Differentiation, Tangents & Normal - Edexcel Past Exam Questions MARK SCHEME

Question 1 : Jan 05 Q2

Question number	Scheme	Marks
	(i) (a) $15x^2+7$	M1 A1 A1 (3)
	(i) (b) $30x$	B1ft (1)
	(i) (a) A1: 2 terms correctly differentiated. A1: Fully correct.	

Question 2 : Jan 05 Q7

Question number	Scheme	Marks
	(a) $\frac{5-x}{x}=\frac{5}{x}-1 \quad \left(=5x^{-1}-1\right)$	M1
	$\frac{dy}{dx}=8x-5x^{-2}$	M1 A1 A1
	When $x=1$, $\frac{dy}{dx}=3$ (*)	A1 (5)
	(b) At P, $y=8$	B1
	Equation of tangent: $y-8=3(x-1)$ ($y=3x+5$) (or equiv.)	M1 A1ft (3)
	(c) Where $y=0$, $x=-\frac{5}{3}$ $(=k)$ (or exact equiv.)	M1 A1 (2)
		10
	(a) First M1 can also be scored by an attempt to use the quotient or product rule to differentiate $\frac{5-x}{x}$. (b) The B mark may be earned in part (a).	

Question 3 : June 05 Q2

(a)	$\frac{dy}{dx} = 6 + 8x^{-3}$ $\qquad x^n \to x^{n-1}$ both	M1 A1 (2)

Question 4 : June 05 Q10

Question Number	Scheme	Marks
(a)	$x = 3, \quad y = 9 - 36 + 24 + 3 = 0$ $\qquad$ (9 – 36 + 27=0 is OK)	B1 (1)
(b)	$\frac{dy}{dx} = \frac{3}{3}x^2 - 2 \times 4 \times x + 8 \qquad (= x^2 - 8x + 8)$	M1 A1
	When $x = 3$, $\frac{dy}{dx} = 9 - 24 + 8 \Rightarrow m = -7$	M1
	Equation of tangent: $\quad y - 0 = -7(x - 3)$ $y = -7x + 21$	M1 A1 c.a.o (5)
(c)	$\frac{dy}{dx} = m \quad$ gives $\quad x^2 - 8x + 8 = -7$	M1
	$(x^2 - 8x + 15 = 0)$ $(x-5)(x-3) = 0$ $x = (3) \quad$ or 5 $\qquad x = 5$	M1 A1
	$\therefore y = \frac{1}{3}5^3 - 4 \times 5^2 + 8 \times 5 + 3$	M1
	$y = -15\frac{1}{3} \quad$ or $\quad -\frac{46}{3}$	A1 (5)
(b)	1^{st} M1 some correct differentiation ($x^n \to x^{n-1}$ for one term) 1^{st} A1 correct unsimplified (all 3 terms) 2^{nd} M1 substituting $x_p (= 3)$ in their $\frac{dy}{dx}$ clear evidence 3^{rd} M1 using their m to find tangent at p.	
(c)	1^{st} M1 forming a correct equation " their $\frac{dy}{dx}$ = gradient of their tangent" 2^{nd} M1 for solving a quadratic based on their $\frac{dy}{dx}$ leading to $x =$ 3^{rd} M1 for using their x value in y to obtain y coordinate	
MR	For misreading (0, 3) for (3, 0) award B0 and then M1A1 as in scheme. Then allow all M marks but no A ft. (Max 7)	

Question 5 : Jan 06 Q4

Question number	Scheme	Marks
	(a) $\frac{\mathrm{d}y}{\mathrm{d}x} = 4x + 18x^{-4}$ M1: $x^2 \to x$ or $x^{-3} \to x^{-4}$	M1 A1 (2)

Question 6 : June 06 Q5

Question number	Scheme	Marks
(a)	$(y = x^4 + 6x^{\frac{1}{2}} \Rightarrow y' =)\ 4x^3 + 3x^{-\frac{1}{2}}$ or $4x^3 + \dfrac{3}{\sqrt{x}}$	M1A1A1 (3)
(b)	$(x+4)^2 = x^2 + 8x + 16$	M1
	$\dfrac{(x+4)^2}{x} = x + 8 + 16x^{-1}$ (allow 4+4 for 8)	A1
	$(y = \dfrac{(x+4)^2}{x} \Rightarrow y' =)\ 1 - 16x^{-2}$ o.e.	M1A1 (4)
		7

