Circles - Edexcel Past Exam Questions MARK SCHEME

Question 1 June 05 Q8

Question number	Scheme	Marks
	(a) Centre (5, 0) (or $x = 5, y = 0$)	B1 B1 (2)
	(b) $(x \pm a)^2 \pm b \pm 9 + (y \pm c)^2 = 0 \Rightarrow r^2 = \ldots$ or $r = \ldots$, Radius = 4	M1, A1 (2)
	(c) (1, 0), (9, 0) Allow just $x = 1$, $x = 9$	B1ft, B1ft (2)
	(d) Gradient of $AT = -\frac{2}{7}$	B1
	$y = -\frac{2}{7}(x-5)$	M1 A1ft (3)
		9
	(a) (0, 5) scores B1 B0. (d) M1: Equation of straight line through centre, <u>any</u> gradient (except 0 or ∞) (The equation can be in any form). A1ft: Follow through from centre, but gradient must be $-\frac{2}{7}$.	

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Question 2 Jan 05 Q2

Question Number	Scheme	Marks
	(a) $\left(\frac{5+13}{2}, \frac{-1+11}{2}\right), = \underline{\underline{(9,5)}}$ (b) $r^2 = (9-5)^2 + (5--1)^2 (=52)$ Equation of circle: $(x-9)^2 + (y-5)^2 = 52$	M1, A1 (2) M1 M1, A1$\sqrt{}$A1 (4) (6)
	(a) M1 for some use of correct formula. can be implied Use of $\left(\frac{1}{2}(x_A - x_B), \frac{1}{2}(y_A - y_B)\right) \to (4,6)$ is M0A0 (b) M1 attempt to find r or r^2. $\sqrt{}$ their (9,5) $r = AB = \sqrt{208}$ is M0 2[nd] M1 for $(x-9)^2 + (y-5)^2 =$ constant. ($\sqrt{}$ their (9,5) A1$\sqrt{}$ for $(x-9)^2 + (y-5)^2 =$ their r^2. ($\sqrt{}$their (9,5) and r^2) A1 for $(x-9)^2 + (y-5)^2 = 52$ only.	

Question 3 Jan 06 Q3

	(a) $(AB)^2 = (4-3)^2 + (5)^2$ [= 26]	M1	
	$AB = \underline{\sqrt{26}}$	A1	(2)
	(b) $p = \left(\frac{4+3}{2}, \frac{5}{2}\right)$	M1	
	$= \underline{\left(\frac{7}{2}, \frac{5}{2}\right)}$	A1	(2)
	(c) $(x - x_p)^2 + (y - y_p)^2 = \left(\frac{AB}{2}\right)^2$ LHS	M1	
	RHS	M1	
	$(x-3.5)^2 + (y-2.5)^2 = 6.5$ oe	A1 c.a.o	(3)
			7
	(a) M1 for an expression for AB or AB^2 N.B. $(x_1 + x_2)^2 + \ldots$ is M0 (b) M1 for a full method for x_p (c) 1st M1 for using their x_p and y_p in LHS 2nd M1 for using their AB in RHS N.B. $x^2 + y^2 - 7x - 5y + 12 = 0$ scores, of course, 3/3 for part (c). Condone use of calculator approximations that lead to correct answer given.		

Question 4 June 06 Q7

Question number	Scheme	Marks	
	(a) Gradient of PQ is $-\frac{1}{3}$	B1	
	$y-2=-\frac{1}{3}(x-2)$ $\quad (3y+x=8)$	M1 A1	(3)
	(b) $y=1$: $\quad 3+x=8 \quad x=5$ $\quad$ (*)	B1	(1)
	(c) $("5"-2)^2+(1-2)^2$ $\quad$ M: Attempt PQ^2 or PQ	M1 A1	
	$(x-5)^2+(y-1)^2=10$ $\quad$ M: $(x\pm a)^2+(y\pm b)^2=k$	M1 A1	(4)
			8
	(a) M1: eqn. of a straight line through (2, 2) with any gradient except 3, 0 or ∞. Alternative: Using (2, 2) in $y=mx+c$ to find a value of c scores M1, but an equation (general or specific) must be seen. If the given value $x=5$ is used to find the gradient of PQ, maximum marks are (a) B0 M1 A1 (b) B0. (c) For the first M1, condone one slip, numerical or sign, inside a bracket. The first M1 can be scored if their x-coord. is used instead of 5. For the second M1, allow any equation in this form, with non-zero a, b and k.		

