Name:

Total Marks:

Pure Mathematics 1

Advanced Subsidiary

Practice Paper A

Time: 2 hours

Written by
K.Naiker

Information for Candidates

- This practice paper follows the Edexcel GCE AS level Specifications
- There are 15 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

1. Given that $y = \frac{1}{16}x^4$, express each of the following in the form kx^n , where k and n are constants.

(a) $y^{\frac{1}{4}}$ **(1)**

(b) $2y^{-1}$ **(1)**

(c) $(4y)^{\frac{3}{2}}$ **(1)**

(Total 3 marks)

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2. Find all the roots of the function $f(x) = 4x - 14x^{\frac{1}{2}} + 6$ **(Total 3 marks)**

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3. Find the set of values of k for which the equation $kx^2 - (k+2)x + 3k = 2$ has no real roots, except for one value which must be stated. Give your answer in set notation and in exact form **(4)**

(Total 4 marks)

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4. A ball is being projected from a point on the top of a building above the horizontal ground. The height, in metres, of the ball after t seconds can be modelled by the function:

$$h(t) = 13.7t + 7.8 - 2.9t^2$$

(a) Use the model to find the height of the tower **(1)**

(b) After how many seconds does the ball hit the ground **(2)**

(c) Rewrite $h(t)$ in the form $A - B(t - C)^2$ where A, B and C are constants **(3)**

(d) With reference to your answer in part **(c)** or otherwise, find the maximum height of the ball above the ground, and the time at which this maximum height is reached **(2)**

(Total 8 marks)

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5. (a) (i) Sketch the graph of $y = x(x-2)(x+1)^2$ stating clearly the points of intersection with the axes **(3)**

(ii) State the range of values for which $f(x) \leq 0$ **(1)**

(iii) The point with coordinates $(-2, 0)$ lies on the curve with equation $y = (x+k)(x+k-2)(x+k+1)^2$ where k is a constant.
Find the possible values of k **(1)**

(b) On seperate axes, sketch the graph $3y = -x(x-2)(x+1)^2$ stating clearly the points of intersection with the axes **(3)**

(Total 8 marks)

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6. A has position vector $5\mathrm{i} - 2\mathrm{j}$ and the point B has position vector $-4\mathrm{i} + 3\mathrm{j}$.

Given that C is the point such that $\overrightarrow{AC} = 2\overrightarrow{AB}$

(a) find the unit vector in the direction of $\overrightarrow{OC}$ **(3)**

D is a point with position vector $ai - 3j$, where a is a constant.
Given that $\overrightarrow{OD} = b\overrightarrow{OA} + \overrightarrow{OB}$, where b is a constant.

(b) Find the values of a and b **(2)**

(Total 5 marks)

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7. (a) Find the first 3 terms of the expansion $(1+3px)^9$ in ascending powers of x leaving each term in its simplest form where p is a non-zero constant **(2)**

(b) Given that, in the expansion of $(1+3px)^9$, the coefficient of x is q and the coefficient of x^2 is $4q$, find the value of p and q **(3)**

(Total 5 marks)

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8. The circle C has equation $x^2 + y^2 + 2x - 20y = -51$
The line L with equation $3y - 4x = 9$ intersects the circle at the points P and Q.
Given that the x coordinate of Q is > 0

(a) Find the centre and radius of the circle **(2)**

(b) Find the equation of the tangent at the point P and the point Q **(4)**

Points P and Q form the chord PQ of the circle

(c) Find the equation of the perpendicular bisector of the chord PQ giving your answers in the form $ax + by + c = 0$ where a, b and c are constants **(3)**

(d) The perpendicular bisector and the two tangents intersect at a single point. Find the coordinate of the point of intersection. **(3)**

(Total 12 marks)

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9. (a) Prove that for any positive values of a and b

$$\frac{4a}{b} + \frac{b}{a} \geq 4$$ **(3)**

(b) By use of a counter-example, show that this inequality is not true when either a or b is not positive. **(2)**

(Total 5 marks)

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10. (a) Show that $\dfrac{\sin^4 x + \sin^2 x \cos^2 x}{\cos^2 x - 1} \equiv -1$ **(3)**

(b) Hence solve the equation $\dfrac{\sin^4 x + \sin^2 x \cos^2 x}{\cos^2 x - 1} + 4 = 2\sin^2 x + 3\cos x$ in the interval $0 \leq x \leq 360^o$ **(5)**

(Total 8 marks)