(a)	M1	For some attempt to differentiate $x^n \rightarrow x^{n-1}$
	1st A1	For one correct term as printed.
	2nd A1	For both terms correct as printed.
		$4x^3 + 3x^{-\frac{1}{2}} + c$ scores M1A1A0
(b)	1st M1	For attempt to expand $(x+4)^2$, must have x^2, x, x^0 terms and at least 2 correct e.g. $x^2 + 8x + 8$ or $x^2 + 2x + 16$
	1st A1	Correct expression for $\dfrac{(x+4)^2}{x}$. As printed but allow $\dfrac{16}{x}$ and $8x^0$.
	2nd M1	For some correct differentiation, any term. Can follow through their simplification.
		N.B. $\dfrac{x^2+8x+16}{x}$ giving rise to $(2x + 8)/1$ is M0A0
ALT	Product or Quotient rule (If in doubt send to review)	
	M2	For correct use of product or quotient rule. Apply usual rules on formulae.
	1st A1	For $\dfrac{2(x+4)}{x}$ or $\dfrac{2x(x+4)}{x^2}$
	2nd A1	for $-\dfrac{(x+4)^2}{x^2}$

Question 7 : Jan 06 Q9

Question number	Scheme	Marks
	(a) -2 (P), 2 (Q) (± 2 scores B1 B1)	B1, B1
		(2)
	(b) $y = x^3 - x^2 - 4x + 4$ (May be seen earlier) Multiply out, giving 4 terms	M1
	$\frac{dy}{dx} = 3x^2 - 2x - 4$ (*)	M1 A1cso
		(3)
	(c) At $x = -1$: $\frac{dy}{dx} = 3(-1)^2 - 2(-1) - 4 = 1$	
	Eqn. of tangent: $y - 6 = 1(x - (-1))$, $y = x + 7$ (*)	M1 A1cso
		(2)
	(d) $3x^2 - 2x - 4 = 1$ (Equating to "gradient of tangent")	M1
	$3x^2 - 2x - 5 = 0$ $(3x-5)(x+1) = 0$ $x = \ldots$	M1
	$x = \frac{5}{3}$ or equiv.	A1
	$y = \left(\frac{5}{3} - 1\right)\left(\frac{25}{9} - 4\right)$, $= \frac{2}{3} \times \left(-\frac{11}{9}\right) = -\frac{22}{27}$ or equiv.	M1, A1
		(5)
		Total 12 marks

Question 8 : Jan 07 Q1

Question number	Scheme	Marks
	$4x^3 \to kx^2$ or $2x^{\frac{1}{2}} \to kx^{-\frac{1}{2}}$ (k a non-zero constant) $12x^2, +x^{-\frac{1}{2}}$......., $(-1 \to 0)$	M1 A1, A1, B1 (4) **4**
	Accept equivalent alternatives to $x^{-\frac{1}{2}}$, e.g. $\frac{1}{x^{1/2}}$, $\frac{1}{\sqrt{x}}$, $x^{-0.5}$. M1: $4x^3$ 'differentiated' to give kx^2, or… $2x^{\frac{1}{2}}$ 'differentiated' to give $kx^{-\frac{1}{2}}$ (but not for just $-1 \to 0$). 1st A1: $12x^2$ (Do not allow just $3 \times 4x^2$) 2nd A1: $x^{-\frac{1}{2}}$ or equivalent. (Do not allow just $\frac{1}{2} \times 2x^{-\frac{1}{2}}$, but allow $1x^{-\frac{1}{2}}$ or $\frac{2}{2}x^{-\frac{1}{2}}$). B1: -1 differentiated to give zero (or 'disappearing'). Can be given provided that at least one of the other terms has been changed. Adding an extra term, e.g. $+ C$, is B0.	

Question 9 : Jan 07 Q8

Question number	Scheme	Marks	
	(a) $4x \to k$ or $3x^{3/2} \to kx^{1/2}$ or $-2x^2 \to kx$	M1	
	$\frac{dy}{dx} = 4 + \frac{9}{2}x^{1/2} - 4x$	A1 A1	(3)
	(b) For $x = 4$, $y = (4 \times 4) + (3 \times 4\sqrt{4}) - (2 \times 16) = 16 + 24 - 32 = 8$ (*)	B1	(1)
	(c) $\frac{dy}{dx} = 4 + 9 - 16 = -3$ M: Evaluate their $\frac{dy}{dx}$ at $x = 4$	M1	
	Gradient of normal $= \frac{1}{3}$	A1ft	
	Equation of normal: $y - 8 = \frac{1}{3}(x-4)$, $3y = x + 20$ (*)	M1, A1	(4)
	(d) $y = 0$: $x = ...\ (-20)$ and use $(x_2 - x_1)^2 + (y_2 - y_1)^2$	M1	
	$PQ = \sqrt{24^2 + 8^2}$ or $PQ^2 = 24^2 + 8^2$ Follow through from $(k, 0)$ May also be scored with $(-24)^2$ and $(-8)^2$.	A1ft	
	$= 8\sqrt{10}$	A1	(3)

Question 10 : June 07 Q3

Question number	Scheme	Marks
	(a) $\left(\frac{dy}{dx}\right) = \underline{6x^1 + \frac{4}{2}x^{-\frac{1}{2}}}$ or $\left(6x + 2x^{-\frac{1}{2}}\right)$	M1 A1 (2)
	(b) $\underline{6 + -x^{-\frac{3}{2}}}$ or $\underline{6 + -1 \times x^{-\frac{3}{2}}}$	M1 A1ft (2)
	3	

(a)	M1	for <u>some</u> attempt to differentiate: $x^n \to x^{n-1}$ Condone missing $\frac{dy}{dx}$ or $y = \ldots$
	A1	for both terms correct, as written or better. No $+C$ here. Of course $\frac{2}{\sqrt{x}}$ is acceptable.
(b)	M1	for some attempt to differentiate again. Follow through their $\frac{dy}{dx}$, at least one term correct or correct follow through.
	A1f.t.	as written or better, follow through must have 2 <u>distinct</u> terms and simplified e.g. $\frac{4}{4} = 1$.

