Differentiation : Modelling & Stationary Points - Edexcel Past Exam Questions

1.

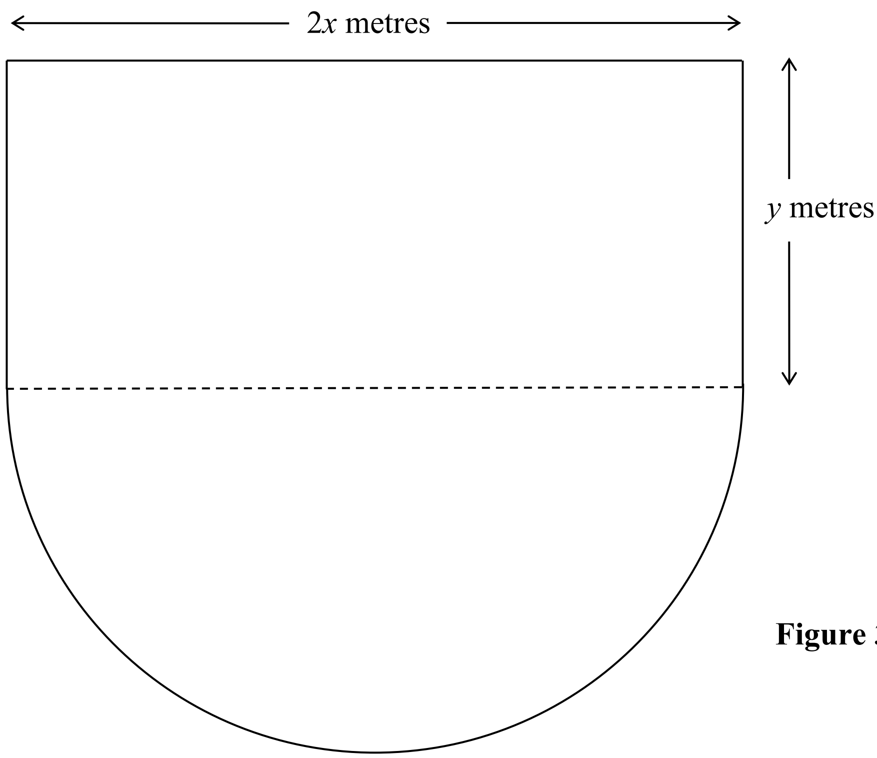

Figure 3

Figure 3 shows the plan of a stage in the shape of a rectangle joined to a semicircle. The length of the rectangular part is $2x$ metres and the width is y metres. The diameter of the semicircular part is $2x$ metres. The perimeter of the stage is 80 m.

(*a*) Show that the area, A m^2, of the stage is given by

$$A = 80x - \left(2 + \frac{\pi}{2}\right)x^2.$$

(4)

(*b*) Use calculus to find the value of x at which A has a stationary value. **(4)**

(*c*) Prove that the value of x you found in part (*b*) gives the maximum value of A. **(2)**

(*d*) Calculate, to the nearest m^2, the maximum area of the stage. **(2)**

Jan 05 Q9

2. Find the coordinates of the stationary point on the curve with equation $y = 2x^2 - 12x$. **(4)**

June 05 Q1

3. The curve C has equation

$$y = 2x^3 - 5x^2 - 4x + 2.$$

(a) Find $\frac{dy}{dx}$. **(2)**

(b) Using the result from part (a), find the coordinates of the turning points of C. **(4)**

(c) Find $\frac{d^2y}{dx^2}$. **(2)**

(d) Hence, or otherwise, determine the nature of the turning points of C. **(2)**

Jan 06 Q7

4. A diesel lorry is driven from Birmingham to Bury at a steady speed of v kilometres per hour. The total cost of the journey, £C, is given by

$$C = \frac{1400}{v} + \frac{2v}{7}.$$

(a) Find the value of v for which C is a minimum **(5)**

(b) Find $\frac{d^2C}{dv^2}$ and hence verify that C is a minimum for this value of v. **(2)**

(c) Calculate the minimum total cost of the journey. **(2)**

Jan 07 Q8

5.

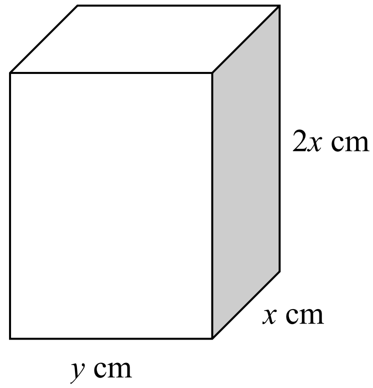

Figure 4

Figure 4 shows a solid brick in the shape of a cuboid measuring $2x$ cm by x cm by y cm.

The total surface area of the brick is 600 cm^2.

(a) Show that the volume, V cm^3, of the brick is given by

$$V = 200x - \frac{4x^3}{3}.$$

(4)

Given that x can vary,

(b) use calculus to find the maximum value of V, giving your answer to the nearest cm^3. **(5)**

(c) Justify that the value of V you have found is a maximum. **(2)**

June 07 Q10

6.

