## Question 1: June 05 Q1

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
|  | $\begin{aligned} (4-9 x)^{\frac{1}{2}} & =2\left(1-\frac{9 x}{4}\right)^{\frac{1}{2}} \\ & =2\left(1+\frac{\frac{1}{2}}{1}\left(-\frac{9 x}{4}\right)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1.2}\left(-\frac{9 x}{4}\right)^{2}+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2 .3}\left(-\frac{9 x}{4}\right)^{3}+\ldots\right) \\ & =2\left(1-\frac{9}{8} x-\frac{81}{128} x^{2}-\frac{729}{1024} x^{3}+\ldots\right) \\ & =2-\frac{9}{4} x,-\frac{81}{64} x^{2},-\frac{729}{512} x^{3}+\ldots \end{aligned}$ <br> Note The M1 is gained for $\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1.2}(\ldots)^{2}$ or $\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2 .3}(\ldots)^{3}$ <br> Special Case <br> If the candidate reaches $=2\left(1-\frac{9}{8} x-\frac{81}{128} x^{2}-\frac{729}{1024} x^{3}+\ldots\right)$ and goes no further allow A 1 A 0 A 0 | B1 <br> M1 <br> A1, A1, A1 <br> [5] |

Question 2: Jan 06 Q5

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| (a) | Considers $3 x^{2}+16=A(2+x)^{2}+B(1-3 x)(2+x)+C(1-3 x)$ and substitutes $x=-2$, or $x=1 / 3$, <br> or compares coefficients and solves simultaneous equations | M1 |
|  | To obtain $\mathrm{A}=3$, and $\mathrm{C}=4$ | A1, A1 |
|  | Compares coefficients or uses simultaneous equation to show $\mathrm{B}=0$. | B1 <br> (4) |
| (b) | Writes $3(1-3 x)^{-1}+4(2+x)^{-2}$ | M1 |
|  | $=3\left(1+3 x,+9 x^{2}+27 x^{3}+\ldots \ldots\right)+$ | (M1, A1) |
|  | $\frac{4}{4}\left(1+\frac{(-2)}{1}\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{1.2}\left(\frac{x}{2}\right)^{2}+\frac{(-2)(-3)(-4)}{1.2 .3}\left(\frac{x}{2}\right)^{3}+\ldots .\right)$ | ( M1 A1) |
|  | $=4+8 x,+27 \frac{3}{4} x^{2}+80 \frac{1}{2} x^{3}+\ldots$ | A1, A1 ${ }^{(7)}$ |
|  | Or uses $\left(3 x^{2}+16\right)(1-3 x)^{-1}(2+x)^{-2}$ | M1 |
|  | $\left(3 x^{2}+16\right)\left(1+3 x,+9 x^{2}+27 x^{3}+\right) \times$ | (M1A1) $\times$ |
|  | $1 / 4\left(1+\frac{(-2)}{1}\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{1.2}\left(\frac{x}{2}\right)^{2}+\frac{(-2)(-3)(-4)}{1.2 .3}\left(\frac{x}{2}\right)^{3}\right)$ | (M1A1) |
|  | $=4+8 x,+27 \frac{3}{4} x^{2}+80 \frac{1}{2} x^{3}+\ldots$ | A1, A1 |

## Question 3: June 06 Q2

\begin{tabular}{|c|c|c|c|}
\hline Question Number \& Scheme \& \& Marks \\
\hline \multirow[t]{2}{*}{(a)} \& \begin{tabular}{l}
\[
3 x-1 \equiv A(1-2 x)+B
\] \\
Let \(\mathrm{X}=\frac{1}{2} ; \quad \frac{3}{2}-1=\mathrm{B} \quad \Rightarrow \mathrm{B}=\frac{1}{2}\)
\end{tabular} \& Considers this identity and either substitutes \(x=\frac{1}{2}\), equates coefficients or solves simultaneous equations \& \begin{tabular}{l}
complete \\
M1
\end{tabular} \\
\hline \& \begin{tabular}{l}
Equate \(x\) terms; \(3=-2 \mathrm{~A} \Rightarrow \mathrm{~A}=-\frac{3}{2}\) \\
(No working seen, but A and B correctly stated \(\Rightarrow\) award all three marks. If one of A or B correctly stated give two out of the three marks available for this part.)
\end{tabular} \& \(\mathrm{A}=-\frac{3}{2} ; \mathrm{B}=\frac{1}{2}\) \& A1;A1