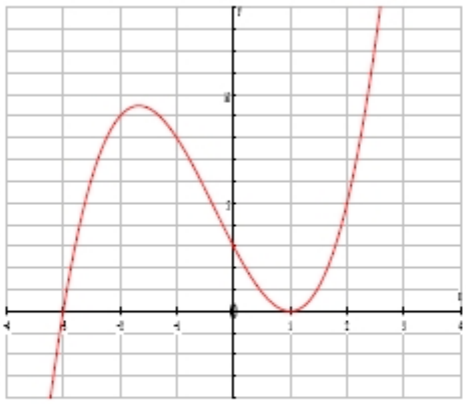
Question 11 : June 07 Q10

Question number	Scheme	Marks
	(a) $x=1:\ y=-5+4=\underline{-1}$, $\quad x=2:\ y=-16+2=\underline{-14}$ (can be given in (b) or (c))	1st B1 for – 1 2nd B1 for - 14
	$PQ=\sqrt{(2-1)^2+(-14-(-1))^2}=\sqrt{170}$ (*)	M1 A1cso (4)
	(b) $y=x^3-6x^2+4x^{-1}$	M1
	$\frac{dy}{dx}=3x^2-12x-4x^{-2}$	M1 A1
	$x=1:\ \frac{dy}{dx}=3-12-4=-13$ M: Evaluate at one of the points	M1
	$x=2:\ \frac{dy}{dx}=12-24-1=-13$ $\therefore$ Parallel A: Both correct + conclusion	A1 (5)
	(c) Finding gradient of normal $\left(m=\frac{1}{13}\right)$	M1
	$y--1=\frac{1}{13}(x-1)$	M1 A1ft
	$\underline{x-13y-14=0}$ o.e.	A1cso (4)
		13
(a)	M1 for attempting PQ or PQ^2 using their P and their Q. Usual rules about quoting formulae. We must see attempt at $1^2+(y_P-y_Q)^2$ for M1. $PQ^2=\sqrt{\ldots}$ etc could be M1A0. A1cso for proceeding to the correct answer with no incorrect working seen.	
(b)	1st M1 for multiplying by x^2, the x^3 or $-6x^2$ must be correct. 2nd M1 for some correct differentiation, at least one term must be correct as printed. 1st A1 for a fully correct derivative. **These 3 marks can be awarded anywhere when first seen.** 3rd M1 for attempting to substitute $x=1$ or $x=2$ in their derivative. Substituting in y is M0. 2nd A1 for -13 from both substitutions and a brief comment. The -13 must come from their derivative.	
(c)	1st M1 for use of the perpendicular gradient rule. Follow through their -13. 2nd M1 for full method to find the equation of the normal or tangent at P. If formula is quoted allow slips in substitution, otherwise a correct substitution is required. 1st A1ft for a correct expression. Follow through their -1 and their changed gradient. 2nd A1cso for a correct equation with $=0$ and integer coefficients. This mark is dependent upon the -13 coming from their derivative in (b) hence cso. Tangent can get M0M1A0A0, changed gradient can get M0M1A1A0orM1M1A1A0.	
MR	Condone confusion over terminology of tangent and normal, mark gradient and equation. Allow for $-\frac{4}{x}$ or $(x+6)$ but not omitting $4x^{-1}$ or treating it as $4x$.	

Question 12 : Jan 08 Q10

Question number	Scheme	Marks
	(a) $\left(2x^{-\frac{1}{2}} + 3x^{-1}\right)$ $\quad p = -\frac{1}{2}, \quad q = -1$	B1, B1 (2)
	(b) $\left(y = 5x - 7 + 2x^{-\frac{1}{2}} + 3x^{-1}\right)$	
	$\left(\frac{dy}{dx} =\right)$ $\quad 5 \quad (\text{or } 5x^0)$ $\quad$ ($5x - 7$ correctly differentiated)	B1
	Attempt to differentiate either $2x^p$ with a fractional p, giving kx^{p-1} $(k \neq 0)$, (the fraction p could be in decimal form) or $3x^q$ with a negative q, giving kx^{q-1} $(k \neq 0)$.	M1
	$\left(-\frac{1}{2} \times 2x^{-\frac{3}{2}} - 1 \times 3x^{-2} =\right)$ $\quad -x^{-\frac{3}{2}}, \; -3x^{-2}$	A1ft, A1ft (4)
		6
	(b): N.B. It is possible to 'start again' in (b), so the p and q may be different from those seen in (a), but note that the M mark is for the attempt to differentiate $\underline{\underline{2}}x^p$ or $\underline{\underline{3}}x^q$. However, marks for part (a) <u>cannot</u> be earned in part (b). 1st A1ft: ft their $2x^p$, but p must be a fraction and coefficient must be simplified (the fraction p could be in decimal form). 2nd A1ft: ft their $3x^q$, but q must be negative and coefficient must be simplified. 'Simplified' coefficient means $\frac{a}{b}$ where a and b are integers with no common factors. Only a single + or − sign is allowed (e.g. − − must be replaced by +). Having $+C$ loses the B mark.	