Question 5 Jan 07 Q3

Question Number	Scheme	Marks
	Centre $\left(\dfrac{-1+3}{2}, \dfrac{6+4}{2}\right)$, i.e. (1, 5)	M1, A1
	$r = \dfrac{\sqrt{(3-(-1))^2 + (6-4)^2}}{2}$ or $r^2 = (1-(-1))^2 + (5-4)^2$ or $r^2 = (3-1)^2 + (6-5)^2$ o.e.	M1
	$(x-1)^2 + (y-5)^2 = 5$	M1,A1,A1 **(6)**

Some use of correct formula in x or y coordinate. Can be implied. Use of $\left(\frac{1}{2}(x_A - x_B), \frac{1}{2}(y_A - y_B)\right) \to (-2, -1)$ or (2, 1) is M0 A0 but watch out for use of $x_A + \frac{1}{2}(x_A - x_B)$ etc which is okay.	M1
(1, 5) (5, 1) gains M1 A0.	A1
Correct method to find r or r^2 using given points or f.t. from their centre. Does not need to be simplified. Attempting radius $= \sqrt{\dfrac{(\text{diameter})^2}{2}}$ is an incorrect method, so M0. N.B. Be careful of labelling: candidates may not use d for diameter and r for radius. Labelling should be ignored. Simplification may be incorrect – mark awarded for correct method. Use of $\sqrt{(x_1 - x_2)^2 - (y_1 - y_2)^2}$ is M0.	M1
Write down $(x \pm a)^2 + (y \pm b)^2$ = any constant (a letter or a number). Numbers do not have to be substituted for a, b and if they are they can be wrong.	M1
LHS is $(x-1)^2 + (y-5)^2$. Ignore RHS.	A1
RHS is 5. Ignore subsequent working. Condone use of decimals that leads to exact 5.	A1
Or correct equivalents, e.g. $x^2 + y^2 - 2x - 10y + 21 = 0$.	

Alternative – note the order of the marks needed for ePEN.	
As above.	M1
As above.	A1
$x^2 + y^2$ + (constant)x + (constant)y + constant = 0. Numbers do not have to be substituted for the constants and if they are they can be wrong.	3rd M1
Attempt an appropriate substitution of the coordinates of their centre (i.e. working with coefficient of x and coefficient of y in equation of circle) and substitute (–1, 4) or (3, 6) into equation of circle.	2nd M1
$-2x - 10y$ part of the equation $x^2 + y^2 - 2x - 10y + 21 = 0$.	A1
$+21 = 0$ part of the equation $x^2 + y^2 - 2x - 10y + 21 = 0$.	A1
Or correct equivalents, e.g. $(x-1)^2 + (y-5)^2 = 5$.	