[3] <br>
\hline \multirow[t]{5}{*}{(b)} \& $f(x)=-\frac{3}{2}(1-2 x)^{-1}+\frac{1}{2}(1-2 x)^{-2}$ \& Moving powers to top on any one of the two expressions \& M1 <br>

\hline \& $$
=-\frac{3}{2}\left\{1+(-1)(-2 x) ;+\frac{(-1)(-2)}{2!}(-2 x)^{2}+\frac{(-1)(-2)(-3)}{3!}(-2 x)^{3}+\ldots\right\}
$$ \& Either $1 \pm 2 x$ or $1 \pm 4 x$ from either first or second expansions respectively \& dM1; <br>

\hline \& $$
+\frac{1}{2}\left\{1+(-2)(-2 x) ;+\frac{(-2)(-3)}{2!}(-2 x)^{2}+\frac{(-2)(-3)(-4)}{3!}(-2 x)^{3}+\ldots\right\}
$$

\[
=-\frac{3}{2}\left\{1+2 x+4 x^{2}+8 x^{3}+···\right\}+\frac{1}{2}\left\{1+4 x+12 x^{2}+32 x^{3}+···\right\}

\] \& | Ignoring $-\frac{3}{2}$ and $\frac{1}{2}$, any one correct $\qquad$ \} expansion. |
| :--- |
| Both $\qquad$ \} correct. | \& | A1 |
| :--- |
| A1 | <br>


\hline \& \multirow[t]{2}{*}{$=-1-x ;+0 x^{2}+4 x^{3}$} \& \multirow[t]{2}{*}{$-1-x ;\left(0 x^{2}\right)+4 x^{3}$} \& | A1; A1 |
| :--- |
| [6] | <br>

\hline \& \& \& 9 marks <br>
\hline
\end{tabular}

Question 4: Jan 07 Q1


Question 5: June 07 Q1

| Question Number | Scheme |  | Marks |
| :---: | :---: | :---: | :---: |
| (a) | ${ }^{* *}$ represents a constant $f(x)=(3+2 x)^{-3}=\underline{(3)^{-3}}\left(1+\frac{2 x}{3}\right)^{-3}=\frac{1}{\underline{27}}\left(1+\frac{2 x}{3}\right)^{-3}$ $=\frac{1}{27}\left\{1+(-3)\left({ }^{* *} x\right) ;+\frac{(-3)(-4)}{2!}\left({ }^{* *} x\right)^{2}+\frac{(-3)(-4)(-5)}{3!}\left(^{* *} x\right)^{3}+\ldots\right\}$ <br> with ** $\neq 1$ $=\frac{1}{27}\left\{1+(-3)\left(\frac{2 x}{3}\right)+\frac{(-3)(-4)}{2!}\left(\frac{2 x}{3}\right)^{2}+\frac{(-3)(-4)(-5)}{3!}\left(\frac{2 x}{3}\right)^{3}+\ldots\right\}$ $=\frac{1}{27}\left\{1-2 x+\frac{8 x^{2}}{3}-\frac{80}{27} x^{3}+\ldots\right\}$ $=\frac{1}{27}-\frac{2 x}{27} ;+\frac{8 x^{2}}{81}-\frac{80 x^{3}}{729}+\ldots$ | Takes 3 outside the bracket to give any of <br> (3) ${ }^{-3}$ or $\frac{1}{27}$. <br> See note below. <br> Expands $\left(1+{ }^{* *} x\right)^{-3}$ to give a simplified or an un-simplified $1+(-3)\left(^{* *} x\right) \text {; }$ <br> A correct simplified or an un-simplified $\qquad$ expansion with candidate's followed thro' (**x) <br> Anything that cancels to $\frac{1}{27}-\frac{2 \mathrm{x}}{27}$; Simplified $\frac{8 x^{2}}{81}-\frac{80 x^{3}}{729}$ | B1 <br> M1; <br> $\mathrm{A} 1 \sqrt{ }$ <br> A1; <br> A1 <br> [5] |
|  |  |  | 5 marks |
| $\begin{aligned} & \text { Note: You would award: B1M1A0 for } \\ & =\frac{1}{27}\left\{1+(-3)\left(\frac{2 x}{3}\right)+\frac{(-3)(-4)}{2!}(2 x)^{2}+\frac{(-3)(-4)(-5)}{3!}(2 x)^{3}+\ldots\right\} \end{aligned}$ <br> Special Case: If you see the constant $\frac{1}{27}$ in a candidate's final binomial expression, then you can award B1 <br> because ** is not consistent. |  |  |  |