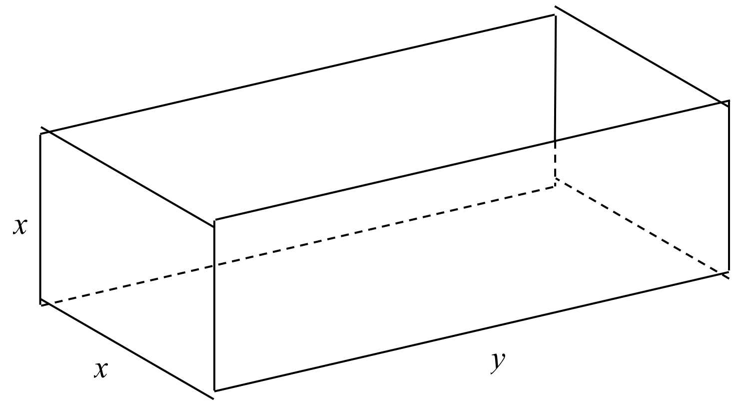

Figure 4

Figure 4 shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is x metres.

The capacity of the tank is 100 m^3.

(a) Show that the area A m^2 of the sheet metal used to make the tank is given by

$$A = \frac{300}{x} + 2x^2.$$ **(4)**

(b) Use calculus to find the value of x for which A is stationary. **(4)**

(c) Prove that this value of x gives a minimum value of A. **(2)**

(d) Calculate the minimum area of sheet metal needed to make the tank. **(2)**

Jan 08 Q9

7. A solid right circular cylinder has radius r cm and height h cm.

The total surface area of the cylinder is 800 cm^2 .

(a) Show that the volume, V cm^3 , of the cylinder is given by

$$V = 400r - \pi r^3.$$ **(4)**

Given that r varies,

(b) use calculus to find the maximum value of V, to the nearest cm^3. **(6)**

(c) Justify that the value of V you have found is a maximum. **(2)**

Jan 09 Q10

8. The curve C has equation $y = 12\sqrt{(x)} - x^{\frac{3}{2}} - 10,\ x > 0$.

(a) Use calculus to find the coordinates of the turning point on C. **(7)**

(b) Find $\frac{d^2y}{dx^2}$. **(2)**

(c) State the nature of the turning point. **(1)**

Jan 10 Q9

9. $y = x^2 - k\sqrt{x}$, where k is a constant.

(a) Find $\frac{dy}{dx}$. **(2)**

(b) Given that y is decreasing at $x = 4$, find the set of possible values of k. **(2)**

June 10 Q3

10.

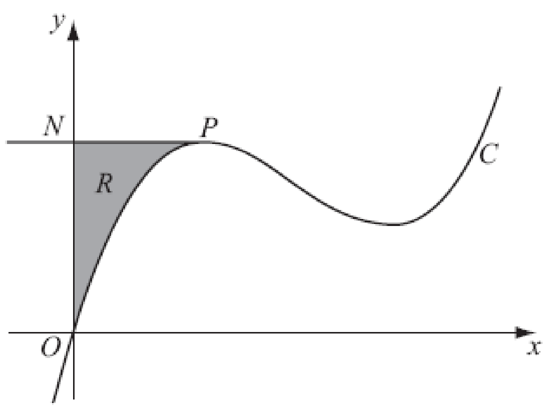

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx,$$

where k is a constant.

The point P on C is the maximum turning point.

Given that the x-coordinate of P is 2,

(a) show that $k = 28$. **(3)**

June 10 Q8

11. The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5 - x)^2, \quad 0 < x < 5.$$

(a) Find $\dfrac{dV}{dx}$. **(4)**

(b) Hence find the maximum volume of the box. **(4)**

(c) Use calculus to justify that the volume that you found in part (b) is a maximum. **(2)**

Jan 11 Q10

12.

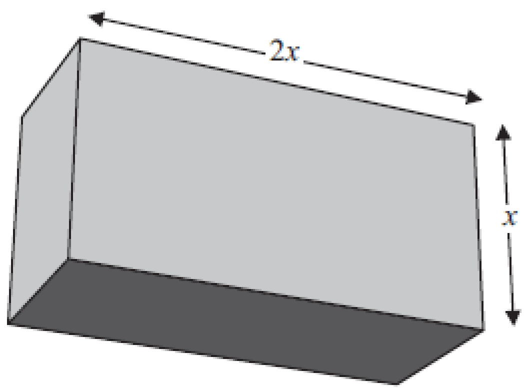

Figure 2

A cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cm, as shown in Figure 2.

The volume of the cuboid is 81 cubic centimetres.

(a) Show that the total length, L cm, of the twelve edges of the cuboid is given by

$$L = 12x + \frac{162}{x^2}.$$ **(3)**

(b) Use calculus to find the minimum value of L. **(6)**

(c) Justify, by further differentiation, that the value of L that you have found is a minimum. **(2)**

June 11 Q8
