Question 1: June 05 Q3

Question number	Scheme	Marks
	(a) Attempt to evaluate f(–4) or f(4)	M1
	$f(-4) = 2(-4)^3 + (-4)^2 - 25(-4) + 12 \quad (= 128 + 16 + 100 + 12) \; = 0$, so ….. is a factor.	A1 (2)
	(b) $(x+4)(2x^2 - 7x + 3)$	M1 A1
	……...$(2x-1)(x-3)$	M1 A1 (4)
		6
	(b) First M requires $(2x^2 + ax + b)$, $a \neq 0, b \neq 0$. Second M for the attempt to factorise the quadratic. <u>Alternative:</u> $(x+4)(2x^2 + ax + b) = 2x^3 + (8+a)x^2 + (4a+b)x + 4b = 0$, then compare coefficients to find <u>values</u> of a and b. [M1] $a = -7, \; b = 3$ [A1] <u>Alternative:</u> Factor theorem: Finding that $f\left(\frac{1}{2}\right) = 0, \therefore (2x-1)$ is a factor [M1, A1] n.b. Finding that $f\left(\frac{1}{2}\right) = 0, \therefore (x - \frac{1}{2})$ is a factor scores M1, A0 ,unless the factor 2 subsequently appears. Finding that $f(3) = 0, \therefore (x-3)$ is a factor [M1, A1]	

Question 2: Jan 06 Q1

Question number	Scheme		Marks	
	(a) $2+1-5+c=0$ or $-2+c=0$		M1	
	$\underline{c=2}$		A1	(2)
	(b) $\text{f}(x) = (x-1)(2x^2+3x-2)$	$(x-1)$	B1	
		division	M1	
	$= \dots \underline{(2x-1)(x+2)}$		M1 A1	(4)
	(c) $\text{f}\left(\frac{3}{2}\right) = 2\times\frac{27}{8}+\frac{9}{4}-\frac{15}{2}+c$		M1	
	Remainder $= c + 1.5$ $= \underline{3.5}$	ft their c	A1ft	(2)
				8

	(a)	M1	for evidence of substituting $x = 1$ leading to linear equation in c
	(b)	B1	for identifying $(x-1)$ as a factor
	1st	M1	for attempting to divide.
			Other factor must be at least $(2x^2 + \text{ one other term})$
	2nd	M1	for attempting to factorise a quadratic resulting from attempted division
		A1	for just $(2x-1)(x+2)$.
	(c)	M1	for attempting $\text{f}(\pm\frac{3}{2})$. If not implied by $1.5 + c$, we must see some substitution of $\pm\frac{3}{2}$.
		A1	follow through their c only, but it must be a number.

Question 3: June 06 Q4

Question number	Scheme	Marks
	(a) $f(-2) = 2(-2)^3 + 3(-2)^2 - 29(-2) - 60$ M: Attempt f(2) or f(–2)	M1
	$= -16 + 12 + 58 - 60 = -6$	A1 (2)
	(b) $f(-3) = 2(-3)^3 + 3(-3)^2 - 29(-3) - 60$ M: Attempt f(3) or f(–3)	M1
	$(= -54 + 27 + 87 - 60)$ $= 0 \ \therefore (x+3)$ is a factor	A1 (2)
	(c) $(x+3)(2x^2 - 3x - 20)$	M1 A1
	$= (x+3)(2x+5)(x-4)$	M1 A1 (4)
		8
	(a) Alternative (long division): Divide by $(x+2)$ to get $(2x^2 + ax + b)$, $a \neq 0, b \neq 0$. [M1] $(2x^2 - x - 27)$, remainder = – 6 [A1] (b) A1 requires zero and a simple conclusion (even just a tick, or Q.E.D.). (c) First M requires division by $(x+3)$ to get $(2x^2 + ax + b)$, $a \neq 0, b \neq 0$. Second M for the attempt to factorise their quadratic. Usual rule: $(2x^2 + ax + b) = (2x + c)(x + d)$, where $\lvert cd \rvert = \lvert b \rvert$. Alternative (first 2 marks): $(x+3)(2x^2 + ax + b) = 2x^3 + (6+a)x^2 + (3a+b)x + 3b = 0$, then compare coefficients to find values of a and b. [M1] $a = -3, \ b = -20$ [A1] Alternative: Factor theorem: Finding that $f\left(-\frac{5}{2}\right) = 0 \ \therefore$ factor is, $(2x+5)$ [M1, A1] Finding that $f(4) = 0 \ \therefore$ factor is, $(x-4)$ [M1, A1] "Combining" all 3 factors is not required. If just one of these is found, score the first 2 marks M1 A1 M0 A0. Losing a factor of 2: $(x+3)\left(x + \frac{5}{2}\right)(x-4)$ scores M1 A1 M1 A0. Answer only, one sign wrong: e.g. $(x+3)(2x-5)(x-4)$ scores M1 A1 M1 A0	