Question 6 June 07 Q7

Question number	Scheme	Marks	
	(a) Gradient of AM: $\frac{1-(-2)}{3-1}=\frac{3}{2}$ or $\frac{-3}{-2}$	B1	
	Gradient of l: $=-\frac{2}{3}$ M: use of $m_1 m_2 = -1$, or equiv.	M1	
	$y-1=-\frac{2}{3}(x-3)$ or $\frac{y-1}{x-3}=-\frac{2}{3}$ $[3y=-2x+9]$ (Any equiv. form)	M1 A1	(4)
	(b) $x=6$: $3y=-12+9=-3$ $y=-1$ (or show that for $y=-1$, $x=6$) (*) (A conclusion is <u>not</u> required).	B1	(1)
	(c) $(r^2=)$ $(6-1)^2+(-1-(-2))^2$ M: Attempt r^2 or r	M1 A1	
	N.B. Simplification is <u>not</u> required to score M1 A1		
	$(x\pm 6)^2+(y\pm 1)^2=k$, $k\neq 0$ (<u>Value</u> for k not needed, could be r^2 or r)	M1	
	$(x-6)^2+(y+1)^2=26$ (or equiv.)	A1	(4)
	Allow $(\sqrt{26})^2$ or other exact equivalents for 26. (But… $(x-6)^2+(y--1)^2=26$ scores M1 A0)		
	(Correct answer with no working scores full marks)		**9**
	(a) 2^{nd} M1: eqn. of a straight line through (3, 1) with any gradient except 0 or ∞. <u>Alternative:</u> Using (3, 1) in $y=mx+c$ to find a value of c scores M1, but an equation (general or specific) must be seen. Having coords the <u>wrong way round</u>, e.g. $y-3=-\frac{2}{3}(x-1)$, loses the 2^{nd} M mark <u>unless</u> a correct general formula is seen, e.g. $y-y_1=m(x-x_1)$. If the point $P(6,-1)$ is used to find the gradient of MP, maximum marks are (a) B0 M0 M1 A1 (b) B0. (c) 1^{st} M1: Condone <u>one</u> slip, numerical or sign, <u>inside</u> a bracket. Must be attempting to use points $P(6, -1)$ and $A(1, -2)$, or perhaps P and B. (Correct coordinates for B are (5, 4)). 1^{st} M alternative is to use a complete Pythag. method on triangle MAP, n.b. $MP=MA=\sqrt{13}$. <u>Special case:</u> If candidate persists in using <u>their</u> value for the y-coordinate of P instead of the given -1, allow the M marks in part (c) if earned.		

Question 7 June 08 Q5

Question Number	Scheme	Marks
(a)	$(8-3)^2+(3-1)^2$ or $\sqrt{(8-3)^2+(3-1)^2}$	M1 A1
	$(x\pm3)^2+(y\pm1)^2=k$ or $(x\pm1)^2+(y\pm3)^2=k$ (k a positive value)	M1
	$(x-3)^2+(y-1)^2=29$	A1 (4)
(b)	Gradient of radius $=\dfrac{2}{5}$ (or exact equivalent)	B1
	Gradient of tangent $=\dfrac{-5}{2}$	M1
	$y-3=\dfrac{-5}{2}(x-8)$	M1 A1 ft
	$5x+2y-46=0$ or equivalent	A1 (5)
		(9 marks)

Question 8 Jan 09 Q5

Question Number	Scheme	Marks
(a)	PQ: $m_1 = \frac{10-2}{9-(-3)}$ $(=\frac{2}{3})$ **and** QR: $m_2 = \frac{10-4}{9-a}$	M1
(b)	$m_1 m_2 = -1$: $\frac{8}{12} \times \frac{6}{9-a} = -1$ $a = 13$ (*)	M1 A1 (3)
Alt for (a)	(a) Alternative method (Pythagoras) Finds **all three** of the following	
	$(9-(-3))^2 + (10-2)^2$, (i.e. 208), $(9-a)^2 + (10-4)^2$, $(a-(-3))^2 + (4-2)^2$	M1
	Using Pythagoras (correct way around) e.g. $a^2 + 6a + 9 = 240 + a^2 - 18a + 81$ to form equation	M1
	Solve (or verify) for a, $a = 13$ (*)	A1 (3)
	(b) Centre is at (5, 3)	B1
	$(r^2 =)$ $(10-3)^2 + (9-5)^2$ or equiv., or $(d^2 =)$ $(13-(-3))^2 + (4-2)^2$	M1 A1
	$(x-5)^2 + (y-3)^2 = 65$ or $x^2 + y^2 - 10x - 6y - 31 = 0$	M1 A1 (5)
Alt for (b)	Uses $(x-a)^2 + (y-b)^2 = r^2$ or $x^2 + y^2 + 2gx + 2fy + c = 0$ and substitutes (-3, 2), (9, 10) and (13, 4) then eliminates one unknown	M1
	Eliminates second unknown	M1
	Obtains $g = -5, f = -3, c = -31$ or $a = 5, b = 3$, $r^2 = 65$	A1, A1, B1cao (5) [8]
Notes (a)	**M1**-considers gradients of PQ and QR -must be y difference / x difference (or considers three lengths as in alternative method) **M1** Substitutes gradients into product = -1 (or lengths into Pythagoras' Theorem the correct way round) **A1** Obtains $a = 13$ with no errors by solution or verification. Verification can score 3/3.	
(b)	Geometrical method: **B1** for coordinates of centre – can be implied by use in part (b) **M1** for attempt to find r^2, d^2, r or d (allow one slip in a bracket). **A1** cao. These two marks may be gained implicitly from circle equation **M1** for $(x \pm 5)^2 + (y \pm 3)^2 = k^2$ or $(x \pm 3)^2 + (y \pm 5)^2 = k^2$ ft their (5,3) Allow k^2 non numerical. **A1** cao for whole equation and rhs must be 65 or $(\sqrt{65})^2$, (similarly B1 must be 65 or $(\sqrt{65})^2$, in alternative method for (b))	