Question 13 : Jan 08 Q10

Question number	Scheme	Marks
	(a) 	
	Shape (drawn anywhere)	B1
	Minimum at (1, 0) (perhaps labelled 1 on x-axis)	B1
	$(-3, 0)$ (or -3 shown on $-$ve x-axis)	B1
	(0, 3) (or 3 shown on +ve y-axis)	B1 (4)
	N.B. The max. can be anywhere.	
	(b) $y = (x+3)(x^2 - 2x + 1)$ [Marks can be awarded if this is seen in part (a)]	M1
	$= x^3 + x^2 - 5x + 3$ $\quad (k = 3)$	A1cso (2)
	(c) $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 2x - 5$	M1 A1
	$3x^2 + 2x - 5 = 3$ or $3x^2 + 2x - 8 = 0$	M1
	$(3x-4)(x+2) = 0 \quad x = \ldots$	M1
	$x = \frac{4}{3}$ (or exact equiv.) , $x = -2$	A1, A1 (6)
		12
	(a) The individual marks are independent, <u>but</u> the 2nd, 3rd and 4th B's are dependent upon a sketch having been attempted. B marks for coordinates: Allow (0, 1), etc. (coordinates the wrong way round) <u>if</u> marked in the correct place on the sketch. (b) M: Attempt to multiply out $(x-1)^2$ and write as a product with $(x+3)$, or attempt to multiply out $(x+3)(x-1)$ and write as a product with $(x-1)$, or attempt to expand $(x+3)(x-1)(x-1)$ directly (at least 7 terms). The $(x-1)^2$ or $(x+3)(x-1)$ expansion must have 3 (or 4) terms, so should not, for example, be just $x^2 + 1$. A: It is not necessary to state explicitly '$k = 3$'. Condone missing brackets if the intention seems clear and a fully correct expansion is seen. (c) 1st M: Attempt to differentiate (correct power of x in at least one term). 2nd M: Setting their derivative equal to 3. 3rd M: Attempt to solve a 3-term quadratic based on their derivative. The equation <u>could</u> come from $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. N.B. After an incorrect k value in (b), full marks are still possible in (c).	

Question 14 : June 08 Q4

Question Number	Scheme	Marks
(a)	$f'(x) = 3 + 3x^2$	M1 A1 (2)
(b)	$3 + 3x^2 = 15$ and start to try and simplify	M1
	$x^2 = k \rightarrow x = \sqrt{k}$ (ignore $\pm$)	M1
	$x = 2$ (ignore $x = -2$)	A1 (3)
		(5 marks)

Question 15 : June 08 Q9

Question Number	Scheme	Marks
(a)	$\left[\frac{\mathrm{d}y}{\mathrm{d}x} =\right] 3kx^2 - 2x + 1$	M1 A1 (2)
(b)	Gradient of line is $\frac{7}{2}$	B1
	When $x = -\frac{1}{2}$: $3k \times \left(\frac{1}{4}\right) - 2 \times \left(-\frac{1}{2}\right) + 1, = \frac{7}{2}$	M1
	$\frac{3k}{4} = \frac{3}{2} \Rightarrow k = 2$	A1 A1 (4)
(c)	$x = -\frac{1}{2} \Rightarrow y = k \times \left(-\frac{1}{8}\right) - \left(\frac{1}{4}\right) - \frac{1}{2} - 5, = -6$	M1 A1 (2)
		(8 marks)

