Question 1 : June 05 Q7

(a)	Setting $p = 300$ at $t = 0 \Rightarrow 300 = \dfrac{2800a}{1+a}$	M1
	$(300 = 2500a)$; $a = 0.12$ (c.s.o) *	dM1A1 (3)
(b)	$1850 = \dfrac{2800(0.12)e^{0.2t}}{1+0.12e^{0.2t}}$; $e^{0.2t} = 16.2...$	M1A1
	Correctly taking logs to $0.2t = \ln k$	M1
	$t = 14$ (13.9..)	A1 (4)
(c)	Correct derivation: (Showing division of num. and den. by $e^{0.2t}$; using a)	B1 (1)
(d)	Using $t \to \infty$, $e^{-0.2t} \to 0$,	M1
	$p \to \dfrac{336}{0.12} = 2800$	A1 (2)
		[10]

Question 2 : June 06 Q4

(a)	425 ºC	B1	(1)
(b)	$300 = 400e^{-0.05t} + 25 \Rightarrow 400e^{-0.05t} = 275$ sub. $T = 300$ and attempt to rearrange to $e^{-0.05t} = a$, where $a \in$ Q	M1	
	$e^{-0.05t} = \dfrac{275}{400}$	A1	
	M1 correct application of logs	M1	
	$t = 7.49$	A1	(4)
(c)	$\dfrac{dT}{dt} = -20\ e^{-0.05t}$ (M1 for $ke^{-0.05t}$)	M1 A1	
	At $t = 50$, rate of decrease = (±) 1.64 ºC/min	A1	(3)
(d)	$T > 25$, (since $e^{-0.05t} \to 0$ as $t \to \infty$)	B1	(1)
		(9 marks)	

Question 3 : June 07 Q1

Question Number	Scheme	Marks
(*a*)	$\ln 3x = \ln 6$ or $\ln x = \ln\left(\frac{6}{3}\right)$ or $\ln\left(\frac{3x}{6}\right) = 0$	M1
	$x = 2$ (only this answer)	A1 (cso) (2)
(*b*)	$(e^x)^2 - 4e^x + 3 = 0$ (any 3 term form)	M1
	$(e^x - 3)(e^x - 1) = 0$	
	$e^x = 3$ or $e^x = 1$ Solving quadratic	M1 dep
	$x = \ln 3$, $x = 0$ (or ln 1)	M1 A1 (4)
		(6 marks)

Notes: (a) Answer $x = 2$ with no working or no incorrect working seen: M1A1

Note: $x = 2$ from $\ln x = \frac{\ln 6}{\ln 3} = \ln 2$ M0A0

$\ln x = \ln 6 - \ln 3 \Rightarrow x = e^{(\ln 6 - \ln 3)}$ allow M1, $x = 2$ (no wrong working) A1

(b) 1st M1 for attempting to multiply through by e^x : Allow y, X, even x, for e^x

2nd M1 is for solving quadratic as far as getting two values for e^x or y or X etc

3rd M1 is for converting their answer(s) of the form $e^x = k$ to $x = \ln k$ (must be exact)

A1 is for ln3 **and** ln1 or 0 (Both required and no further solutions)

Question 4 : June 07 Q8

Question Number	Scheme	Marks
(a)	$D=10,\ t=5,\quad x=10e^{-\frac{1}{8}\times 5}$ $=5.353$ awrt	M1 A1 (2)
(b)	$D=10+10e^{-\frac{5}{8}},\ t=1,\quad x=15.3526\ldots\times e^{-\frac{1}{8}}$ $x=13.549$ (✱)	M1 A1 cso (2)
Alt.(b)	$x=10e^{-\frac{1}{8}\times 6}+10e^{-\frac{1}{8}\times 1}$ M1 $\quad x=13.549$ (✱) A1 cso	
(c)	$15.3526\ldots e^{-\frac{1}{8}T}=3$ $e^{-\frac{1}{8}T}=\dfrac{3}{15.3526\ldots}=0.1954\ldots$ $-\dfrac{1}{8}T=\ln 0.1954\ldots$ $T=13.06\ldots$ or 13.1 or 13	M1 M1 A1 (3) **(7 marks)**

Notes: (b) (main scheme) M1 is for $(10+10e^{-\frac{5}{8}})e^{-\frac{1}{8}}$, or $\{10+\text{their(a)}\}\,e^{-\frac{1}{8}}$

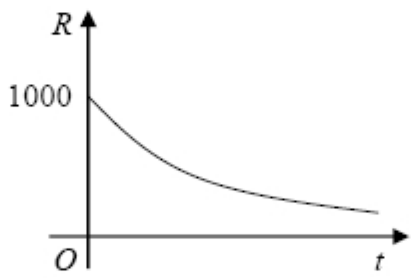
N.B. The answer is given. There are many correct answers seen which deserve M0A0 or M1A0

(c) 1st M is for $(10+10e^{-\frac{5}{8}})\,e^{-\frac{T}{8}}=3$ o.e.