Question Number	Scheme	Marks
Further alternatives	(i) A number of methods find gradient of PQ = 2/3 then give perpendicular gradient is –3/2 This is **M1**	M1
	They then proceed using equations of lines through point Q or by using gradient QR to obtain equation such as $\frac{4-10}{a-9} = -\frac{3}{2}$ **M1** (may still have x in this equation rather than a and there may be a small slip)	M1
	They then complete to give $(a) = 13$ **A1**	A1
	(ii) A long involved method has been seen finding the coordinates of the centre of the circle first. This can be done by a variety of methods Giving centre as (c, 3) and using an equation such as $(c-9)^2 + 7^2 = (c+3)^2 + 1^2$ (equal radii) or $\frac{3-6}{c-3} = -\frac{3}{2}$ **M1** (perpendicular from centre to chord bisects chord)	M1
	Then using c (= 5) to find a is **M1**	M1
	Finally $a = 13$ **A1**	A1
	(iii) Vector Method: States **PQ**. **QR** = 0, with vectors stated 12**i** +8**j** and $(9-a)$**i** + 6**j** is **M1**	M1
	Evaluates scalar product so $108 - 12a + 48 = 0$ **(M1)**	M1
	solves to give $a = 13$ **(A1)**	A1

Question 9 June 09 Q6

Question Number	Scheme	Marks
Q (a)	$(x-3)^2-9+(y+2)^2-4=12$ Centre is $(3, -2)$	M1 A1, A1
	$(x-3)^2+(y+2)^2=12+"9"+"4"$ $r=\sqrt{12+"9"+"4"}=5$ (or $\sqrt{25}$)	M1 A1 (5)
(b)	$PQ=\sqrt{(7--1)^2+(-5-1)^2}$ or $\sqrt{8^2+6^2}$	M1
	$=10=2\times$radius, $\therefore$ diam. (N.B. For A1, need a comment or conclusion)	A1 (2)
	[ALT: midpt. of PQ $\left(\frac{7+(-1)}{2},\frac{1+(-5)}{2}\right)$: M1, $=(3,-2)=$ centre: A1]	
	[ALT: eqn. of PQ $3x+4y-1=0$: M1, verify $(3,-2)$ lies on this: A1]	
	[ALT: find two grads, e.g. PQ and P to centre: M1, equal $\therefore$ diameter: A1]	
	[ALT: show that point $S(-1,-5)$ or $(7, 1)$ lies on circle: M1 because $\angle PSQ=90°$, semicircle $\therefore$ diameter: A1]	
(c)	R must lie on the circle (angle in a semicircle theorem)… often implied by a diagram with R on the circle or by subsequent working)	B1
	$x=0\Rightarrow\ y^2+4y-12=0$	M1
	$(y-2)(y+6)=0$ $y=.....$ (M is dependent on previous M)	dM1
	$y=-6$ or 2 (Ignore $y=-6$ if seen, and 'coordinates' are not required))	A1 (4) [11]
(a)	1st M1 for attempt to complete square. Allow $(x\pm 3)^2\pm k$, or $(y\pm 2)^2\pm k$, $k\neq 0$. 1st A1 x-coordinate 3, 2nd A1 y-coordinate -2 2nd M1 for a full method leading to $r=$ …, with their 9 and their 4, 3rd A1 5 or $\sqrt{25}$ The 1st M can be implied by $(\pm 3, \pm 2)$ but a full method must be seen for the 2nd M. Where the 'diameter' in part (b) has clearly been used to answer part (a), no marks in (a), but in this case the M1 (not the A1) for part (b) can be given for work seen in (a). Alternative 1st M1 for comparing with $x^2+y^2+2gx+2fy+c=0$ to write down centre $(-g,-f)$ directly. Condone sign errors for this M mark. 2nd M1 for using $r=\sqrt{g^2+f^2-c}$. Condone sign errors for this M mark.	
