## Arc Length and Areas of Sectors - Edexcel Past Exam Questions MARK SCHEME

## Question 1: Jan 05 Q7

| Question | Scheme | Marks |
| :---: | :---: | :---: |
|  | (a) $r \theta=8 \times 0.7=5.6(\mathrm{~cm})$ <br> (b) $B C^{2}=8^{2}+11^{2}-2 \times 8 \times 11 \times \cos 0.7$ <br> $\Rightarrow B C=7.098$ or $7.10(\mathrm{Awrt})$ or $\sqrt{(50.4)}$ or better Perimeter $=(a)+(11-8)+B C,=15.7(\mathrm{~cm})$ <br> (c) $\Delta=\frac{1}{2} a b \sin c=\frac{1}{2} \times 11 \times 8 \times \sin 0.7$ <br> Sector $=\frac{1}{2} r^{2} \theta=\frac{1}{2} \times 8^{2} \times 0.7$ <br> Area of $R=28.345 \ldots . .-22.4=5.9455=5.95\left(\mathrm{~cm}^{2}\right)$ | M1, A1 <br> (2) <br> M1 <br> A1 <br> M1, A1cao <br> (4) <br> M1, A1 <br> M1, A1 <br> A1 <br> (5) |
|  | (c) Final A1 accept 3 sf or better <br> (a) and (c) M1 for quoting and attempting to use correct formula <br> (b) $1^{\text {st }} \mathrm{M} 1$ for attempting to use cosine rule (formula given) |  |

## Question 2: June 05 Q7



Question 3: Jan 06 Q5

| Question | Scheme | Marks |
| :---: | :---: | :---: |
|  | (a) $\cos A \hat{O} B=\frac{5^{2}+5^{2}-6^{2}}{2 \times 5 \times 5}$ or <br> $\sin \theta=\frac{3}{5}$ with use of $\cos 2 \theta=1-2 \sin ^{2} \theta$ attempted $=\frac{7}{25} \quad *$ <br> (b) $A \hat{O} B=1.2870022 \ldots \quad$ radians <br> 1.287 or better <br> (c) Sector $=\frac{1}{2} \times 5^{2} \times(b),=16.087 \ldots$ <br> (AWRT) 16.1 <br> (d) Triangle $=\frac{1}{2} \times 5^{2} \times \sin (b)$ or $\frac{1}{2} \times 6 \times \sqrt{5^{2}-3^{2}}$ <br> Segment $=$ (their sector) - their triangle $=(\text { sector from } \mathrm{c})-12=(\mathrm{AWRT}) \underline{4.1}$ | M1  <br> A1cso (2) <br> B1 (1) <br> M1 A1  <br> M1  <br> dM1  <br> A1ft  |
|  | (a) M1 for a full method leading to $\cos A \hat{O} B[$ N.B. Use of calculator is M 0$]$ (usual rules about quoting formulae) <br> (b) Use of (b) in degrees is M0 <br> (d) $1^{\text {st }} \mathrm{M} 1$ for full method for the area of triangle $A O B$ <br> $2^{\text {nd }} \mathrm{M} 1$ for their sector - their triangle. Dependent on $1^{\text {st }} \mathrm{M} 1$ in part (d). <br> A1ft for their sector from part (c) -12 [or 4.1 following a correct restart]. |  |

Question 4: June 06 Q8



## Notes

| $9\left(\right.$ a) N.B. $a^{2}=b^{2}+c^{2}-2 b c \cos A$ is in the formulae book. |  |
| :--- | :--- |
| Use of cosine rule for $\cos P Q R$. Allow $A, \theta$ or other symbol for angle. <br> (i) $(6 \sqrt{3})^{2}=6^{2}+6^{2}-2.6 .6 \cos P Q R$ : Apply usual rules for formulae: (a) formula not stated, <br> must be correct, (b) correct formula stated, allow one sign slip when substituting. <br> or (ii) $\cos P Q R=\frac{ \pm 6^{2} \pm 6^{2} \pm(6 \sqrt{3})^{2}}{ \pm 2 \times 6 \times 6}$ <br> Also allow invisible brackets [so allow $6 \sqrt{3}^{2}$ ] in (i) or (ii) |  |
| Correct expression $\frac{6^{2}+6^{2}-(6 \sqrt{3})^{2}}{2 \times 6 \times 6}$ o.e. (e.g. $-\frac{36}{72}$ or $-\frac{1}{2}$ ) |  |
| $\frac{2 \pi}{3}$ | A1 |

