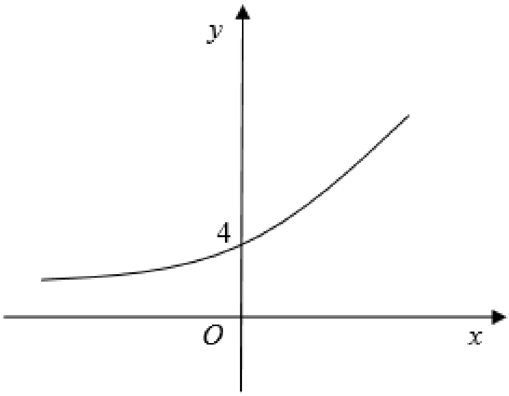
Functions- Edexcel Past Exam Questions MARK SCHEME

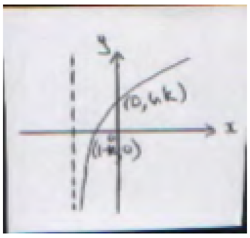
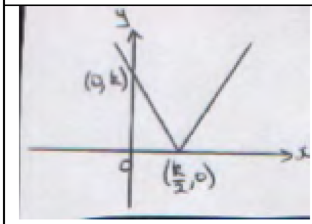
Question 1: June 05 Q3

Question Number	Scheme	Marks
(a)	$\dfrac{5x+1}{(x+2)(x-1)} - \dfrac{3}{x+2}$	B1
	$= \dfrac{5x+1-3(x-1)}{(x+2)(x-1)}$	M1
	M1 for combining fractions even if the denominator is not lowest common	
	$= \dfrac{2x+4}{(x+2)(x-1)} = \dfrac{2(x+2)}{(x+2)(x-1)} = \dfrac{2}{x-1}$ *	M1 A1 cso
	M1 must have linear numerator	(4)
(b)	$y = \dfrac{2}{x-1} \Rightarrow xy - y = 2 \Rightarrow xy = 2 + y$	M1A1
	$f^{-1}(x) = \dfrac{2+x}{x}$ o.e.	A1
		(3)
(c)	$fg(x) = \dfrac{2}{x^2+4}$ (attempt) $[\dfrac{2}{"g"-1}]$	M1
	Setting $\dfrac{2}{x^2+4} = \dfrac{1}{4}$ and finding $x^2 = \ldots$; $x = \pm 2$	M1; A1
		(3)
		[10]

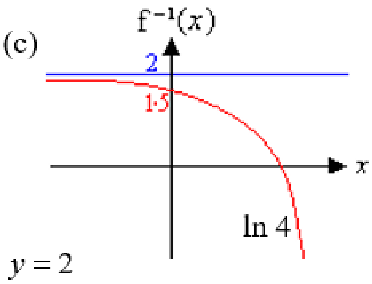
Question 2: Jan 06 Q8

Question Number	Scheme	Marks
	(a) $\text{gf}(x) = e^{2(2x+\ln 2)}$	M1
	$= e^{4x}e^{2\ln 2}$	M1
	$= e^{4x}e^{\ln 4}$	M1
	$= 4e^{4x}$ Give mark at this point, cso	A1 **(4)**
	$\left(\text{Hence } \text{gf}: x \mapsto 4e^{4x}, \quad x \in \mathbb{R}\right)$	
	(b)  Shape and point	B1 **(1)**
	(c) Range is $\mathbb{R}_+$ Accept $\text{gf}(x) > 0$, $y > 0$	B1 **(1)**