Question 4: Jan 07 Q5

Question Number	Scheme	Marks
(a)	$f(-2) = (-2)^3 + 4(-2)^2 + (-2) - 6$ $\{= -8 + 16 - 2 - 6\}$ $= 0, \therefore x + 2$ is a factor	M1 A1 (2)
(b)	$x^3 + 4x^2 + x - 6 = (x + 2)(x^2 + 2x - 3)$ $= (x + 2)(x + 3)(x - 1)$	M1, A1 M1, A1 (4)
(c)	$-3, -2, 1$	B1 (1) **(7)**

Question 5: June 07 Q2

Question number	Scheme	Marks	
	f(2) = 24 – 20 – 32 + 12 = –16 (M: Attempt f(2) or f(–2))	M1 A1	(2)
	(If continues to say ‘remainder = 16’, isw)		
	Answer must be seen in part (a), not part (b).		
	$(x+2)(3x^2-11x+6)$	M1 A1	
	$(x+2)(3x-2)(x-3)$	M1 A1	(4)
	(If continues to ‘solve an equation’, isw)		
			6
	(a) Answer only (if correct) scores both marks. (16 as ‘answer only’ is M0 A0). <u>Alternative (long division):</u> Divide by $(x-2)$ to get $(3x^2+ax+b)$, $a \neq 0, b \neq 0$. [M1] $(3x^2+x-14)$, and – 16 seen. [A1] (If continues to say ‘remainder = 16’, isw) (b) First M requires division by $(x+2)$ to get $(3x^2+ax+b)$, $a \neq 0, b \neq 0$. Second M for attempt to factorise <u>their</u> quadratic, even if wrongly obtained, perhaps with a remainder from their division. Usual rule: $(kx^2+ax+b)=(px+c)(qx+d)$, where $\lvert pq \rvert = \lvert k \rvert$ and $\lvert cd \rvert = \lvert b \rvert$. Just solving their quadratic by the formula is M0. “Combining” all 3 factors is <u>not</u> required. <u>Alternative (first 2 marks):</u> $(x+2)(3x^2+ax+b)=3x^3+(6+a)x^2+(2a+b)x+2b=0$, then compare coefficients to find <u>values</u> of a and b. [M1] $a=-11,\ b=6$ [A1] <u>Alternative:</u> Factor theorem: Finding that $f(3)=0$ $\therefore$ factor is, $(x-3)$ [M1, A1] Finding that $f\left(\frac{2}{3}\right)=0$ $\therefore$ factor is, $(3x-2)$ [M1, A1] If just one of these is found, score the <u>first 2 marks</u> M1 A1 M0 A0. <u>Losing a factor of 3:</u> $(x+2)\left(x-\frac{2}{3}\right)(x-3)$ scores M1 A1 M1 A0. <u>Answer only, one sign wrong:</u> e.g. $(x+2)(3x-2)(x+3)$ scores M1 A1 M1 A0.		

Question 6: June 08 Q1

<table>
<tr><th>Question number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td></td><td>(a) Attempt to find f(–4) or f(4). $\left(f(-4) = 2(-4)^3 - 3(-4)^2 - 39(-4) + 20\right)$
$(= -128 - 48 + 156 + 20) \quad = 0$, so $(x + 4)$ is a factor.

(b) $2x^3 - 3x^2 - 39x + 20 = (x+4)(2x^2 - 11x + 5)$

$.....(2x-1)(x-5)$ (The 3 brackets need not be written together)
or $.....\left(x - \frac{1}{2}\right)(2x - 10)$ or equivalent</td><td>M1
A1 (2)

M1 A1

M1 A1cso (4)

6</td></tr>
<tr><td></td><td>(a) Long division scores no marks in part (a). The <u>factor theorem</u> is required.
However, the first two marks in (b) can be earned from division seen in (a)…
… but if a different long division result is seen in (b), the work seen in (b) takes precedence for marks in (b).