Question 6: Jan 08 Q8

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| (a) | $(x-6)^{2}+(y-4)^{2}=; 3^{2}$ | B1; B1 (2) |
| (b) | Complete method for MP: $=\sqrt{(12-6)^{2}+(6-4)^{2}}$ | M1 |
|  | $=\sqrt{40}$ or awrt 6.325 | A1 |
|  | [These first two marks can be scored if seen as part of solution for (c)] |  |
|  | Complete method for $\cos \theta, \sin \theta$ or $\tan \theta$ <br> e.g. $\cos \theta=\frac{\mathrm{MT}}{\mathrm{MP}}=\frac{3}{\text { candidate' } s \sqrt{40}} \quad(=0.4743) \quad\left(\theta=61.6835^{\circ}\right)$ <br> [If TP $=6$ is used, then M0] | M1 |
|  | $\theta=1.0766 \mathrm{rad}$ AG | A1 (4) |
| (c) | Complete method for area TMP; e.g. $=\frac{1}{2} \times 3 \times \sqrt{40} \sin \theta$ | M1 |
|  | $=\frac{3}{2} \sqrt{31} \quad(=8.3516 .$.$) allow awrt 8.35$ | A1 |
|  | Area (sector) $M T Q=0.5 \times 3^{2} \times 1.0766 \quad(=4.8446 \ldots)$ | M1 |
|  | Area $T P Q=$ candidate' $\mathrm{s}(8.3516 . .-4.8446 .$. | M1 |
|  | $\begin{aligned} & =3.507 \text { awrt } \\ & {[\text { Note: } 3.51 \text { is A0] }} \end{aligned}$ | $\begin{array}{cc} \text { A1 } \\ {[11]} \end{array}$ |
| Notes |  |  |
|  | (b) First M1 can be implied by $\sqrt{ }$ 40or $\sqrt{ } 31$ |  |
|  | For second M1: |  |
|  | May find TP $=\sqrt{(\sqrt{40})^{2}-3^{2}}=\sqrt{31}$, then either $\sin \theta=\frac{T P}{M P}=\frac{\sqrt{31}}{\sqrt{40}}(=0.8803 \ldots)$ or $\tan \theta=\frac{\sqrt{31}}{3}$ (1.8859..) or cos rule |  |
|  | NB. Answer is given, but allow final Al if all previous work is correct. |  |
|  | (c) First M1: (alternative) $\frac{1}{2} \times 3 \times \sqrt{40-9}$ |  |
|  | Second M1: allow even if candidate' s value of $\theta$ used. (Despite being given !) |  |

## Question 7: June 08 Q7

| Question <br> Number | Scheme | Marks |
| :---: | :---: | :---: |
| (a) | $r \theta=7 \times 0.8=5.6 \quad(\mathrm{~cm})$ | M1 A1 (2) |
| (b) | $\frac{1}{2} r^{2} \theta=\frac{1}{2} \times 7^{2} \times 0.8=19.6\left(\mathrm{~cm}^{2}\right)$ | M1 A1 (2) |
| (c) | $B D^{2}=7^{2}+(\text { their } A D)^{2}-(2 \times 7 \times($ their $A D) \times \cos 0.8)$ | M1 |
|  | $B D^{2}=7^{2}+3.5^{2}-(2 \times 7 \times 3.5 \times \cos 0.8) \quad$ (or awrt $46^{\circ}$ for the angle) | A1 |
|  | Perimeter $=($ their $D C)+" 5.6 "+" 5.21 "=14.3(\mathrm{~cm})$ | M1 A1 (4) |
| (d) | $\triangle A B D=\frac{1}{2} \times 7 \times($ their $A D) \times \sin 0.8 \quad(\mathrm{ft}$ their $A D) \quad(=8.78 \ldots)$ | M1 A1 ft |
|  | Area $=$ "19.6" -"8.78 $\ldots$ " $=10.8\left(\mathrm{~cm}^{2}\right)$ | M1 A1 (4) |
|  |  | (12 marks) |