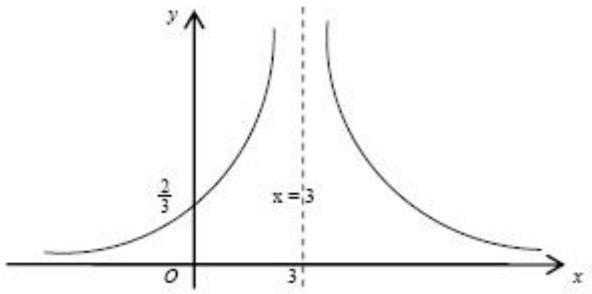
Question 3: June 06 Q7

Question Number	Scheme		Marks	
(*a*)		Log graph: Shape	B1	
		Intersection with –ve x-axis	dB1	
		$(0, \ln k)$, $(1-k, 0)$	B1	
		Mod graph :V shape, vertex on +ve x-axis	B1	
		$(0, k)$ and $\left(\frac{k}{2}, 0\right)$	B1	(5)
(*b*)	$f(x) \in \mathbb{R}$, $-\infty < f(x) < \infty$, $-\infty < y < \infty$		B1	(1)
(*c*)	$fg\left(\frac{k}{4}\right) = \ln\{k + \lvert\frac{2k}{4} - k\rvert\}$ or $f\left(\lvert -\frac{k}{2}\rvert\right)$		M1	
	$= \ln\left(\frac{3k}{2}\right)$		A1	(2)

Question 4: Jan 07 Q6

Question Number	Scheme		Marks	
	(a) $y = \ln(4-2x)$			
	$e^y = 4 - 2x$ leading to $x = 2 - \frac{1}{2}e^y$	Changing subject and removing ln	M1 A1	
	$y = 2 - \frac{1}{2}e^x \Rightarrow f^{-1} \mapsto 2 - \frac{1}{2}e^x$ *	cso	A1	
	Domain of f^{-1} is		B1	**(4)**
	(b) Range of f^{-1} is $f^{-1}(x) < 2$ (and $f^{-1}(x) \in$)		B1	**(1)**
	(c) 	Shape	B1	
		1.5	B1	
		ln 4	B1	
			B1	**(4)**

Question 5: June 07 Q5

Question Number	Scheme		Marks
(a)	Finding g(4) = k and f(k) = …. or $fg(x) = \ln\left(\frac{4}{x-3} - 1\right)$		M1
	[f(2) = ln(2x2 – 1) fg(4) = ln(4 – 1)] = ln 3		A1 (2)
(b)	$y = \ln(2x-1) \Rightarrow e^y = 2x-1$ or $e^x = 2y - 1$		M1, A1
	$f^{-1}(x) = \frac{1}{2}(e^x + 1)$ Allow $y = \frac{1}{2}(e^x + 1)$		A1
	Domain $x \in \Re$ [Allow $\Re$, all reals, $(-\infty, \infty)$] independent		B1 (4)
(c)		Shape, and x-axis should appear to be asymptote	B1
		Equation $x = 3$ needed, may see in diagram (ignore others)	B1 ind.
		Intercept $(0, \frac{2}{3})$ no other; accept y = ⅔ (0.67) or on graph	B1 ind (3)
(d)	$\frac{2}{x-3} = 3 \Rightarrow x = 3\frac{2}{3}$ or exact equiv.		B1
	$\frac{2}{x-3} = -3$, $\Rightarrow x = 2\frac{1}{3}$ or exact equiv.		M1, A1 (3)
	Note: $2 = 3(x + 3)$ or $2 = 3(-x - 3)$ o.e. is M0A0		
Alt:	Squaring to quadratic $(9x^2 - 54x + 77 = 0)$ **and** solving M1; B1A1		**(12 marks)**