A1 requires zero and a simple <u>conclusion</u> (even just a tick, or Q.E.D.), or may be scored by a <u>preamble</u>, e.g. 'If f(–4) = 0, (x + 4) is a factor…..'

(b) First M requires use of $(x + 4)$ to obtain $(2x^2 + ax + b)$, $a \neq 0, b \neq 0$, even with a remainder. Working need not be seen… this could be done 'by inspection'.
Second M for the attempt to factorise their three-term quadratic.
Usual rule: $(kx^2 + ax + b) = (px + c)(qx + d)$, where $|cd| = |b|$ and $|pq| = |k|$.
If 'solutions' appear before or after factorisation, ignore…
… but factors must be seen to score the second M mark.

<u>Alternative (first 2 marks):</u>
$(x+4)(2x^2 + ax + b) = 2x^3 + (8+a)x^2 + (4a+b)x + 4b = 0$, then compare coefficients to find <u>values</u> of a and b. [M1]
$a = -11, \; b = 5$ [A1]

<u>Alternative:</u>
Factor theorem: Finding that $f\left(\frac{1}{2}\right) = 0 \;\therefore$ factor is, $(2x - 1)$ [M1, A1]
Finding that $f(5) = 0 \;\therefore$ factor is, $(x - 5)$ [M1, A1]
"Combining" all 3 factors is <u>not</u> required.
If just one of these is found, score the <u>first 2 marks</u> M1 A1 M0 A0.

<u>Losing a factor of 2:</u> $(x+4)\left(x - \frac{1}{2}\right)(x-5)$ scores M1 A1 M1 A0.

<u>Answer only, one sign wrong:</u> e.g. $(x+4)(2x-1)(x+5)$ scores M1 A1 M1 A0</td><td></td></tr>
</table>

Question 7: June 10 Q2

Question Number	Scheme	Marks
	(a) Attempting to find $\mathrm{f}(3)$ or $\mathrm{f}(-3)$	M1
	$\mathrm{f}(3) = 3(3)^3 - 5(3)^2 - (58 \times 3) + 40 = 81 - 45 - 174 + 40 = -98$	A1 (2)
	(b) $\{3x^3 - 5x^2 - 58x + 40 = (x-5)\}\ (3x^2 + 10x - 8)$	M1 A1
	Attempt to <u>factorise</u> 3-term quadratic, or to use the quadratic formula (see general principles at beginning of scheme). This mark may be implied by the correct solutions to the quadratic.	M1
	$(3x-2)(x+4) = 0 \qquad x = ...$ <u>or</u> $x = \dfrac{-10 \pm \sqrt{100+96}}{6}$	A1 ft
	$\dfrac{2}{3}$ (or exact equiv.), -4, 5 (Allow 'implicit' solns, e.g. $\mathrm{f}(5) = 0$, etc.)	A1 (5)
	Completely correct solutions without working: full marks.	7

(a) <u>Alternative (long division):</u>

Divide by $(x-3)$ to get $(3x^2 + ax + b)$, $a \neq 0, b \neq 0$. [M1]

$(3x^2 + 4x - 46)$, and -98 seen. [A1]

(If continues to say 'remainder = 98', isw)

<u>'Grid' method</u>

$$\begin{array}{c|cccc} 3 & 3 & -5 & -58 & 40 \\ & 0 & 9 & 12 & -138 \\ \hline & 3 & 4 & -46 & -98 \end{array}$$

(b) 1st M requires use of $(x-5)$ to obtain $(3x^2 + ax + b)$, $a \neq 0, b \neq 0$.

(Working need not be seen… this could be done 'by inspection'.)

<u>'Grid' method</u>

$$\begin{array}{c|cccc} 3 & 3 & -5 & -58 & 40 \\ & 0 & 15 & 50 & -40 \\ \hline & 3 & 10 & -8 & 0 \end{array}$$

$(3x^2 + 10x - 8) \longleftarrow$

2nd M for the attempt to <u>factorise</u> their 3-term quadratic, or to solve it using the quadratic formula.

Factorisation: $(3x^2 + ax + b) = (3x + c)(x + d)$, where $|cd| = |b|$.

A1ft: Correct factors for their 3-term quadratic <u>followed by a solution</u> (at least one value, which might be incorrect), <u>or</u> numerically correct expression from the quadratic formula for their 3-term quadratic.

<u>Note</u> therefore that if the quadratic is correctly factorised but no solutions are given, the last 2 marks will be lost.

