

Name:.....

Total Marks:.....

# GCSE (9-1) Grade 8/9

## Nth Term of a Quadratic Sequence



### Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name.
- Answer **all** questions.
- Answer the questions in the spaces provided
  - there may be more space than you need.
- **Show all your working out**

### Information

- The total mark for this paper is 85.
- The marks for **each** question are shown in brackets.
  - use this as a guide as to how much time to spend on each question.
- Questions labelled with an **asterisk** (\*) are ones where the quality of your written communication will be assessed

### Advice

- Read each question carefully before you start to answer it
- Attempt every question
- Check your answers if you have time at the end



1. Write down the next two terms in the following sequence

$$6, 9, 14, 21, \underline{30}, \underline{41}$$

$\underbrace{+3}_{\text{+3}}$     $\underbrace{+5}_{\text{+5}}$     $\underbrace{+7}_{\text{+7}}$     $\underbrace{+9}_{\text{+9}}$     $\underbrace{+11}_{\text{+11}}$

.....  
30 and 41

(2 marks)

- 
2. Write down the next two terms in the following sequence

$$4, -5, -18, -35, \dots \underline{-56}, \underline{-81}$$

$\underbrace{-9}_{\text{-9}}$     $\underbrace{-13}_{\text{-13}}$     $\underbrace{-17}_{\text{-17}}$     $\underbrace{-21}_{\text{-21}}$     $\underbrace{-25}_{\text{-25}}$

.....  
-56 and -81

(2 marks)



3. The nth term of a sequence is

$$2n^2 + 3n - 2$$

Work out the 8<sup>th</sup> term of the sequence

$$\begin{aligned} n = 8 &\Rightarrow 2(8)^2 + 3(8) - 2 \\ &= 2(64) + 24 - 2 \\ &= 128 + 24 - 2 \\ &= 150 \end{aligned}$$

150

(2 marks)

4. The nth term of a sequence is

$$n^2 + 3n - 1$$

Write down the first 5 terms in the sequence

$$1^{\text{st}} \text{ term} \Rightarrow n = 1 \Rightarrow (1)^2 + 3(1) - 1 = 3$$

$$2^{\text{nd}} \text{ term} \Rightarrow n = 2 \Rightarrow (2)^2 + 3(2) - 1 = 9$$

$$3^{\text{rd}} \text{ term} \Rightarrow n = 3 \Rightarrow (3)^2 + 3(3) - 1 = 17$$

$$4^{\text{th}} \text{ term} \Rightarrow n = 4 \Rightarrow (4)^2 + 3(4) - 1 = 27$$

$$5^{\text{th}} \text{ term} \Rightarrow n = 5 \Rightarrow (5)^2 + 3(5) - 1 = 39$$

3, 9, 17, 27, 39

(2 marks)



5. Find an expression, in terms of  $n$ , for the  $n$ th term of the quadratic sequence:

$$\begin{array}{ccccccc} 2, & \underbrace{3, 6}, & \underbrace{11, 18} \\ +1 & +3 & +5 & +7 \\ \underbrace{+2} & \underbrace{+2} & \underbrace{+2} & \\ \end{array} \Rightarrow \frac{n^2 - 1}{2} = n^2$$

$$\begin{array}{ccccc} n^2 & 1 & 4 & 9 & 16 & 25 \\ \downarrow +1 & \downarrow -1 & \downarrow -3 & \downarrow -5 & \downarrow -7 \\ 2 & 3 & 6 & 11 & 18 \end{array}$$

$$+1, -1, -3, -5, -7$$

$$\text{N}^{\text{th}} \text{ term of linear sequence} = -2n + 3$$

$$\therefore \text{N}^{\text{th}} \text{ term of quadratic sequence} = n^2 - 2n + 3 \quad \dots \quad n^2 - 2n + 3 \quad (4 \text{ marks})$$

6. Find an expression, in terms of  $n$ , for the  $n$ th term of the quadratic sequence:

$$\begin{array}{ccccccc} 3, & \underbrace{10, 19}, & \underbrace{30, 43} \\ +7 & +9 & +11 & +13 \\ \underbrace{+2} & \underbrace{+2} & \underbrace{+2} & \\ \end{array} \therefore n^2$$

$$\begin{array}{ccccc} n^2 & 1 & 4 & 9 & 16 & 25 \\ \downarrow +2 & \downarrow +6 & \downarrow +10 & \downarrow +14 & \downarrow +18 \\ 3 & 10 & 19 & 30 & 43 \end{array}$$

$$2, \underbrace{6, 10}, \underbrace{14, 18} \Rightarrow \text{N}^{\text{th}} \text{ term} = 4n - 2$$

$$\therefore \text{N}^{\text{th}} \text{ term of sequence} = n^2 + 4n - 2$$

$$\dots \quad n^2 + 4n - 2$$

(4 marks)