Question 6: Jan 08 Q8

Question Number	Scheme		Marks	
	(a) $x = 1 - 2y^3 \Rightarrow y = \left(\frac{1-x}{2}\right)^{1/3}$ or $\sqrt[3]{\frac{1-x}{2}}$		M1 A1	(2)
	$f^{-1} : x \mapsto \left(\frac{1-x}{2}\right)^{1/3}$	Ignore domain		
	(b) $gf(x) = \frac{3}{1-2x^3} - 4$		M1 A1	
	$= \frac{3 - 4(1-2x^3)}{1-2x^3}$		M1	
	$= \frac{8x^3 - 1}{1-2x^3}$ *	cso	A1	(4)
	$gf : x \mapsto \frac{8x^3-1}{1-2x^3}$	Ignore domain		
	(c) $8x^3 - 1 = 0$	Attempting solution of numerator = 0	M1	
	$x = \frac{1}{2}$	Correct answer and no additional answers	A1	(2)
	(d) $\frac{dy}{dx} = \frac{(1-2x^3) \times 24x^2 + (8x^3-1) \times 6x^2}{(1-2x^3)^2}$		M1 A1	
	$= \frac{18x^2}{(1-2x^3)^2}$		A1	
	Solving their numerator = 0 and substituting to find y.		M1	
	$x = 0,\ y = -1$		A1	(5) [13]

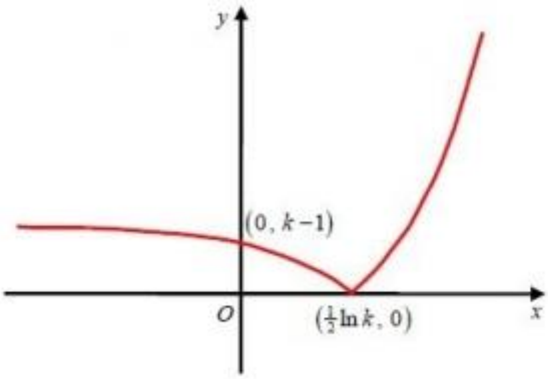
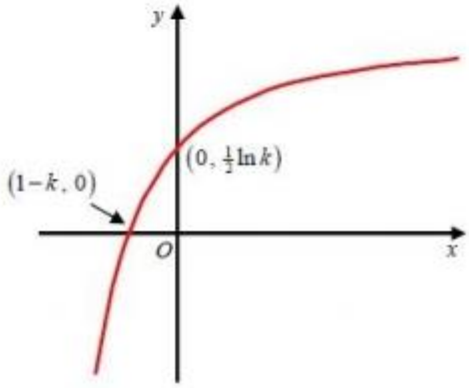
Question 7: June 08 Q4

Question Number	Scheme	Marks
(a)	$x^2-2x-3=(x-3)(x+1)$	B1
	$f(x)=\frac{2(x-1)-(x+1)}{(x-3)(x+1)}\quad\left(or\ \frac{2(x-1)}{(x-3)(x+1)}-\frac{x+1}{(x-3)(x+1)}\right)$	M1 A1
	$=\frac{x-3}{(x-3)(x+1)}=\frac{1}{x+1}$ *	A1 cso (4)
(b)	$\left(0,\frac{1}{4}\right)$ Accept $0<y<\frac{1}{4},\ 0<f(x)<\frac{1}{4}$ etc.	B1 B1 (2)
(c)	Let $y=f(x)$ $y=\frac{1}{x+1}$	
	$x=\frac{1}{y+1}$	
	$yx+x=1$	
	$y=\frac{1-x}{x}$ or $\frac{1}{x}-1$	M1 A1
	$f^{-1}(x)=\frac{1-x}{x}$	
	Domain of f^{-1} is $\left(0,\frac{1}{4}\right)$	B1 ft (3)
(d)	$fg(x)=\frac{1}{2x^2-3+1}$	
	$\frac{1}{2x^2-2}=\frac{1}{8}$	M1
	$x^2=5$	A1
	$x=\pm\sqrt{5}$ both	A1 (3)
		(12 marks)

Question 8: Jan 09 Q5

Question Number	Scheme	Marks	
(a)	$g(x) \geq 1$	B1	(1)
(b)	$fg(x) = f\left(e^{x^2}\right) = 3e^{x^2} + \ln e^{x^2}$	M1	
	$= x^2 + 3e^{x^2}$ ✱	A1	(2)
	$\left(fg : x \mapsto x^2 + 3e^{x^2}\right)$		
(c)	$fg(x) \geq 3$	B1	(1)