Question 16 : Jan09 Q6

Question Number	Scheme	Marks
(a)	$2x^{3/2}$ or $p = \frac{3}{2}$ (<u>Not</u> $2x\sqrt{x}$)	B1
	$-x$ or $-x^1$ or $q = 1$	B1 (2)
(b)	$\left(\frac{dy}{dx} =\right) 20x^3 + 2 \times \frac{3}{2}x^{1/2} - 1$	M1
	$= \underline{20x^3 + 3x^{\frac{1}{2}} - 1}$	A1A1ftA1ft (4) [6]
(a)	1st B1 for $p = 1.5$ or exact equivalent 2nd B1 for $q = 1$	
(b)	M1 for an attempt to differentiate $x^n \to x^{n-1}$ (for any of the 4 terms) 1st A1 for $20x^3$ (the -3 must 'disappear') 2nd A1ft for $3x^{\frac{1}{2}}$ or $3\sqrt{x}$. Follow through their p but they must be differentiating $2x^p$, where p is a <u>fraction</u>, and the coefficient must be simplified if necessary. 3rd A1ft for -1 (<u>not</u> the unsimplified $-x^0$), or follow through for correct differentiation of their $-x^q$ (i.e. coefficient of x^q is -1). If ft is applied, the coefficient must be simplified if necessary. 'Simplified' coefficient means $\frac{a}{b}$ where a and b are integers with no common factors. Only a single + or − sign is allowed (e.g. − − must be replaced by +). If there is a 'restart' in part (b) it can be marked independently of part (a), but marks for part (a) cannot be scored for work seen in (b). <u>Multiplying</u> by $\sqrt{x}$: (assuming this is a restart) e.g. $y = 5x^4\sqrt{x} - 3\sqrt{x} + 2x^2 - x^{3/2}$ $\left(\frac{dy}{dx} =\right) \frac{45}{2}x^{7/2} - \frac{3}{2}x^{-1/2} + 4x - \frac{3}{2}x^{1/2}$ scores M1 A0 A0 (p not a fraction) A1ft. <u>Extra term</u> included: This invalidates the final mark. e.g. $y = 5x^4 - 3 + 2x^2 - x^{3/2} - x^{1/2}$ $\left(\frac{dy}{dx} =\right) 20x^3 + 4x - \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-1/2}$ scores M1 A1 A0 (p not a fraction) A0. <u>Numerator and denominator differentiated separately</u>: For this, neither of the last two (ft) marks should be awarded. <u>Quotient/product rule</u>: Last two terms must be correct to score the last 2 marks. (If the M mark has not already been earned, it can be given for the quotient/product rule attempt.)	

Question 17 : Jan 09 Q11

Question Number	Scheme	Marks
(a)	$\left(\frac{dy}{dx}=\right) -4+8x^{-2}$ (4 or $8x^{-2}$ for M1… sign can be wrong)	M1A1
	$x=2 \Rightarrow \quad m=-4+2=-2$	M1
	$y=9-8-\frac{8}{2}=-3$ The first 4 marks <u>could</u> be earned in part (b)	B1
	Equation of tangent is: $y+3=-2(x-2) \rightarrow y=1-2x$ (*)	M1 A1cso (6)
(b)	Gradient of normal = $\frac{1}{2}$	B1ft
	Equation is: $\frac{y+3}{x-2}=\frac{1}{2}$ or better equivalent, e.g. $y=\frac{1}{2}x-4$	M1A1 (3)
(c)	$(A:)\ \frac{1}{2}$, $(B:)\ 8$	B1, B1
	Area of triangle is: $\frac{1}{2}(x_B \pm x_A)\times y_P$ with values for all of x_B, x_A and y_P	M1
	$\frac{1}{2}\left(8-\frac{1}{2}\right)\times 3 = \frac{45}{4}$ or 11.25	A1 (4) [13]
(a)	1st M1 for 4 or $8x^{-2}$ (ignore the signs). 1st A1 for both terms correct (including signs). 2nd M1 for substituting $x=2$ into their $\frac{dy}{dx}$ (must be different from their y) B1 for $y_P=-3$, but not if clearly found from the given equation of the <u>tangent</u>. 3rd M1 for attempt to find the equation of tangent at P, follow through their m and y_P. Apply general principles for straight line equations (see end of scheme). <u>NO DIFFERENTIATION ATTEMPTED</u>: Just assuming $m=-2$ at this stage is M0 2nd A1cso for correct work leading to printed answer (allow equivalents with $2x$, y, and 1 terms… such as $2x+y-1=0$).	
(b)	B1ft for correct use of the perpendicular gradient rule. Follow through their m, but if $m \neq -2$ there must be clear evidence that the m is thought to be the gradient of the tangent. M1 for an attempt to find normal at P using their changed gradient and their y_P. Apply general principles for straight line equations (see end of scheme). A1 for any correct form as specified above (correct answer only).	
(c)	1st B1 for $\frac{1}{2}$ and 2nd B1 for 8. M1 for a full method for the area of triangle ABP. Follow through their x_A, x_B and their y_P, but the mark is to be awarded 'generously', condoning sign errors.. The final answer must be positive for A1, with negatives in the working condoned. <u>Determinant</u>: Area $=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2 & -3 & 1 \\ 0.5 & 0 & 1 \\ 8 & 0 & 1 \end{vmatrix} = \ldots$ (Attempt to multiply out required for M1) <u>Alternative</u>: $AP=\sqrt{(2-0.5)^2+(-3)^2}$, $BP=\sqrt{(2-8)^2+(-3)^2}$, Area $=\frac{1}{2}AP\times BP=\ldots$ M1 <u>Intersections with y-axis instead of x-axis</u>: Only the M mark is available B0 B0 M1 A0.	