## Sequences and Series

## Aliter <br> Way 2

$$
f(x)=(3+2 x)^{-3}
$$

$$
\left.\frac{1}{27} \text { or }(3)^{-3} \text { (See note } \downarrow\right)
$$

$$
=\left\{\begin{array}{c}
(3)^{-3}+(-3)(3)^{-4}\left({ }^{* *} x\right) ;+\frac{(-3)(-4)}{2!}(3)^{-5}\left({ }^{* *} x\right)^{2} \\
+\frac{(-3)(-4)(-5)}{3!}(3)^{-6}\left({ }^{*} x\right)^{3}+\ldots
\end{array}\right\}
$$

Expands $(3+2 x)^{-3}$ to give an un-simplified or simplified

A correct un-simplified or simplified

$$
\text { with ** } \neq 1
$$



Anything that

$$
=\frac{1}{27}-\frac{2 x}{27} ;+\frac{8 x^{2}}{81}-\frac{80 x^{3}}{729}+\ldots
$$



Attempts using Maclaurin expansions need to be escalated up to your team leader.
If you feel the mark scheme does not apply fairly to a candidate please escalate the response up to your team leader.

Special Case: If you see the constant $\frac{1}{27}$ in a candidate's final binomial expression, then you can award B1

## Question 6: Jan 08 Q2



You would award B1M1A0 for
$=2\left\{1+\left(\frac{1}{3}\right)\left(-\frac{3 x}{8}\right)+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}\left(-\frac{3 x}{8}\right)^{2}+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}(-3 x)^{3}+\ldots\right\}$

If you see the constant term " 2 " in a candidate's final binomial expansion, then you can award B1.
because ** is not consistent.

Be wary of calculator value of $(7.7)^{\frac{1}{4}}=1.974680822 \ldots$

| Question Number | Scheme |  | Marks |
| :---: | :---: | :---: | :---: |
| Aliter <br> (a) | $(8-3 x)^{\frac{1}{3}}$ |  |  |
| Way 2 |  | 2 or $(8)^{\frac{1}{3}}$ (See note $\downarrow$ ) | B1 |
|  | $\begin{aligned} & =\left\{\begin{array}{r} \left.(8)^{\frac{1}{3}+\left(\frac{1}{3}\right)(8)^{-\frac{2}{3}}(* * x) ;+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}(8)^{-\frac{-}{3}(* * x)^{2}}} \begin{array}{r} +\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)}{3!}(8)^{-\frac{1}{3}}(* * x)^{3}+\ldots \end{array}\right\} \\ \\ \text { with } * * \neq 1 \end{array}\right. \end{aligned}$ | Expands $(8-3 x)^{\frac{1}{5}}$ to give an un-simplified or simplified $(8)^{\frac{1}{1}}+\left(\frac{1}{3}\right)(8)^{-\frac{1}{3}}\left({ }^{* *} x\right) ;$ <br> A correct un-simplified or simplified $\qquad$ expansion with candidate's followed through ( ${ }^{*} x$ ) | M1; A1 $\sqrt{5}$ |
|  | $=\left\{\begin{array}{c} (8)^{\frac{1}{2}}+\left(\frac{1}{3}\right)(8)^{-\frac{2}{3}}(-3 x) ;+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}(8)^{-\frac{1}{3}}(-3 x)^{2} \\ +\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}(8)^{-\frac{1}{3}}(-3 x)^{3}+\ldots \end{array}\right\}$ | Award SC M1 if you see |  |
|  | $=\left\{2+\left(\frac{1}{3}\right)\left(\frac{1}{4}\right)(-3 x)+\left(-\frac{1}{9}\right)\left(\frac{1}{32}\right)\left(9 x^{2}\right)+\left(\frac{3}{31}\right)\left(\frac{1}{256}\right)\left(-27 x^{3}\right)+\ldots\right\}$ |  |  |
|  | $=2-\frac{1}{4} x ;-\frac{1}{32} x^{2}-\frac{5}{768} x^{3}-\ldots$ | Anything that cancels to $2-\frac{1}{4} x$; or $2\left\{1-\frac{1}{8} x \ldots \ldots ..\right\}$ | A1; |
|  |  | Simplified $-\frac{1}{32} x^{2}-\frac{5}{768} x^{3}$ | A1 [5] |

Be wary of calculator value of $(7.7)^{\frac{1}{5}}=1.974680822 \ldots$

If you see the constant term " 2 " in a candidate's final binomial expansion, then you can award B1.