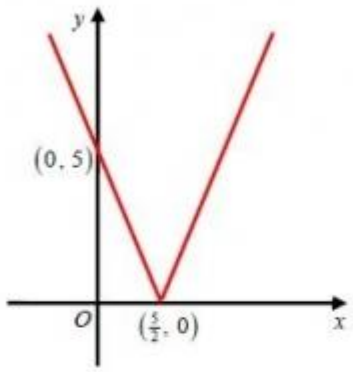
Question 9: June 09 Q5

Question Number	Scheme		Marks
Q (a)		Curve retains shape when $x > \frac{1}{2}\ln k$	B1
		Curve reflects through the x-axis when $x < \frac{1}{2}\ln k$	B1
		$(0, k-1)$ and $(\frac{1}{2}\ln k, 0)$ marked in the correct positions.	B1
			(3)
(b)		Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote)	B1
		$(1-k, 0)$ and $(0, \frac{1}{2}\ln k)$	B1
			(2)
(c)	Range of f: $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k, \infty)}$	Either $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k, \infty)}$ or $\underline{f > -k}$ or $\underline{\text{Range} > -k}$.	B1
			(1)
(d)	$y = e^{2x} - k \Rightarrow y + k = e^{2x}$	Attempt to make x (or swapped y) the subject	M1
	$\Rightarrow \ln(y+k) = 2x$ $\Rightarrow \frac{1}{2}\ln(y+k) = x$	Makes e^{2x} the subject and takes ln of both sides	M1
	Hence $f^{-1}(x) = \underline{\frac{1}{2}\ln(x+k)}$	$\underline{\frac{1}{2}\ln(x+k)}$ or $\underline{\ln\sqrt{(x+k)}}$	$\underline{\text{A1}}$ cao
			(3)
(e)	$f^{-1}(x)$: Domain: $\underline{x > -k}$ or $\underline{(-k, \infty)}$	Either $\underline{x > -k}$ or $\underline{(-k, \infty)}$ or Domain $> -k$ or x "ft one sided inequality" their part (c) RANGE answer	B1 $\sqrt{}$
			(1)
			[10]

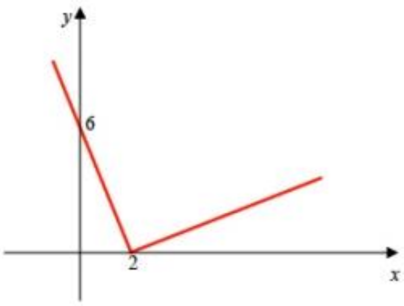
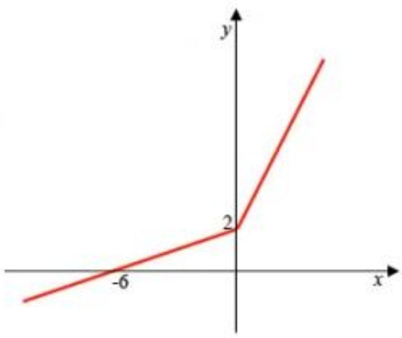
Question 10: Jan 10 Q9