2nd M is for converting $e^{-\frac{T}{8}}=k$ $(k>0)$ to $-\dfrac{T}{8}=\ln k$. This is independent of 1st M.

Trial and improvement: M1 as scheme,
M1 correct process for their equation (two equal to 3 s.f.)
A1 as scheme

Question 5 : Jan 08 Q5

Question Number	Scheme		Marks	
	(a) 1000		B1	(1)
	(b) $1000\,e^{-5730c} = 500$		M1	
	$e^{-5730c} = \frac{1}{2}$		A1	
	$-5730c = \ln\frac{1}{2}$		M1	
	$c = 0.000121$	cao	A1	(4)
	(c) $R = 1000\,e^{-22920c} = 62.5$	Accept 62-63	M1 A1	(2)
	(d) 	Shape 1000	B1 B1	(2) **[9]**

Question 6 : June 09 Q3

Question Number	Scheme		Marks	
Q	$P = 80e^{\frac{t}{5}}$			
(a)	$t = 0 \Rightarrow P = 80e^{\frac{0}{5}} = 80(1) = \underline{80}$	$\underline{80}$	B1	
				(1)
(b)	$P = 1000 \Rightarrow 1000 = 80e^{\frac{t}{5}} \Rightarrow \frac{1000}{80} = e^{\frac{t}{5}}$	Substitutes $P = 1000$ and rearranges equation to make $e^{\frac{t}{5}}$ the subject.	M1	
	$\therefore\ t = 5\ln\left(\frac{1000}{80}\right)$			
	$t = 12.6286...$	$\boxed{\text{awrt 12.6 or 13 years}}$	A1	
		Note $t = 12$ or $t = \text{awrt } 12.6 \Rightarrow t = 12$ will score A0		(2)
(c)	$\frac{dP}{dt} = 16e^{\frac{t}{5}}$	$ke^{\frac{1}{5}t}$ and $k \neq 80$.	M1	
		$16e^{\frac{1}{5}t}$	A1	
				(2)
(d)	$50 = 16e^{\frac{t}{5}}$			
	$\therefore\ t = 5\ln\left(\frac{50}{16}\right)$ $\{= 5.69717...\}$	Using $50 = \frac{dP}{dt}$ and an attempt to solve to find the value of t or $\frac{t}{5}$.	M1	
	$P = 80e^{\frac{1}{5}\left(5\ln\left(\frac{50}{16}\right)\right)}$ or $P = 80e^{\frac{1}{5}(5.69717...)}$	Substitutes their value of t back into the equation for P.	dM1	
	$P = \frac{80(50)}{16} = \underline{250}$	$\underline{250}$ or awrt 250	A1	
				(3)
				[8]

Question 7 : Jan 10 Q9

Question Number	Scheme		Marks
(i)(a)	$\ln(3x-7)=5$		
	$e^{\ln(3x-7)}=e^5$	Takes e of both sides of the equation. This can be implied by $3x-7=e^5$.	M1
	$3x-7=e^5 \Rightarrow x=\dfrac{e^5+7}{3}$ $\{=51.804...\}$	Then rearranges to make x the subject.	dM1
		Exact answer of $\dfrac{e^5+7}{3}$.	A1
			(3)
(b)	$3^x e^{7x+2}=15$		
	$\ln\left(3^x e^{7x+2}\right)=\ln 15$	Takes ln (or logs) of both sides of the equation.	M1
	$\ln 3^x + \ln e^{7x+2} = \ln 15$	Applies the addition law of logarithms.	M1
	$x\ln 3 + 7x + 2 = \ln 15$	$x\ln 3 + 7x+2 = \ln 15$	A1 oe
	$x(\ln 3 + 7) = -2 + \ln 15$	Factorising out at least two x terms on one side and collecting number terms on the other side.	ddM1
	$x=\dfrac{-2+\ln 15}{7+\ln 3}$ $\{=0.0874...\}$	*Exact answer* of $\dfrac{-2+\ln 15}{7+\ln 3}$	A1 oe
			(5)