(c)	1st M1 for setting $x=0$ and getting a 3TQ in y by using eqn. of circle. 2nd M1 (dep.) for attempt to solve a 3TQ leading to at least one solution for y. Alternative 1: (Requires the B mark as in the main scheme) 1st M for using (3, 4, 5) triangle with vertices $(3,-2), (0,-2), (0,y)$ to get a linear or quadratic equation in y (e.g. $3^2+(y+2)^2=25$). 2nd M (dep.) as in main scheme, but may be scored by simply solving a linear equation. Alternative 2: (Not requiring realisation that R is on the circle) B1 for attempt at $m_{PR}\times m_{QR}=-1$, (NOT m_{PQ}) or for attempt at Pythag. in triangle PQR. 1st M1 for setting $x=0$, i.e. $(0, y)$, and proceeding to get a 3TQ in y. Then main scheme. Alternative 2 by 'verification': B1 for attempt at $m_{PR}\times m_{QR}=-1$, (NOT m_{PQ}) or for attempt at Pythag. in triangle PQR. 1st M1 for trying (0, 2). 2nd M1 (dep.) for performing all required calculations. A1 for fully correct working and conclusion.	

Question 10 Jan 10 Q8

Question Number	Scheme	Marks
(a)	$N(2, -1)$	B1, B1 (2)
(b)	$r = \sqrt{\frac{169}{4}} = \frac{13}{2} = 6.5$	B1 (1)
(c)	Complete Method to find x coordinates, $x_2 - x_1 = 12$ and $\frac{x_1 + x_2}{2} = 2$ then solve	M1
	To obtain $x_1 = -4$, $x_2 = 8$	A1ft A1ft
	Complete Method to find y coordinates, using equation of circle or Pythagoras i.e. let d be the distance below N of A *then* $d^2 = 6.5^2 - 6^2 \Rightarrow d = 2.5 \Rightarrow y = ..$	M1
	So $y_2 = y_1 = -3.5$	A1 (5)
(d)	Let $A\hat{N}B = 2\theta \Rightarrow \sin\theta = \frac{6}{\text{"}6.5\text{"}} \Rightarrow \theta = (67.38)...$	M1
	So angle ANB is 134.8 *	A1 (2)
(e)	AP is perpendicular to AN so using triangle ANP $\tan\theta = \frac{AP}{\text{"}6.5\text{"}}$	M1
	Therefore $AP = 15.6$	A1cao (2)
		[12]
(a)	B1 for 2 (α), B1 for -1	
(b)	B1 for 6.5 o.e.	
(c)	1st M1 for finding x coordinates – may be awarded if either x co-ord is correct A1ft,A1ft are for $\alpha - 6$ and $\alpha + 6$ if x coordinate of N is α 2nd M1 for a method to find y coordinates – may be given if y co-ordinate is correct A marks is for -3.5 only.	
(d)	M1 for a full method to find θ or angle ANB (eg sine rule or cosine rule directly or finding another angle and using angles of triangle.) **ft their 6.5 from radius or wrong y.** $\left(\cos ANB = \frac{\text{"}6.5\text{"}^2 + \text{"}6.5\text{"}^2 - 12^2}{2 \times \text{"}6.5\text{"} \times \text{"}6.5\text{"}} = -0.704\right)$ A1 is a printed answer and must be 134.8 – do not accept 134.76.	
(e)	M1 for a full method to find AP <u>Alternative Methods</u> N.B. May use triangle AXP where X is the mid point of AB. Or may use triangle ABP. From circle theorems may use angle BAP = 67.38 or some variation. Eg $\frac{AP}{\sin 67.4} = \frac{12}{\sin 45.2}$, $AP = \frac{6}{\sin 22.6}$ or $AP = \frac{6}{\cos 67.4}$ are each worth M1	