Question Number	Scheme		Marks
(i)(a)	$\ln(3x-7)=5$		
	$e^{\ln(3x-7)}=e^5$	Takes e of both sides of the equation. This can be implied by $3x-7=e^5$.	M1
	$3x-7=e^5 \Rightarrow x=\dfrac{e^5+7}{3}$ $\{=51.804...\}$	Then rearranges to make x the subject.	dM1
		Exact answer of $\dfrac{e^5+7}{3}$.	A1
			(3)
(b)	$3^x e^{7x+2}=15$		
	$\ln\left(3^x e^{7x+2}\right)=\ln 15$	Takes ln (or logs) of both sides of the equation.	M1
	$\ln 3^x + \ln e^{7x+2} = \ln 15$	Applies the addition law of logarithms.	M1
	$x\ln 3 + 7x + 2 = \ln 15$	$x\ln 3 + 7x + 2 = \ln 15$	A1 oe
	$x(\ln 3 + 7) = -2 + \ln 15$	Factorising out at least two x terms on one side and collecting number terms on the other side.	ddM1
	$x=\dfrac{-2+\ln 15}{7+\ln 3}$ $\{=0.0874...\}$	*Exact answer* of $\dfrac{-2+\ln 15}{7+\ln 3}$	A1 oe
			(5)
(ii) (a)	$f(x)=e^{2x}+3,\ x\in\mathbb{R}$		
	$y=e^{2x}+3 \Rightarrow y-3=e^{2x}$	Attempt to make x (or swapped y) the subject	M1
	$\Rightarrow \ln(y-3)=2x$ $\Rightarrow \frac{1}{2}\ln(y-3)=x$	Makes e^{2x} the subject and takes ln of both sides	M1
	Hence $f^{-1}(x)=\underline{\frac{1}{2}\ln(x-3)}$	$\underline{\frac{1}{2}\ln(x-3)}$ or $\underline{\ln\sqrt{(x-3)}}$ or $\underline{f^{-1}(y)=\frac{1}{2}\ln(y-3)}$ (see appendix)	A1 cao
	$f^{-1}(x)$: Domain: $\underline{x>3}$ or $\underline{(3,\infty)}$	Either $\underline{x>3}$ or $\underline{(3,\infty)}$ or $\underline{\text{Domain} > 3}$.	B1
			(4)
(b)	$g(x)=\ln(x-1),\ x\in\mathbb{R},\ x>1$		
	$fg(x)=e^{2\ln(x-1)}+3$ $\{=(x-1)^2+3\}$	An attempt to put function g into function f.	M1
		$e^{2\ln(x-1)}+3$ or $(x-1)^2+3$ or x^2-2x+4.	A1 isw
	$fg(x)$: Range: $\underline{y>3}$ or $\underline{(3,\infty)}$	Either $\underline{y>3}$ or $\underline{(3,\infty)}$ or $\underline{\text{Range} > 3}$ or $\underline{fg(x) > 3}$.	B1
			(3)
			[15]

Question 11: June 10 Q4

Question Number	Scheme	Marks
(a)		M1A1
		(2)
(b)	$\underline{x = 20}$	B1
	$2x - 5 = -(15 + x)$; $\Rightarrow \underline{x = -\frac{10}{3}}$	M1;A1 oe.
		(3)
(c)	$\text{fg}(2) = \text{f}(-3) = \lvert 2(-3) - 5 \rvert ; = \lvert -11 \rvert = 11$	M1;A1
		(2)
(d)	$\text{g}(x) = x^2 - 4x + 1 = (x-2)^2 - 4 + 1 = (x-2)^2 - 3$. Hence $\text{g}_{\text{min}} = -3$	M1
	Either $\text{g}_{\text{min}} = -3$ or $\text{g}(x) \geqslant -3$ or $\text{g}(5) = 25 - 20 + 1 = 6$	B1
	$\underline{-3 \leqslant \text{g}(x) \leqslant 6}$ or $\underline{-3 \leqslant y \leqslant 6}$	A1
		(3) **[10]**
	(a) M1: V or  or  graph with vertex on the x-axis. A1: $(\frac{5}{2}, \{0\})$ and $(\{0\}, 5)$ seen and the graph appears in both the first and second quadrants. (b) M1: Either $2x - 5 = -(15 + x)$ or $-(2x - 5) = 15 + x$ (c) M1: ***Full method*** of inserting $\text{g}(2)$ into $\text{f}(x) = \lvert 2x - 5 \rvert$ or for inserting $x = 2$ into $\lvert 2(x^2 - 4x + 1) - 5 \rvert$. There must be evidence of the modulus being applied. (d) M1: **Full method** to establish the minimum of g. Eg: $(x \pm \alpha)^2 + \beta$ leading to $\text{g}_{\text{min}} = \beta$. Or for candidate to differentiate the quadratic, set the result equal to zero, find x and insert this value of x back into $\text{f}(x)$ in order to find the minimum. B1: For either finding the correct minimum value of g (can be implied by $\text{g}(x) \geqslant -3$ or $\text{g}(x) > -3$) or for stating that $\text{g}(5) = 6$. A1: $\underline{-3 \leqslant \text{g}(x) \leqslant 6}$ or $\underline{-3 \leqslant y \leqslant 6}$ or $\underline{-3 \leqslant \text{g} \leqslant 6}$. **Note that:** $-3 \leqslant x \leqslant 6$ is A0. **Note that:** $-3 \leqslant \text{f}(x) \leqslant 6$ is A0. **Note that:** $-3 \geqslant \text{g}(x) \geqslant 6$ is A0. **Note that:** $\text{g}(x) \geqslant -3$ or $\text{g}(x) > -3$ or $x \geqslant -3$ or $x > -3$ with no working gains M1B1A0. **Note that for the final Accuracy Mark**: If a candidate writes down $-3 < \text{g}(x) < 6$ or $-3 < y < 6$, then award M1B1A0. If, however, a candidate writes down $\text{g}(x) \geqslant -3$, $\text{g}(x) \leqslant 6$, then award A0. If a candidate writes down $\text{g}(x) \geqslant -3$ or $\text{g}(x) \leqslant 6$, then award A0.	