| Question Number | Scheme Marks |
| :---: | :---: |
|  | $\begin{align*} & \frac{1}{2} r^{2} \theta=\frac{1}{2} \times 6^{2} \times 2.2=39.6 \quad\left(\mathrm{~cm}^{2}\right)  \tag{2}\\ & \left(\frac{2 \pi-2.2}{2}=\right) \pi-1.1=2.04(\mathrm{rad}) \tag{2} \end{align*}$ <br> (c) $\triangle D A C=\frac{1}{2} \times 6 \times 4 \sin 2.04 \quad(\approx 10.7)$ <br> Total area $=$ sector +2 triangles $=61 \quad\left(\mathrm{~cm}^{2}\right)$ |
| (a) (b) (c) | M1: Needs $\theta$ in radians for this formula. Could convert to degrees and use degrees formula. <br> A1: Does not need units. Answer should be 39.6 exactly. <br> Answer with no working is M1 A1. <br> This M1A1 can only be awarded in part (a). <br> M1: Needs full method to give angle in radians <br> Al: Allow answers which round to 2.04 (Just writes 2.04 - no working is $2 / 2$ ) <br> M1: Use $\frac{1}{2} \times 6 \times 4 \sin A$ (if any other triangle formula e.g. $\frac{1}{2} b \times h$ is used the method must be complete for this mark) (No value needed for $A$, but should not be using 2.2) <br> Al : ft the value obtained in part (b) - need not be evaluated- could be in degrees <br> M1: Uses Total area $=$ sector +2 triangles or other complete method <br> A1: Allow answers which round to 61 . (Do not need units) <br> Special case degrees: Could get M0A0, M0A0, M1A1M1A0 <br> Special case: Use $\triangle B D C-\triangle B A C$ Both areas needed for first M1 <br> Total area $=$ sector + area found is second M1 <br> NB Just finding lengths $B D, D C$, and angle $B D C$ then assuming area $B D C$ is a sector to find area BDC is $0 / 4$ |

## Question 9: June 09 Q9

| Question Number | Scheme Marks |
| :---: | :---: |
| Q (a) | (Arc length $=) r \theta=r \times 1=r$. Can be awarded by implication from later work, e.g. <br> $3 r h$ or $(2 r h+r h)$ in the $S$ formula. <br> (Requires use of $\theta=1$ ). <br> (Sector area $=$ ) $\frac{1}{2} r^{2} \theta=\frac{1}{2} r^{2} \times 1=\frac{r^{2}}{2}$. Can be awarded by implication from later <br> work, e.g. the correct volume formula. (Requires use of $\theta=1$ ). <br> Surface area $=2$ sectors +2 rectangles + curved face $\left(=r^{2}+3 r h\right) \quad \text { (See notes below for what is allowed here) }$ <br> Volume $=300=\frac{1}{2} r^{2} h$ <br> Sub for $h: S=r^{2}+3 \times \frac{600}{r}=r^{2}+\frac{1800}{r}$ <br> $\frac{\mathrm{d} S}{\mathrm{~d} r}=2 r-\frac{1800}{r^{2}}$ or $2 r-1800 r^{-2}$ or $2 r+-1800 r^{-2}$ <br> $\frac{\mathrm{d} S}{\mathrm{~d} r}=0 \Rightarrow r^{3}=\ldots, \quad r=\sqrt[3]{900}$, or AWRT $9.7 \quad$ (NOT -9.7 or $\pm 9.7$ ) <br> $\frac{\mathrm{d}^{2} S}{\mathrm{~d} r^{2}}=\ldots . \quad$ and consider sign, $\frac{\mathrm{d}^{2} S}{\mathrm{~d} r^{2}}=2+\frac{3600}{r^{3}}>0$ so point is a minimum $S_{\min }=(9.65 \ldots)^{2}+\frac{1800}{9.65 \ldots}$ <br> (Using their value of $r$, however found, in the given $S$ formula) <br> $=279.65 \ldots$ (AWRT: 280) (Dependent on full marks in part (b)) |
| (a) (b) (c) | M1 for attempting a formula (with terms added) for surface area. May be incomplete or wrong and may have extra term(s), but must have an $r^{2}$ (or $r^{2} \theta$ ) term and an $r h$ (or $r h \theta$ ) term. <br> In parts (b). (c) and (d). ignore labelling of parts <br> $1^{\text {st }}$ M1 for attempt at differentiation (one term is sufficient) $r^{n} \rightarrow k r^{n-1}$ <br> $2^{\text {nd }}$ M1 for setting their derivative (a 'changed function') $=0$ and solving as far as $r^{3}=\ldots$ (depending upon their 'changed function', this could be $r=\ldots$ or $r^{2}=\ldots$, etc., but the algebra must deal with a negative power of $r$ and should be sound apart from possible sign errors, so that $r^{n}=\ldots$ is consistent with their derivative). <br> M1 for attempting second derivative (one term is sufficient) $r^{n} \rightarrow k r^{n-1}$, and considering its sign. Substitution of a value of $r$ is not required. (Equating it to zero is M0). <br> Alft for a correct second derivative (or correct ft from their first derivative) and a valid reason (e.g. $>0$ ), and conclusion. The actual value of the second derivative, if found, can be ignored. To score this mark as ft , their second derivative must indicate a minimum. <br> Alternative: <br> M1: Find value of $\frac{\mathrm{d} S}{\mathrm{~d} r}$ on each side of their value of $r$ and consider sign. <br> A1 ft : Indicate sign change of negative to positive for $\frac{\mathrm{d} S}{\mathrm{~d} r}$, and conclude minimum. <br> Alternative: <br> M1: Find value of $S$ on each side of their value of $r$ and compare with their 279.65 . <br> A1 ft: Indicate that both values are more than 279.65 , and conclude minimum. |