Question 18 : June 09 Q3

Question Number	Scheme	Marks
Q (a)	$\frac{dy}{dx} = 6x^2 - 6x^{-3}$	M1 A1 A1 (3)
(a)	M1 for an attempt to differentiate $x^n \to x^{n-1}$ 1^{st} A1 for $6x^2$ 2^{nd} A1 for $-6x^{-3}$ or $-\frac{6}{x^3}$ Condone $+ -6x^{-3}$ here. Inclusion of $+c$ scores A0 here.	

Question 19 : June 09 Q9

Question Number	Scheme	Marks
Q (a)	$\left[(3-4\sqrt{x})^2=\right]9-12\sqrt{x}-12\sqrt{x}+(-4)^2x$	M1
	$9x^{-\frac{1}{2}}+16x^{\frac{1}{2}}-24$	A1, A1 (3)
(b)	$f'(x)=-\frac{9}{2}x^{-\frac{3}{2}},\ +\frac{16}{2}x^{-\frac{1}{2}}$	M1 A1, A1ft (3)
(c)	$f'(9)=-\frac{9}{2}\times\frac{1}{27}+\frac{16}{2}\times\frac{1}{3}=-\frac{1}{6}+\frac{16}{6}=\frac{5}{2}$	M1 A1 (2)
		[8]
(a)	M1 for an attempt to expand $(3-4\sqrt{x})^2$ with at least 3 terms correct- as printed or better Or $9-k\sqrt{x}+16x\ \ (k\neq 0)$. See also the MR rule below 1st A1 for their coefficient of $\sqrt{x}=16$. Condone writing $(\pm)9x^{(\pm)\frac{1}{2}}$ instead of $9x^{-\frac{1}{2}}$ 2nd A1 for B = - 24 or their constant term = - 24	
(b)	M1 for an attempt to differentiate an x term $x^n\to x^{n-1}$ 1st A1 for $-\frac{9}{2}x^{-\frac{3}{2}}$ and their constant B differentiated to zero. NB $-\frac{1}{2}\times 9x^{-\frac{3}{2}}$ is A0 2nd A1ft follow through their $Ax^{\frac{1}{2}}$ but can be scored without a value for A, i.e. for $\frac{A}{2}x^{-\frac{1}{2}}$	
(c)	M1 for some correct substitution of $x=9$ in their expression for $f'(x)$ including an attempt at $(9)^{\pm\frac{k}{2}}$ (k odd) somewhere that leads to some appropriate multiples of $\frac{1}{3}$ or 3 A1 accept $\frac{15}{6}$ or any exact equivalent of 2.5 e.g. $\frac{45}{18},\frac{135}{54}$ or even $\frac{67.5}{27}$ Misread (MR) Only allow MR of the form $\frac{(3-k\sqrt{x})^2}{\sqrt{x}}$ N.B. Leads to answer in (c) of $\frac{k^2-1}{6}$ Score as M1A0A0, M1A1A1ft, M1A1ft	

Question 20 : June 09 Q11

Question Number	Scheme	Marks
Q (a)	$x = 2: \quad y = 8 - 8 - 2 + 9 = 7 \quad (*)$	B1 (1)
(b)	$\frac{dy}{dx} = 3x^2 - 4x - 1$	M1 A1
	$x = 2: \quad \frac{dy}{dx} = 12 - 8 - 1 (= 3)$	A1ft
	$y - 7 = 3(x - 2)$, $\quad \underline{y = 3x + 1}$	M1, $\underline{\text{A1}}$ (5)
(c)	$m = -\frac{1}{3}$ (for $-\frac{1}{m}$ with their m)	B1ft
	$3x^2 - 4x - 1 = -\frac{1}{3}$, $\quad 9x^2 - 12x - 2 = 0$ or $x^2 - \frac{4}{3}x - \frac{2}{9} = 0$ (o.e.)	M1, A1
	$\left(x = \frac{12 + \sqrt{144 + 72}}{18}\right)\left(\sqrt{216} = \sqrt{36}\sqrt{6} = 6\sqrt{6}\right)$ or $(3x - 2)^2 = 6 \rightarrow 3x = 2 \pm \sqrt{6}$	M1
	$x = \frac{1}{3}\left(2 + \sqrt{6}\right) \quad (*)$	A1cso (5)
		[11]
(a)	B1 there must be a clear attempt to substitute $x = 2$ leading to 7 e.g. $2^3 - 2 \times 2^2 - 2 + 9 = 7$	
(b)	1st M1 for an attempt to differentiate with at least one of the given terms fully correct. 1st A1 for a fully correct expression 2nd A1ft for sub. $x = 2$ in $\underline{\text{their}}$ $\frac{dy}{dx} (\neq y)$ accept for a correct expression e.g. $3 \times (2)^2 - 4 \times 2 - 1$ 2nd M1 for use of their "3" (provided it comes from their $\frac{dy}{dx} (\neq y)$ and x=2) to find equation of tangent. Alternative is to use (2, 7) in $y = mx + c$ to $\underline{\text{find a value}}$ for c. Award when $c = \ldots$ is seen. **No attempted use of $\frac{dy}{dx}$ in (b) scores 0/5**	
(c)	1st M1 for forming an equation from their $\frac{dy}{dx} (\neq y)$ and their $-\frac{1}{m}$ (must be changed from m) 1st A1 for a correct 3TQ all terms on LHS (condone missing =0) 2nd M1 for proceeding to $x = \ldots$ or $3x = \ldots$ by formula or completing the square for a 3TQ. Not factorising. Condone $\pm$ 2nd A1 for proceeding to given answer with no incorrect working seen. Can still have $\pm$.	
ALT	$\underline{\text{Verify (for M1A1M1A1)}}$ 1st M1 for attempting to square need ≥ 3 correct values in $\frac{4+6+4\sqrt{6}}{9}$, 1st A1 for $\frac{10+4\sqrt{6}}{9}$ 2nd M1 Dependent on 1st M1 in this case for substituting in all terms of their $\frac{dy}{dx}$ 2nd A1cso for cso $\underline{\text{with a full comment}}$ e.g. "the x co-ord of Q is ..."	