Question 12: Jan 11 Q6

Question Number	Scheme		Marks
(a)	$y = \dfrac{3-2x}{x-5} \Rightarrow y(x-5) = 3-2x$	Attempt to make x (or swapped y) the subject	M1
	$xy - 5y = 3 - 2x$		
	$\Rightarrow xy + 2x = 3 + 5y \Rightarrow x(y+2) = 3 + 5y$	Collect x terms together and factorise.	M1
	$\Rightarrow x = \dfrac{3+5y}{y+2} \quad \therefore f^{-1}(x) = \underline{\dfrac{3+5x}{x+2}}$	$\underline{\dfrac{3+5x}{x+2}}$	A1 oe (3)
(b)	Range of g is $-9 \le g(x) \le 4$ or $-9 \le y \le 4$	<u>Correct Range</u>	B1 **(1)**
(c)		Deduces that g(2) is 0. Seen or implied.	M1
	g g(2)= g (0) = −6, from sketch.	-6	A1 **(2)**
(d)	fg(8) = f(4)	Correct order g followed by f	M1
	$= \dfrac{3-4(2)}{4-5} = \dfrac{-5}{-1} = \underline{5}$	5	A1 **(2)**
(e)(i)		Correct shape 	B1
		$(2, \{0\}), (\{0\}, 6)$	B1
Question Number	Scheme		Marks
(e)(ii)		Correct shape 	B1
		Graph goes through $(\{0\}, 2)$ and $(-6, \{0\})$ which are marked.	B1 (4)
(f)	Domain of g^{-1} is $-9 \le x \le 4$	Either correct answer or a follow through from part (b) answer	B1 $\sqrt{}$ (1) **[13]**

Question 13: June 11 Q4

Question Number	Scheme		Marks	
(a)	$y = 4 - \ln(x+2)$			
	$\ln(x+2) = 4 - y$			
	$x + 2 = e^{4-y}$			
	$x = e^{4-y} - 2$		M1	
	$f^{-1}(x) = e^{4-x} - 2$	oe	M1A1	
				(3)
(b)	$x \leq 4$		B1	
				(1)
(c)	$fg(x) = 4 - \ln(e^{x^2} - 2 + 2)$		M1	
	$fg(x) = 4 - x^2$		dM1A1	
				(3)
(d)	$fg(x) \leq 4$		B1ft	
				(1)
				8 Marks