Question 8 : June 10 Q8

Question Number	Scheme	Marks
(a)	$\dfrac{(x+5)(2x-1)}{(x+5)(x-3)} = \dfrac{(2x-1)}{(x-3)}$	M1 B1 A1 **aef** (3)
(b)	$\ln\left(\dfrac{2x^2+9x-5}{x^2+2x-15}\right)=1$ $\dfrac{2x^2+9x-5}{x^2+2x-15}=\mathrm{e}$ $\dfrac{2x-1}{x-3}=\mathrm{e} \Rightarrow \quad 3\mathrm{e}-1 = x(\mathrm{e}-2)$ $\Rightarrow \ x = \dfrac{3\mathrm{e}-1}{\mathrm{e}-2}$	M1 dM1 M1 A1 **aef cso** (4) **[7]**
	(a) M1: An attempt to factorise the numerator. B1: Correct factorisation of denominator to give $(x+5)(x-3)$. Can be seen anywhere. (b) M1: Uses a correct law of logarithms to combine at least two terms. This usually is achieved by the subtraction law of logarithms to give $\ln\left(\dfrac{2x^2+9x-5}{x^2+2x-15}\right)=1$. The product law of logarithms can be used to achieve $\ln\left(2x^2+9x-5\right)=\ln\left(\mathrm{e}\left(x^2+2x-15\right)\right)$. The product and quotient law could also be used to achieve $\ln\left(\dfrac{2x^2+9x-5}{\mathrm{e}(x^2+2x-15)}\right)=0$. dM1: Removing ln's correctly by the realisation that the anti-ln of 1 is e. Note that this mark is dependent on the previous method mark being awarded. M1: Collect x terms together and factorise. Note that this is not a dependent method mark. A1: $\dfrac{3\mathrm{e}-1}{\mathrm{e}-2}$ or $\dfrac{3\mathrm{e}^1-1}{\mathrm{e}^1-2}$ or $\dfrac{1-3\mathrm{e}}{2-\mathrm{e}}$. aef Note that the answer needs to be in terms of e. The decimal answer is 9.9610559… Note that the solution must be correct in order for you to award this final accuracy mark. **Note: See Appendix for an alternative method of long division.**	

Question 9 : Jan 11 Q4

Question Number	Scheme		Marks
(a)	$\theta = 20 + Ae^{-kt}$ (eqn *)		
	$\{t=0, \theta=90 \Rightarrow\}$ $90 = 20 + Ae^{-k(0)}$	Substitutes $t=0$ and $\theta=90$ into eqn *	M1
	$90 = 20 + A \Rightarrow \underline{A = 70}$	$\underline{A=70}$	A1
			(2)
(b)	$\theta = 20 + 70e^{-kt}$		
	$\{t=5, \theta=55 \Rightarrow\}$ $55 = 20 + 70e^{-k(5)}$ $\frac{35}{70} = e^{-5k}$	Substitutes $t=5$ and $\theta=55$ into eqn * and rearranges eqn * to make $e^{\pm 5k}$ the subject.	M1
	$\ln\left(\frac{35}{70}\right) = -5k$	Takes 'lns' and proceeds to make '$\pm 5k$' the subject.	dM1
	$-5k = \ln\left(\frac{1}{2}\right)$		
	$-5k = \ln 1 - \ln 2 \Rightarrow -5k = -\ln 2 \Rightarrow \underline{k = \frac{1}{5}\ln 2}$	Convincing proof that $k = \frac{1}{5}\ln 2$	A1 *
			(3)
(c)	$\theta = 20 + 70e^{-\frac{1}{5}t\ln 2}$		
	$\frac{d\theta}{dt} = -\frac{1}{5}\ln 2.(70)e^{-\frac{1}{5}t\ln 2}$	$\pm \alpha e^{-kt}$ where $k = \frac{1}{5}\ln 2$ $-14\ln 2e^{-\frac{1}{5}t\ln 2}$	M1 A1 oe
	When $t = 10$, $\frac{d\theta}{dt} = -14\ln 2e^{-2\ln 2}$		
	$\frac{d\theta}{dt} = -\frac{7}{2}\ln 2 = -2.426015132...$		
	Rate of decrease of $\theta = 2.426\ ^{\circ}C/\text{min}$ (3 dp.)	awrt ± 2.426	A1
			(3) **[8]**

Question 10 : June 11 Q5

Question Number	Scheme	Marks	
(a)	$p=7.5$	B1	
			(1)
(b)	$2.5 = 7.5e^{-4k}$	M1	
	$e^{-4k} = \frac{1}{3}$	M1	
	$-4k = \ln(\frac{1}{3})$	dM1	
	$-4k = -\ln(3)$		
	$k = \frac{1}{4}\ln(3)$	A1*	
	See notes for additional correct solutions and the last A1		
			(4)
(c)	$\frac{dm}{dt} = -kpe^{-kt}$ ft on their p and k	M1A1ft	
	$-\frac{1}{4}ln3 \times 7.5e^{-\frac{1}{4}(ln3)t} = -0.6ln3$		
	$e^{-\frac{1}{4}(ln3)t} = \frac{2.4}{7.5} = (0.32)$	M1A1	
	$-\frac{1}{4}(ln3)t = \ln(0.32)$	dM1	
	$t=4.1486....$ 4.15 or awrt 4.1	A1	
			(6)
			11Marks