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11. Solve the following equations, giving your solutions as exact values

(a) $\ln(3x - 5) = 2 - \ln 3$ **(3)**

(b) $e^{x} + 5e^{-x} = 6$ **(3)**

(Total 6 marks)

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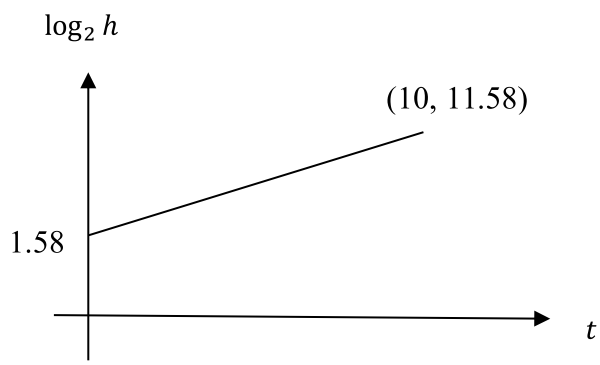
12. A container is being filled with water. After t seconds, the height h mm, of the water present in the container can be modelled by the equation $h = ab^{t}$, where a and b are constants to be found.



The graph passes through the points (0, 1.58) and (10, 11.58)

(a) Sketch the graph of h against t **(2)**

(b) Comparing the graph in part **(a)** to the graph of $\log_2 h$ against t drawn above, state which graph is more useful for calculations. Explain your reasoning **(2)**

(c) Write down an equation of the line **(2)**

(d) Find the values of a and b, giving your answers to 1 significant figures where appropriate **(2)**

(e) Interpret the meaning of a in this model **(1)**

(f) Suggest one reason why this model in unsuitable for long periods of time and suggest an improvement to the model **(2)**

(Total 11 marks)

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13. Given that $f(x) = \frac{2}{3}x^3 - 6x^2 + 20x$

(a) Prove that the function $f(x)$ is increasing for all real values of x **(3)**

(b) Sketch the graph of the gradient function $y = f'(x)$ **(2)**

(Total 5 marks)

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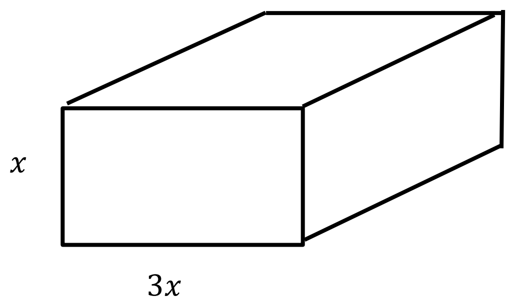
14. The diagram below shows a box in the form of a cuboid. The cuboid has a rectangular cross section where the length of the rectangle is equal to three times its width, x cm. The volume of the cuboid is 144 cm^3 .



(a) Show that the surface area A of the box is $A = \frac{6}{x}(64 + x^3)$ **(3)**

(b) Use calculus to find the minimum value of A **(4)**

(c) Justify that the value of A you have found is a minimum **(2)**

(Total 9 marks)

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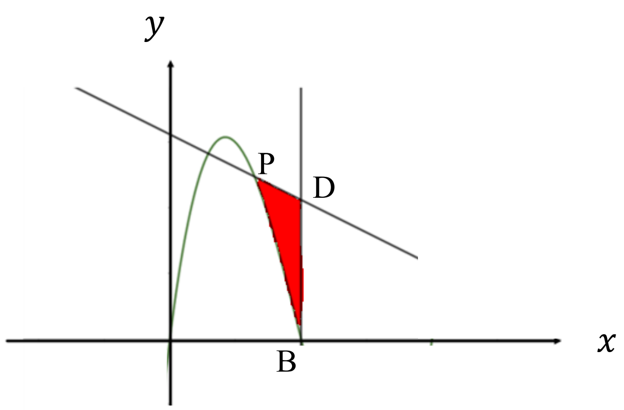
15. The diagram below shows a sketch of part of the curve C with equation

$$y = x(x-2)(x-4)$$



The point P lies on C and has x coordinate $2-\sqrt{2}$. The curve C cuts the x-axis at point B at (2,0). The normal to the curve at P meets the vertical line at B at the point D.

(a) Show that the equation of the normal to the curve at P is $2y + x = 2 + 3\sqrt{2}$ **(4)**

(b) Use calculus to find the exact area of the shaded region **(4)**

(Total 8 marks)

TOTAL FOR PAPER IS 100 MARKS