<u>Alternative (first 2 marks):</u>

$(x-5)(3x^2 + ax + b) = 3x^3 + (a-15)x^2 + (b-5a)x - 5b = 0$,

then compare coefficients to find <u>values</u> of a and b. [M1]

$a = 10$, $b = -8$ [A1]

Question 18: June 11 Q1

Question Number	Scheme		Marks
(a)	$\text{f}(x) = 2x^3 - 7x^2 - 5x + 4$ Remainder $= \text{f}(1) = 2 - 7 - 5 + 4 = -6$ $= -6$	Attempts $\text{f}(1)$ or $\text{f}(-1)$. -6	M1 A1 [2]
(b)	$\text{f}(-1) = 2(-1)^3 - 7(-1)^2 - 5(-1) + 4$ and so $(x+1)$ is a factor.	Attempts $\text{f}(-1)$. $\text{f}(-1) = 0$ with no sign or substitution errors **and for conclusion.**	M1 A1 [2]
(c)	$\text{f}(x) = \{(x+1)\}(2x^2 - 9x + 4)$ $= (x+1)(2x-1)(x-4)$ (**Note**: *Ignore the ePEN notation of (b) (should be (c)) for the final three marks in this part*).		M1 A1 dM1 A1 [4] 8

(a)	M1 for ***attempting*** either $\text{f}(1)$ or $\text{f}(-1)$. Can be implied. Only one slip permitted. M1 can also be given for an attempt (at least two "subtracting" processes) at long division to give a remainder which is independent of x. A1 can be given also for -6 seen at the bottom of long division working. Award A0 for a candidate who finds -6 but then states that the remainder is 6. Award M1A1 for -6 without any working.
(b)	M1: attempting only $\text{f}(-1)$. A1: must correctly show $\text{f}(-1) = 0$ and give a conclusion ***in part (b) only***. **Note**: Stating "hence factor" or "it is a factor" or a "tick" or "QED" is fine for the conclusion. **Note** also that a conclusion can be implied from a <u>preamble</u>, eg: "If $\text{f}(-1) = 0$, $(x+1)$ is a factor…." **Note: Long division scores no marks in part (b). The <u>factor theorem</u> is required.**
(c)	1st M1: Attempts long division or other method, to obtain $(2x^2 \pm ax \pm b)$, $a \neq 0$, even with a remainder. Working need not be seen as this could be done "by inspection." $(2x^2 \pm ax \pm b)$ must be seen ***in part (c) only***. Award 1st M0 if the quadratic factor is clearly found from dividing $\text{f}(x)$ by $(x-1)$. Eg. Some candidates use their $(2x^2 - 5x - 10)$ in part (c) found from applying a long division method in part (a). 1st A1: For seeing $(2x^2 - 9x + 4)$. 2nd dM1: Factorises a 3 term quadratic. (see rule for factorising a quadratic). This is dependent on the previous method mark being awarded. This mark can also be awarded if the candidate applies the quadratic formula correctly. 2nd A1: is cao and needs all three factors on one line. Ignore following work (such as a solution to a quadratic equation.) Note: Some candidates will go from $\{(x+1)\}(2x^2 - 9x + 4)$ to $\{x = -1\}, x = \frac{1}{2}, 4$, and not list all three factors. Award these responses M1A1M1A0. <u>**Alternative:**</u> 1st M1: For finding either $\text{f}(4) = 0$ or $\text{f}\left(\frac{1}{2}\right) = 0$. 1st A1: A second correct factor of usually $(x-4)$ or $(2x-1)$ found. Note that any one of the other correct factors found would imply the 1st M1 mark. 2nd dM1: For using two known factors to find the third factor, usually $(2x \pm 1)$. 2nd A1 for correct answer of $(x+1)(2x-1)(x-4)$. <u>**Alternative: (for the first two marks)**</u> 1st M1: Expands $(x+1)(2x^2 + ax + b)$ $\{$giving $2x^3 + (a+2)x^2 + (b+a)x + b$ $\}$ then compare coefficients to find <u>values</u> for a and b. 1st A1: $a = -9, b = 4$ <u>**Not dealing with a factor of 2:**</u> $(x+1)(x-\frac{1}{2})(x-4)$ or $(x+1)(x-\frac{1}{2})(2x-8)$ scores M1A1M1A0. <u>**Answer only, with one sign error:**</u> eg. $(x+1)(2x+1)(x-4)$ or $(x+1)(2x-1)(x+4)$ scores M1A1M1A0. (c) **Award M1A1M1A1 for Listing all three correct factors with no working.**