Question 11 June 10 Q10

Question Number	Scheme	Marks
	(a) $(10-2)^2+(7-1)^2$ or $\sqrt{(10-2)^2+(7-1)^2}$	M1 A1
	$(x\pm 2)^2+(y\pm 1)^2=k$ (k a positive <u>value</u>)	M1
	$(x-2)^2+(y-1)^2=100$ (Accept 10^2 for 100)	A1
	(Answer only scores full marks)	(4)
	(b) (Gradient of radius =) $\frac{7-1}{10-2}=\frac{6}{8}$ (or equiv.) Must be seen in part (b)	B1
	Gradient of tangent $=\frac{-4}{3}$ (Using perpendicular gradient method)	M1
	$y-7=m(x-10)$ Eqn., in any form, of a line through (10, 7) with any numerical gradient (except 0 or ∞)	M1
	$y-7=\frac{-4}{3}(x-10)$ or equiv (ft gradient of <u>radius</u>, dep. on <u>both</u> M marks)	A1ft
	$\{3y=-4x+61\}$ (N.B. The A1 is only available as <u>ft</u> after B0)	
	The unsimplified version scores the A mark (isw if necessary... subsequent mistakes in simplification are not penalised here.	
	The equation must at some stage be <u>exact</u>, not, e.g. $y=-1.3x+20.3$	(4)
	(c) $\sqrt{r^2-\left(\frac{r}{2}\right)^2}$ Condone sign slip if there is evidence of correct use of Pythag.	M1
	$=\sqrt{10^2-5^2}$ or numerically exact equivalent	A1
	$PQ\left(=2\sqrt{75}\right)=10\sqrt{3}$ Simplest surd form $10\sqrt{3}$ required for final mark	A1
		(3) 11
	(b) 2^{nd} M: Using (10, 7) to find the equation, in any form, of a straight line through (10, 7), with any numerical gradient (except 0 or ∞). <u>Alternative:</u> 2^{nd} M: Using (10, 7) and an m value in $y=mx+c$ to find a value of c. (b) <u>Alternative</u> for first 2 marks (differentiation): $2(x-2)+2(y-1)\frac{dy}{dx}=0$ or equiv. B1 Substitute $x=10$ and $y=7$ to find a value for $\frac{dy}{dx}$ M1 (This M mark can be awarded generously, even if the attempted 'differentiation' is not 'implicit'). (c) <u>Alternatives</u>: To score M1, must be a <u>fully</u> correct method to obtain $\frac{1}{2}PQ$ or PQ. 1^{st} A1: For alternative methods that find PQ directly, this mark is for an <u>exact numerically correct version</u> of PQ.	