Question 21 : Jan 10 Q1

Question number	Scheme	Marks
	$x^4 \to kx^3$ or $x^{1/3} \to kx^{-2/3}$ or $3 \to 0$ (k a non-zero constant)	M1
	$\left(\frac{dy}{dx} =\right)$ $4x^3$.......... , with '3' differentiated to zero (or 'vanishing')	A1
	$\left(\frac{dy}{dx} =\right)$ .... $+\frac{1}{3}x^{-2/3}$ or equivalent, e.g. $\frac{1}{3\sqrt[3]{x^2}}$ or $\frac{1}{3\left(\sqrt[3]{x}\right)^2}$	A1 [3]
	1st A1 requires $4x^3$, <u>and</u> 3 differentiated to zero. Having '$+C$' loses the 1st A mark. Terms not added, but otherwise correct, e.g. $4x^3$, $\frac{1}{3}x^{-2/3}$ loses the 2nd A mark.	

Question 22 : Jan 10 Q6

Question number	Scheme	Marks
	(a) $y = \dfrac{x^2 - 5x - 24}{x} = x - 5 - 24x^{-1}$ (or equiv., e.g. $x + 3 - 8 - \dfrac{24}{x}$)	M1 A1
	$\dfrac{dy}{dx} = 1 + 24x^{-2}$ or $\dfrac{dy}{dx} = 1 + \dfrac{24}{x^2}$	M1 A1 (4)
	(b) $x = 2$: $y = -15$ Allow if seen in part (a).	B1
	$\left(\dfrac{dy}{dx} =\right)$ $1 + \dfrac{24}{4} = 7$ Follow-through from candidate's <u>non-constant</u> $\dfrac{dy}{dx}$. This must be simplified to a "single value".	B1ft
	$y + 15 = 7(x - 2)$ (or equiv., e.g. $y = 7x - 29$) Allow $\dfrac{y+15}{x-2} = 7$	M1 A1 (4) [8]
	(a) 1st M: Mult. out to get $x^2 + bx + c$, $b \neq 0$, $c \neq 0$ <u>and</u> dividing by x (<u>not</u> x^2). Obtaining one correct term, e.g. x........ is sufficient evidence of a division attempt. 2nd M: <u>Dependent on the 1st M</u>: Evidence of $x^n \to kx^{n-1}$ for one x term (i.e. not just the constant term) is sufficient). Note that mark is <u>not</u> given if, for example, the numerator and denominator are differentiated separately. A mistake in the 'middle term', e.g. $x + 5 - 24x^{-1}$, does not invalidate the 2nd A mark, so M1 A0 M1 A1 is possible. (b) B1ft: For evaluation, using $x = 2$, of their $\dfrac{dy}{dx}$, even if unlabelled or called y. M: For the equation, in any form, of a straight line through (2, '-15') with candidate's $\dfrac{dy}{dx}$ value as gradient. Alternative is to use (2, '-15') in $y = mx + c$ to <u>find a value</u> for c, in which case $y = 7x + c$ leading to $c = -29$ is sufficient for the A1). (See general principles for straight line equations at the end of the scheme). Final A: 'Unsimplified' forms are acceptable, but... $y - (-15) = 7(x - 2)$ is A0 (unresolved 'minus minus').	