Question 11: June 10 Q6


## Question 12: Jan 11 Q2

| Question Number | Scheme ${ }^{\text {a }}$ Marks |
| :---: | :---: |
| (a) | $\begin{aligned} & 11^{2}=8^{2}+7^{2}-(2 \times 8 \times 7 \cos C) \\ & \cos C=\frac{8^{2}+7^{2}-11^{2}}{2 \times 8 \times 7} \text { (or equivalent) } \\ & \{\hat{C}=1.64228 \ldots\} \Rightarrow \hat{C}=\text { awrt } 1.64 \end{aligned}$ |
| (b) |  |
|  | Notes |
| (a) | M1 is also scored for $8^{2}=7^{2}+11^{2}-(2 \times 7 \times 11 \cos C)$ or $7^{2}=8^{2}+11^{2}-(2 \times 8 \times 11 \cos C)$ $\text { or } \cos C=\frac{7^{2}+11^{2}-8^{2}}{2 \times 7 \times 11} \quad \text { or } \quad \cos C=\frac{8^{2}+11^{2}-7^{2}}{2 \times 8 \times 11}$ <br> $1^{\text {st }} \mathrm{A} 1$ : Rearranged correctly to make $\cos C=\ldots$ and numerically correct (possibly unsimplified). Award A1 for any of $\cos C=\frac{8^{2}+7^{2}-11^{2}}{2 \times 8 \times 7}$ or $\cos C=\frac{-8}{112}$ or $\cos C=-\frac{1}{14}$ or $\cos C=$ awrt -0.071 . <br> SC: Also allow $1^{\text {st }} \mathrm{A} 1$ for $112 \cos C=-8$ or equivalent. <br> Also note that the $1^{\text {st }} \mathrm{A} 1$ can be implied for $\hat{C}=$ awrt 1.64 or $\hat{C}=$ awrt $94.1^{\circ}$. <br> Special Case: $\cos C=\frac{1}{14}$ or $\cos C=\frac{11^{2}-8^{2}-7^{2}}{2 \times 8 \times 7}$ scores a SC: M1A0A0. <br> $2^{\text {nd }} \mathrm{A} 1$ : for awrt 1.64 cao <br> Note that $A=0.6876 \ldots{ }^{c}$ ( or $39.401 \ldots{ }^{\circ}$ ), $B=0.8116 \ldots{ }^{c}$ (or $46.503 \ldots{ }^{\circ}$ ) |
| (b) | M1: alternative methods must be fully correct to score the M1. <br> For any (or both) of the M1 or the $1^{\text {st }} \mathrm{A} 1$; their $C$ can either be in degrees or radians. <br> Candidates who use $\cos C=\frac{1}{14}$ to give $C=1.499 \ldots$, can achieve the correct answer of awrt <br> 27.9 in part (b). These candidates will score M1A1A0cso, in part (b). <br> Finding $C=1.499 \ldots$ in part (a) and achieving awrt 27.9 with no working scores M1A1A0. <br> Otherwise with no working in part (b), awrt 27.9 scores M1A1A1. <br> Special Case: If the candidate gives awrt 27.9 from any of the below then award Mlalal. $\frac{1}{2}(7 \times 11) \sin \left(0.8116^{\text {c }} \text { or } 46.503^{\circ}\right)=\text { awrt } 27.9, \frac{1}{2}(8 \times 11) \sin \left(0.6876 \ldots .{ }^{c} \text { or } 39.401 \ldots{ }^{\circ}\right)=\text { awrt } 27.9 .$ <br> Alternative: Hero's Formula: $A=\sqrt{13(13-11)(13-8)(13-7)}=$ awrt 27.9 , where M1 is attempt to apply $A=\sqrt{s(s-11)(s-8)(s-7)}$ and the first A 1 is for the correct application of the formula. |