Question 12 Jan 11 Q9

Question Number	Scheme		Marks
(a)	$C\left(\frac{-2+8}{2}, \frac{11+1}{2}\right) = C(3, 6)$ **AG**	Correct method (no errors) for finding the mid-point of AB giving $(3, 6)$	B1*
			(1)
(b)	$(8-3)^2+(1-6)^2$ or $\sqrt{(8-3)^2+(1-6)^2}$ or	Applies distance formula in order to find the radius.	M1
	$(-2-3)^2+(11-6)^2$ or $\sqrt{(-2-3)^2+(11-6)^2}$	Correct application of formula.	A1
	$(x-3)^2+(y-6)^2 = 50$ $\left(\text{or } \left(\sqrt{50}\right)^2 \text{ or } \left(5\sqrt{2}\right)^2\right)$	$(x \pm 3)^2+(y \pm 6)^2 = k$, k is a positive <u>value</u>.	M1
		$(x-3)^2+(y-6)^2 = 50$ (Not 7.07^2)	A1
			(4)
(c)	{For $(10, 7)$,} $\underline{(10-3)^2+(7-6)^2 = 50}$, {so the point lies on C.}		<u>B1</u>
			(1)
(d)	{Gradient of radius} $= \frac{7-6}{10-3}$ or $\frac{1}{7}$	This must be seen in part (d).	B1
	Gradient of tangent $= \frac{-7}{1}$	Using a perpendicular gradient method.	M1
	$y-7 = -7(x-10)$	$y-7 = (\text{their gradient})(x-10)$	M1
	$y = -7x+77$	$y = -7x+77$ or $y = 77-7x$	A1 cao
			(4)
			[10]

	<u>**Notes**</u>
(a)	Alternative method: $C\left(-2+\frac{8--2}{2}, 11+\frac{1-11}{2}\right)$ or $C\left(8+\frac{-2-8}{2}, 1+\frac{11-1}{2}\right)$
(b)	You need to be convinced that the candidate is attempting to work out the radius and not the diameter of the circle to award the first M1. Therefore allow 1st M1 generously for $\frac{(-2-8)^2+(11-1)^2}{2}$ Award 1st M1A1 for $\frac{(-2-8)^2+(11-1)^2}{4}$ or $\frac{\sqrt{(-2-8)^2+(11-1)^2}}{2}$. Correct answer in (b) with no working scores full marks.
(c)	B1 awarded for correct verification of $\underline{(10-3)^2+(7-6)^2 = 50}$ with no errors. Also to gain this mark candidates need to have the correct equation of the circle either from part (b) or re-attempted in part (c). They cannot verify $(10, 7)$ lies on C without a correct C. Also a candidate could either substitute $x = 10$ in C to find $y = 7$ or substitute $y = 7$ in C to find $x = 10$.

Question Number	Scheme	Marks
(d)	2nd M1 mark also for the complete method of applying $7 = (\text{their gradient})(10) + c$, finding c. **Note**: Award 2nd M0 in (d) if their numerical gradient is either 0 or ∞. <u>**Alternative:**</u> For first two marks (differentiation): $2(x-3)+2(y-6)\frac{dy}{dx} = 0$ (or equivalent) scores B1. 1st M1 for substituting **both** $x = 10$ and $y = 7$ to find a value for $\frac{dy}{dx}$, which must contain both x and y. (This M mark can be awarded generously, even if the attempted "differentiation" is not "implicit".) <u>**Alternative:**</u> $(10-3)(x-3)+(7-6)(y-6) = 50$ scores B1M1M1 which leads to $y = -7x+77$.	

Question 13 June 11 Q4

Question Number	Scheme		Marks
	$x^2+y^2+4x-2y-11=0$		
(a)	$\{\underline{(x+2)^2-4}+\underline{(y-1)^2-1}-11=0\}$	$(\pm 2, \pm 1)$, see notes.	M1
	Centre is $(-2, 1)$.	$(-2, 1)$.	A1 cao [2]
(b)	$(x+2)^2+(y-1)^2=11+1+4$	$r=\sqrt{11\pm "1" \pm "4"}$	M1
	So $r=\sqrt{11+1+4} \Rightarrow r=4$	4 or $\sqrt{16}$ (Award A0 for ± 4).	A1 [2]
(c)	When $x=0$, $y^2-2y-11=0$	Putting $x=0$ in C or their C.	M1
		$y^2-2y-11=0$ or $(y-1)^2=12$, etc	A1 aef
	$y=\dfrac{2\pm\sqrt{(-2)^2-4(1)(-11)}}{2(1)}$ $\left\{=\dfrac{2\pm\sqrt{48}}{2}\right\}$	Attempt to use formula or a method of completing the square in order to find $y=\ldots$	M1
	So, $y=1\pm 2\sqrt{3}$	$1\pm 2\sqrt{3}$	A1 cao cso
			[4]
			8