Question 23 : June 10 Q7

Question Number	Scheme	Marks
	$\frac{3x^2+2}{x} = 3x + 2x^{-1}$	M1 A1
	$(y' =) 24x^2, -2x^{-\frac{1}{2}}, +3 - 2x^{-2}$	M1 A1 A1A1
	$\left[24x^2 - 2x^{-\frac{1}{2}} + 3 - 2x^{-2}\right]$	6
	Notes	
	1st M1 for attempting to divide(one term correct) 1st A1 for both terms correct on the same line, accept $3x^1$ for $3x$ or $\frac{2}{x}$ for $2x^{-1}$ These first two marks may be implied by a correct differentiation at the end. 2nd M1 for an attempt to differentiate $x^n \to x^{n-1}$ for at least one term of their expression "Differentiating" $\frac{3x^2+2}{x}$ and getting $\frac{6x}{1}$ is M0 2nd A1 for $24x^2$ only 3rd A1 for $-2x^{-\frac{1}{2}}$ allow $\frac{-2}{\sqrt{x}}$. Must be simplified to this, not e.g. $\frac{-4}{2}x^{-\frac{1}{2}}$ 4th A1 for $3-2x^{-2}$ allow $\frac{-2}{x^2}$. Both terms needed. Condone $3+(-2)x^{-2}$ If "+c" is included then they lose this final mark They do not need one line with all terms correct for full marks. Award marks when first seen in this question and apply ISW. Condone a mixed line of some differentiation and some division e.g. $24x^2 - 4x^{\frac{1}{2}} + 3x + 2x^{-1}$ can score 1st M1A1 and 2nd M1A1	
Quotient /Product Rule	$\frac{x(6x)-(3x^2+2)\times 1}{x^2}$ or $6x(x^{-1})+(3x^2+2)(-x^{-2})$ $\frac{3x^2-2}{x^2}$ or $3-\frac{2}{x^2}$ (o.e.)	1st M1 for an attempt: $\frac{P-Q}{x^2}$ or $R+(-S)$ with one of P,Q or R,S correct. 1st A1 for a correct expression 4th A1 same rules as above

Question 24 : Jan 11 Q1

(a)	$\left(\frac{dy}{dx}=\right)\frac{3}{2}x^2-\frac{27}{2}x^{\frac{1}{2}}-8x^{-2}$	M1A1A1A1 (4)
(b)	$x=4 \;\Rightarrow\; y=\frac{1}{2}\times 64-9\times 2^3+\frac{8}{4}+30$	M1
	$= 32 - 72 + 2 + 30 \qquad = \underline{-8}$ *	A1cso (2)
(c)	$x=4 \;\Rightarrow\; y'=\frac{3}{2}\times 4^2-\frac{27}{2}\times 2-\frac{8}{16}$	M1
	$= 24-27-\frac{1}{2}= \qquad -\frac{7}{2}$	A1
	Gradient of the normal = $-1\div"\frac{7}{2}"$	M1
	Equation of normal: $y--8=\frac{2}{7}(x-4)$	M1A1ft
	$\underline{7y-2x+64=0}$	A1 (6) 12

Question Number	Scheme	Marks
	Notes	
(a)	1st M1 for an attempt to differentiate $x^n \to x^{n-1}$ 1st A1 for one correct term in x 2nd A1 for 2 terms in x correct 3rd A1 for all correct x terms. No 30 term and no $+c$.	
(b)	M1 for substituting $x = 4$ into $y =$ and attempting $4^{\frac{3}{2}}$ A1 note this is a printed answer	
(c)	1st M1 Substitute x = 4 into y' (allow slips) A1 Obtains −3.5 or equivalent 2nd M1 for correct use of the perpendicular gradient rule using their gradient. (May be slip doing the division) Their gradient must have come from y' 3rd M1 for an attempt at equation of tangent or normal at P 2nd A1ft for correct use of their changed gradient to find **normal** at P. Depends on 1st, 2nd and 3rd Ms 3rd A1 for any equivalent form with integer coefficients	

Question 25 : June 11 Q2

Question Number	Scheme	Marks
(a)	$\frac{dy}{dx} = 10x^4 - 3x^{-4}$ or $10x^4 - \frac{3}{x^4}$	M1 A1 A1 (3)
(b)	$\left(\int =\right) \frac{2x^6}{6} + 7x + \frac{x^{-2}}{-2} = \frac{x^6}{3} + 7x - \frac{x^{-2}}{2}$ $+C$	M1 A1 A1 B1 (4) 7

Notes

(a) M1: Attempt to differentiate $x^n \to x^{n-1}$ (for any of the 3 terms)
i.e. ax^4 or ax^{-4}, where a is any non-zero constant or
the 7 differentiated to give 0 is sufficient evidence for M1

1st A1: One correct (non-zero) term, possibly unsimplified.

2nd A1: Fully correct **simplified** answer.

(b) M1: Attempt to integrate $x^n \to x^{n+1}$
(i.e. ax^6 or ax or ax^{-2}, where a is any non-zero constant).

1st A1: Two correct terms, possibly unsimplified.

2nd A1: All three terms correct and **simplified**.

Allow correct equivalents to printed answer , e.g. $\frac{x^6}{3} + 7x - \frac{1}{2x^2}$ or $\frac{1}{3}x^6 + 7x - \frac{1}{2}x^{-2}$

Allow $\frac{1x^6}{3}$ or $7x^1$

B1: $+ C$ appearing at any stage in part (b) (independent of previous work)