Question 13: June 11 Q5

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| (a) |  |  |
| (b) | $\sin \left(\frac{\pi}{6}\right)=\frac{r}{6-r}$ $\sin \left(\frac{\pi}{6}\right)$ or $\sin 30^{\circ}=\frac{r}{6-r}$ <br> $\frac{1}{2}=\frac{r}{6-r}$ Replaces $\sin$ by numeric value <br> $6-r=2 r \Rightarrow r=2$ $r=2$ | M1 <br> dM1 |
|  |  |  |
| (c) | Area $=6 \pi-\pi(2)^{2}=2 \pi$ or awrt $6.3(\mathrm{~cm})^{2} \quad$ \|heir area of sector $-\pi r^{2}$ | M1 |
| (a) | M1: Needs $\theta$ in radians for this formula. <br> Candidate could convert to degrees and use the degrees formula. <br> A1: Does not need units. Answer should be either $6 \pi$ or 18.85 or awrt 18.8 <br> Correct answer with no working is M1A1. <br> This M1A1 can only be awarded in part (a). <br> M1: Also allow $\cos \left(\frac{\pi}{3}\right)$ or $\cos 60^{\circ}=\frac{r}{6-r}$. <br> $1^{\text {t }}$ M1: Needs correct trigonometry method. Candidates could state $\sin \left(\frac{\pi}{6}\right)=\frac{r}{x}$ and $x+r=6$ or equivalent in their working to gain this method mark. <br> $\mathrm{dM1}$ : Replaces $\sin$ by numerical value. $0.009 \ldots=\frac{r}{6-r}$ from working "incorrectly" in degrees is fine <br> here for dM 1 . <br> A1: For $r=2$ from correct solution only. <br> Alternative: $1^{\text {st }} \mathrm{M} 1$ for $\frac{r}{O C}=\sin 30$ or $\frac{r}{O C}=\cos 60.2^{\text {nd }} \mathrm{M} 1$ for $O C=2 r$ and then A 1 for $r=2$. <br> Note seeing $O C=2 r$ is M1M1. <br> Special Case: If a candidate states an answer of $r=2$ (must be in part (b)) as a guess or from an incorrect method then award SC: M0M0B1. Such a candidate could then go on to score M1A1 in part (c). <br> M1: For "their area of sector - their area of circle", where $r>0$ is ft from their answer to part (b). Allow the method mark if "their area of sector" < "their area of circle". The candidate must show somewhere in their working that they are subtracting the correct way round, even if their answer is negative. <br> Some candidates in part (c) will either use their value of $r$ from part (b) or even introduce a value of $r$ in part (c). You can apply the scheme to award either M0A0 or M1A0 or M1A1 to these candidates. <br> Note: Candidates can get M1 by writing "their part (a) answer $-\pi r^{2 n}$, where the radius of the circle is not substituted. <br> A1: cao - accept exact answer or awrt 6.3 <br> Correct answer only with no working in (c) gets M1A1 <br> Beware: The answer in (c) is the same as the arc length of the pendant |  |
| (b) |  |  |
|  |  |  |
| (c) |  |  |