Question 7: June 08 Q5


## Question 8: Jan 09 Q3

| Question Number | Scheme |  | Marks |
| :---: | :---: | :---: | :---: |
| (a) | $27 x^{2}+32 x+16=A(3 x+2)(1-x)+B(1-x)+C(3 x+2)^{2}$ | Forming this identity | M1 |
|  | $\begin{array}{ll} x=-\frac{2}{3} \cdot & 12-\frac{s 4}{3}+16-\left(\frac{5}{3}\right) B \Rightarrow \frac{20}{3}-\left(\frac{5}{3}\right) B \Rightarrow B=4 \\ x=1, & 27+32+16=25 C \Rightarrow 75=25 C \Rightarrow C=3 \end{array}$ | Substitutes either $x=-\frac{2}{3}$ or $x=1$ into their identity or equates 3 terms or substitutes in values to write down three simultaneous equations. Both $B=4$ and $C=3$ (Note the Al is dependent on both method marks in this part.) | M1 A1 |
|  | $\begin{aligned} \text { Equate } x^{2}: & 27--3 A+9 C \Rightarrow 27--3 A+27 \Rightarrow 0--3 A \\ & \Rightarrow A-0 \end{aligned} \begin{aligned} & x-0 . \quad 16-2 A+B+4 C \\ & \Rightarrow 16-2 A+4+12 \Rightarrow 0-2 A \Rightarrow A-0 \end{aligned}$ | Compares coefficients or substitutes in a third $x$-value or uses simultaneous equations to show $A=0$. | $\begin{array}{rr}\text { B1 } \\ \\ \\ & \text { (4) }\end{array}$ |
| (b) | $f(x)=\frac{4}{(3 x+2)^{2}}+\frac{3}{(1-x)}$ |  |  |
|  | $\begin{aligned} & =4(3 x+2)^{-2}+3(1-x)^{-1} \\ & =4\left[2\left(1+\frac{3}{2} x\right)^{-2}\right]+3(1-x)^{-1} \\ & =1\left(1+\frac{3}{2} x\right)^{-2}+3(1-x)^{-1} \end{aligned}$ | Moving powers to top on any one of the two expressions | M1 |
|  |  | Either $1 \pm(-2)\left(\frac{3 \mathrm{r}}{2}\right)$ or $1 \pm(-1)(-x)$ from either first or second expansions respectively Ignoring 1 and 3 , any one correct $\{$ $\qquad$ \} expansion. <br> Both \{ $\square$ correct. | dM1; <br> A1 <br> A1 |
|  | $\begin{aligned} & =\left\{1-3 x+\frac{37}{4} x^{2}+\ldots\right\}+3\left\{1+x+x^{2}+\ldots\right\} \\ & =4+0 x ;+\frac{3 p}{4} x^{2} \end{aligned}$ | $4+(0 x) ; \frac{39}{4} x^{2}$ | A1; A1 <br> (6) |


| Question Ilumber | Scheme |  | Marks |
| :---: | :---: | :---: | :---: |
| (c) | $\begin{aligned} \text { Actual }-f(0.2) & =\frac{1.08+6.4+16}{(6.76)(0.8)} \\ & =\frac{23.48}{5.408}=4.341715976 \ldots=\frac{2935}{676} \end{aligned}$ | Attempt to find the actual value of $f(0.2)$ or seeing awrt 4.3 and believing it is candidate's actual $\mathrm{f}(0.2)$. | M1 |
|  | Or $\begin{aligned} \text { Actual }=f(0.2) & =\frac{4}{(3(0.2)+2)^{2}}+\frac{3}{(1-0.2)} \\ & =\frac{4}{6.76}+3.75=4.341715976 \ldots=\frac{2935}{676} \end{aligned}$ | Candidates can also attempt to find the actual value by using $\frac{A}{(3 x+2)}+\frac{B}{(3 x+2)^{2}}+\frac{C}{(1-x)}$ <br> with their $A, B$ and $C$. |  |
|  | $\begin{aligned} & \begin{array}{r} \text { Estimate }-f(0.2)-4+3.3(0.2)^{2} \\ \\ =4+0.39=4.39 \end{array} \\ & \text { \%age error }=\frac{\|4.39-4.341715976\|}{4.341715976 \ldots} \times 100 \end{aligned}$ | Attempt to find an estimate for $f(0,2)$ using their answer to (b) | M1 $\sqrt{ }$ |
|  |  | $\left\|\frac{\text { their estimate-actual }}{\text { actual }}\right\| \times 100$ | M1 |
|  | $-1.112095408 . .1 .1 \%(2 s f)$ | 1.1\% | A1 cao <br> (4) |
|  |  |  | [14] |