Note: Please mark parts (a) and (b) together. Answers only in (a) and/or (b) get full marks.
Note in part (a) the marks are now M1A1 and not B1B1 as on ePEN.

(a) M1: for $(\pm 2, \pm 1)$. Otherwise, M1 for an attempt to complete the square eg. $\underline{(x\pm 2)^2 \pm \alpha}$, $\alpha \neq 0$ or $\underline{(y\pm 1)^2 \pm \beta}$, $\beta \neq 0$. M1A1: Correct answer of $(-2, 1)$ stated from any working gets M1A1.

(b) M1: to find the radius using 11, "1" and "4", ie. $r=\sqrt{11\pm "1" \pm "4"}$. By applying this method candidates will usually achieve $\sqrt{16}$, $\sqrt{6}$, $\sqrt{8}$ or $\sqrt{14}$ and not 16, 6, 8 or 14.

<u>Note:</u> $(x+2)^2+(y-1)^2=-11-5=-16 \Rightarrow r=\sqrt{16}=4$ should be awarded M0A0.

<u>Alternative:</u> M1 in part (a): For comparing with $x^2+y^2+2gx+2fy+c=0$ to write down centre $(-g, -f)$ directly. Condone sign errors for this M mark. M1 in part (b): For using $r=\sqrt{g^2+f^2-c}$. Condone sign errors for this method mark.

$(x+2)^2+(y-1)^2=16 \Rightarrow r=8$ scores M0A0, but $r=\sqrt{16}=8$ scores M1A1 isw.

(c) 1st M1: Putting $x=0$ in either $x^2+y^2+4x-2y-11=0$ or their circle equation usually given in part (a) or part (b). 1st A1 for a correct equation in y **in any form** which can be implied by later working.

2nd M1: See rules for using the formula. Or completing the square on a 3TQ to give $y=a\pm\sqrt{b}$, where $\sqrt{b}$ is a surd, $b \neq$ their 11 and $b>0$. This mark should not be given for an attempt to factorise.

2nd A1: Need exact pair in simplified surd form of $\{y=\}\, 1\pm 2\sqrt{3}$. This mark is also cso.

Do not need to see $(0, 1+2\sqrt{3})$ and $(0, 1-2\sqrt{3})$. Allow 2nd A1 for bod $(1+2\sqrt{3}, 0)$ and $(1-2\sqrt{3}, 0)$.

Any incorrect working in (c) gets penalised the final accuracy mark. So, <u>beware:</u> incorrect $(x-2)^2+(y-1)^2=16$ leading to $y^2-2y-11=0$ and then $y=1\pm 2\sqrt{3}$ scores M1A1M1A0.

<u>Special Case for setting $y=0$:</u> Award SC: M0A0M1A0 for an attempt at applying the formula

$$x=\frac{-4\pm\sqrt{(-4)^2-4(1)(-11)}}{2(1)} \quad \left\{=\frac{-4\pm\sqrt{60}}{2}=-2\pm\sqrt{15}\right\}$$

Award SC: M0A0M1A0 for completing the square to their equation in x which will usually be $x^2+4x-11=0$ to give $a\pm\sqrt{b}$, where $\sqrt{b}$ is a surd, $b \neq$ their 11 and $b>0$.

<u>Special Case</u>: For a candidate not using $\pm$ but achieving one of the correct answers then award SC: M1A1 M1A0 for one of either $y=1+2\sqrt{3}$ or $y=1-2\sqrt{3}$ or $y=1+\sqrt{12}$ or $y=1-\sqrt{12}$.