the line l.

The straight line m passes through the points with coordinates $(-1, -2)$ and $(4, 1)$.

b Find the equation of m in the form $ax + by + c = 0$, where a, b and c are integers.

The lines l and m intersect at the point P.

c Find the coordinates of P.

2 Given that the straight line passing through the points A $(2, -3)$ and B $(7, k)$ has gradient $\frac{3}{2}$,

a find the value of k,

b show that the perpendicular bisector of AB has the equation $8x + 12y - 45 = 0$.

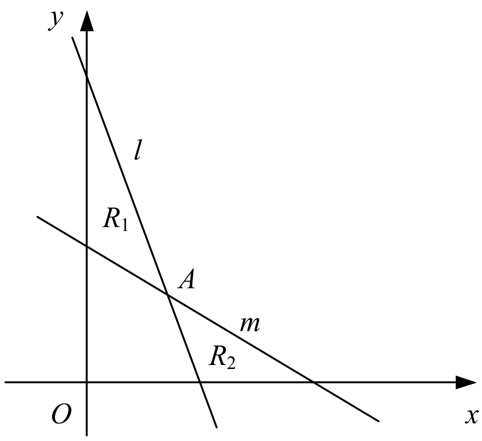
3 The vertices of a triangle are the points A $(5, 4)$, B $(-5, 8)$ and C $(1, 11)$.

a Find the equation of the straight line passing through A and B, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

b Find the coordinates of the point M, the mid-point of AC.

c Show that OM is perpendicular to AB, where O is the origin.

4



The line l with equation $3x + y - 9 = 0$ intersects the line m with equation $2x + 3y - 12 = 0$ at the point A as shown in the diagram above.

a Find, as exact fractions, the coordinates of the point A.

The region R_1 is bounded by l, m and the y-axis.

The region R_2 is bounded by l, m and the x-axis.

b Show that the ratio of the area of R_1 to the area of R_2 is $25 : 18$

5 The straight line l has the equation $2x + 5y + 10 = 0$.

The straight line m has the equation $6x - 5y - 30 = 0$.

a Sketch the lines l and m on the same set of axes showing the coordinates of any points at which each line crosses the coordinate axes.

The points where line m crosses the coordinate axes are denoted by A and B.

b Show that l passes through the mid-point of AB.

6 The straight line l passes through the points with coordinates $(-10, -4)$ and $(5, 4)$.

a Find the equation of l in the form $ax + by + c = 0$, where a, b and c are integers.

The line l crosses the coordinate axes at the points P and Q.

b Find, as an exact fraction, the area of triangle OPQ, where O is the origin.

c Show that the length of PQ is $2\frac{5}{6}$.

7 The point A has coordinates $(-8, 1)$ and the point B has coordinates $(-4, -5)$.

a Find the equation of the straight line passing through A and B, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

b Show that the distance of the mid-point of AB from the origin is $k\sqrt{10}$ where k is an integer to be found.

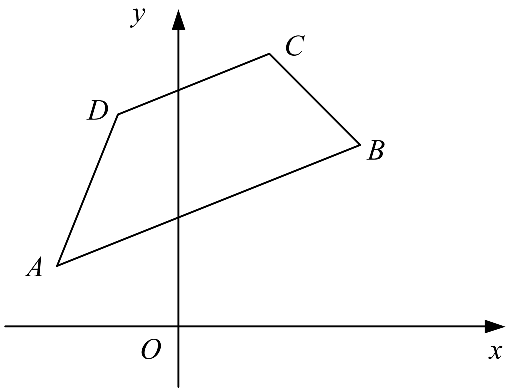
8 The straight line l_1 has gradient $\frac{1}{3}$ and passes through the point with coordinates $(-3, 4)$.

a Find the equation of l_1 in the form $ax + by + c = 0$, where a, b and c are integers.

The straight line l_2 has the equation $5x + py - 2 = 0$ and intersects l_1 at the point with coordinates $(q, 7)$.

b Find the values of the constants p and q.

9



The diagram shows trapezium $ABCD$ in which sides AB and DC are parallel. The point A has coordinates $(-4, 2)$ and the point B has coordinates $(6, 6)$.

a Find the equation of the straight line passing through A and B, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Given that the gradient of BC is -1,

b find an equation of the straight line passing through B and C.

Given also that the point D has coordinates $(-2, 7)$,

c find the coordinates of the point C,

d show that $\angle ACB = 90°$.

10 The straight line l passes through the points A $(1, 2\sqrt{3})$ and B $(\sqrt{3}, 6)$.

a Find the gradient of l in its simplest form.

b Show that l also passes through the origin.

c Show that the straight line which passes through A and is perpendicular to l has equation

$$x + 2\sqrt{3}\,y - 13 = 0.$$

1 Find the gradient of a straight line that is

a parallel to the line $y = 3 - 2x$,

b parallel to the line $2x - 5y + 1 = 0$,

c perpendicular to the line $y = 3x + 4$,

d perpendicular to the line $x + 2y - 3 = 0$.

2 Find, in the form $y = mx + c$, the equation of the straight line

a parallel to the line $y = 4x - 1$ which passes through the point with coordinates (1, 7),

b perpendicular to the line $y = 6 - x$ which passes through the point with coordinates (–4, 3),

c perpendicular to the line $x - 3y = 0$ which passes through the point with coordinates (–2, –2).

3 Find, in the form $ax + by + c = 0$, where a, b and c are integers, the equation of the straight line

a parallel to the line $2x - 3y + 5 = 0$ which passes through the point with coordinates (3, –1),

b perpendicular to the line $3x + 4y = 1$ which passes through the point with coordinates (2, 5),

c parallel to the line $3x + 5y = 6$ which passes through the point with coordinates (–4, –7).

4 Find, in the form $ax + by + c = 0$, where a, b and c are integers, the equation of the perpendicular bisector of the line segment joining each pair of points.

a (0, 4) and (8, 0)

b (2, 7) and (4, 1)

c (–3, –2) and (9, 1)

5 The vertices of a triangle are the points A (–6, –3), B (4, –1) and C (3, 4).

a Find the gradient of AB and the gradient of BC.

b Show that $\angle ABC = 90°$.

6 The line with equation $2x - 3y + 5 = 0$ is perpendicular to the line with equation $3x + ky - 1 = 0$.

Find the value of the constant k.

7 The straight line l passes through the points A (–5, 5) and B (1, 7).

a Find an equation of the line l. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

The point M is the mid-point of AB.

b Prove that the line OM, where O is the origin, is perpendicular to line l.

8 The straight line p has the equation $3x - 4y + 8 = 0$.

The straight line q is parallel to p and passes through the point with coordinates (8, 5).

a Find the equation of q in the form $y = mx + c$.

The straight line r is perpendicular to p and passes through the point with coordinates (–4, 6).

b Find the equation of r in the form $ax + by + c = 0$, where a, b and c are integers.

c Find the coordinates of the point where lines q and r intersect.

9 The straight line l_1 passes through the points with coordinates (–3, 7) and (1, –5).

a Find an equation of the line l_1 in the form $ax + by + c = 0$, where a, b and c are integers.

The line l_2 is perpendicular to l_1 and passes through the point with coordinates (4, 6).

b Find, in the form $k\sqrt{5}$, the distance from the origin of the point where l_1 and l_2 intersect.