Question 9: June 09 Q1


Question 10: Jan 10 Q1


## Question 11: June 10 Q5

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
|  | $\text { (a) } \begin{array}{cc} A=2 \\ & 2 x^{2}+5 x-10=A(x-1)(x+2)+B(x+2)+C(x-1) \\ & -3=3 B \Rightarrow B=-1 \\ x \rightarrow-2 & -12=-3 C \Rightarrow C=4 \end{array}$ | B1 M1 A1 <br> A1 <br> (4) |
|  | (b) $\begin{aligned} \frac{2 x^{2}+5 x-10}{(x-1)(x+2)} & =2+(1-x)^{-1}+2\left(1+\frac{x}{2}\right)^{-1} \\ (1-x)^{-1} & =1+x+x^{2}+\ldots \\ \left(1+\frac{x}{2}\right)^{-1} & =1-\frac{x}{2}+\frac{x^{2}}{4}+\ldots \end{aligned}$ | M1 <br> B1 <br> B1 |
|  | $\begin{array}{rlrl} \frac{2 x^{2}+5 x-10}{(x-1)(x+2)} & =(2+1+2)+(1-1) x+\left(1+\frac{1}{2}\right) x^{2}+\ldots \\ & =5+\ldots & \mathrm{ft} \mathrm{their} A-B+\frac{1}{2} C \\ & =\ldots+\frac{3}{2} x^{2}+\ldots & 0 x \text { stated or implied } \end{array}$ | M1 <br> Al ft <br> A1 A1 |
|  |  | [11] |

Question 12: Jan 11 Q5

| Question Number | Scheme | Marks |  |
| :---: | :---: | :---: | :---: |
| (a) | $\begin{aligned} (2-3 x)^{-2}= & 2^{-2}\left(1-\frac{3}{2} x\right)^{-2} \\ \left(1-\frac{3}{2} x\right)^{-2}= & 1+(-2)\left(-\frac{3}{2} x\right)+\frac{-2 .-3}{1.2}\left(-\frac{3}{2} x\right)^{2}+\frac{-2 .-3 .-4}{1.2 .3}\left(-\frac{3}{2} x\right)^{3}+\ldots \\ = & 1+3 x+\frac{27}{4} x^{2}+\frac{27}{2} x^{3}+\ldots \\ & (2-3 x)^{-2}=\frac{1}{4}+\frac{3}{4} x+\frac{27}{16} x^{2}+\frac{27}{8} x^{3}+\ldots \end{aligned}$ | B1 <br> M1 A1 <br> M1 A1 | (5) |
| (b) | $\mathrm{f}(x)=(a+b x)\left(\frac{1}{4}+\frac{3}{4} x+\frac{27}{16} x^{2}+\frac{27}{8} x^{3}+\ldots\right)$ <br> Coefficient of $x ; \quad \frac{3 a}{4}+\frac{b}{4}=0 \quad(3 a+b=0)$ <br> Coefficient of $x^{2} ; \quad \frac{27 a}{16}+\frac{3 b}{4}=\frac{9}{16} \quad(9 a+4 b=3) \quad$ A1 either correct <br> Leading to $a=-1, b=3$ | M1 <br> M1 A1 <br> M1 A1 | (5) |
| (c) | $\begin{aligned} & \text { Coefficient of } x^{3} \text { is } \frac{27 a}{8}+\frac{27 b}{16}=\frac{27}{8} \times(-1)+\frac{27}{16} \times 3 \\ &=\frac{27}{16} \end{aligned}$ | M1 A1ft <br> A1 | (3) |
|  |  |  | [13] |

Question 13: June 11 Q2

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
|  | $\begin{aligned} & \mathrm{f}(x)=(\ldots+\ldots)^{-\frac{1}{-}} \\ & \quad=9^{-\frac{1}{2}}(\ldots+\ldots)^{-} \\ & \left(1+k x^{2}\right)^{n}=1+n k x^{2}+\ldots \quad 3^{-1}, \frac{1}{3} \text { or } \frac{1}{9^{\frac{1}{2}}} \\ & \left(1+k x^{2}\right)^{\frac{-1}{2}}=\ldots+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(k x^{2}\right)^{2} \quad n \text { not a natural number, } k \neq 1 \\ & \left(1+\frac{4}{9} x^{2}\right)^{-\frac{1}{2}}=1-\frac{2}{9} x^{2}+\frac{2}{27} x^{4} \\ & \mathrm{f}(x)=\frac{1}{3}-\frac{2}{27} x^{2}+\frac{2}{81} x^{4} \end{aligned}$ | M1 <br> B1 <br> M1 <br> A1 ft <br> A1 <br> A1 <br> (6) <br> [6] |