7. Find an expression, in terms of  $n$ , for the  $n$ th term of the quadratic sequence:

$$\begin{array}{c} -1, 0, 3, 8, 15 \\ \underbrace{+1}_{+2} \quad \underbrace{+3}_{+2} \quad \underbrace{+5}_{+2} \quad \underbrace{+7}_{+2} \\ \therefore n^2 \end{array}$$

$$\begin{array}{ccccc} & 1 & 4 & 9 & 16 & 25 \\ -2\downarrow & -4\downarrow & -6\downarrow & -8\downarrow & -10\downarrow \\ -1 & 0 & 3 & 8 & 15 \end{array}$$

$$\begin{array}{c} -2, -4, -6, -8, -10 \\ \underbrace{-2}_{-2} \quad \underbrace{-2}_{-2} \quad \underbrace{-2}_{-2} \quad \underbrace{-2}_{-2} \end{array} \Rightarrow -2n + 0$$

$\therefore N^{th}$  term is  $n^2 - 2n$

$$n^2 - 2n$$

(4 marks)

8. Find an expression, in terms of  $n$ , for the  $n$ th term of the quadratic sequence:

$$\begin{array}{c} -2, 4, 14, 28, 46 \\ \underbrace{+6}_{+4} \quad \underbrace{+10}_{+4} \quad \underbrace{+14}_{+4} \quad \underbrace{+18}_{+4} \\ \therefore 2n^2 \quad (4 \div 2) \end{array}$$

$$\begin{array}{ccccc} 2n^2 & 2 & 8 & 18 & 32 & 50 \\ -4\downarrow & -4\downarrow & -4\downarrow & -4\downarrow & -4\downarrow \\ -2 & 4 & 14 & 28 & 46 \end{array}$$

$\therefore N^{th}$  term is  $2n^2 - 4$

$$2n^2 - 4$$

(4 marks)



9. Find an expression, in terms of  $n$ , for the  $n$ th term of the quadratic sequence:

$$6, \underbrace{10, 12}_{+4}, \underbrace{12, 10}_{+0 -2} \Rightarrow -n^2 \quad (-2 \div 2)$$

$$-n^2 \quad \begin{matrix} -1 & -4 & -9 & -16 & -25 \\ +7 \downarrow & +14 \downarrow & +21 \downarrow & +28 \downarrow & +35 \downarrow \\ 6 & 10 & 12 & 12 & 10 \end{matrix}$$

$$7, \underbrace{14, 21}_{+7}, \underbrace{28, 35}_{+7} \Rightarrow 7n - 0$$

$$\therefore N^{\text{th}} \text{ term is } -n^2 + 7n$$

(4 marks)

10. Find an expression, in terms of  $n$ , for the  $n$ th term of the quadratic sequence:

$$-2, \underbrace{-1, 1}_{+1 +2}, \underbrace{4, \dots}_{+1 +1} \Rightarrow 0.5n^2 \quad (1 \div 2)$$

$$0.5n^2 \quad \begin{matrix} 0.5 & 2 & 4.5 & 8 \\ -2.5 \downarrow & -3 \downarrow & -3.5 \downarrow & -4 \downarrow \\ -2 & -1 & 1 & 4 \end{matrix}$$

$$-2.5, \underbrace{-3, -3.5}_{-0.5}, \underbrace{-4}_{-0.5} \Rightarrow -0.5n - 2$$

$$\therefore N^{\text{th}} \text{ term is } 0.5n^2 - 0.5n - 2$$

$$0.5n^2 - 0.5n - 2$$

(4 marks)



11. Here are the first 6 terms of a sequence.

$$9 \quad 4 \quad -5 \quad -18 \quad -35$$

(a) Find an expression, in terms of  $n$ , for the  $n$ th term of this sequence.

$$\begin{array}{ccccccccc}
 9 & 4 & -5 & -18 & -35 \\
 & \underline{-5} & \underline{-9} & \underline{-13} & \underline{-17} \\
 & -4 & -4 & -4 & -4 \\
 \Rightarrow -2n^2 & (-4 \div 2)
 \end{array}$$

$$\begin{array}{cccccc}
 -2n^2 & -2 & -8 & -18 & -32 & -50 \\
 +n & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
 9 & 4 & -5 & -18 & -35
 \end{array}$$

$$\underbrace{11, \quad 12}_{+1}, \quad \underbrace{13, \quad 14}_{+1}, \quad \underbrace{15}_{+1} \Rightarrow n + 10$$

∴  $N^{th}$  term is  $-2n^2 + n + 10$

$$-2n^2 + n + 10$$

(4 marks)

(b) Hence find the term that has value  $-1215$ .

$$-2n^2 + n + 15 = -1215$$

$$-2n^2 + n + 10 + 1215 = 0$$

$$-2n^2 + n + 1225 = 0$$

$$(x-1) \quad \downarrow \quad 2n^2 - n - \underline{1225} = 0$$

$$\text{Using } n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\therefore$  25<sup>th</sup> term

$$n = \frac{1 \pm \sqrt{(-1)^2 - 4(2)(-1225)}}{2(2)}$$

25<sup>th</sup>

$$\lambda = 25 \text{ or } -24.5$$

$\equiv$  Invalid

(2 marks)



12. Here are the first 5 terms of a sequence.

$$7 \quad 16 \quad 29 \quad 46 \quad 67$$

(a) Find an expression, in terms of  $n$ , for the  $n$ th term of this sequence.

$$\begin{array}{ccccccc} 7 & 16 & 29 & 46 & 67 \\ \underbrace{9}_{+4} & \underbrace{13}_{+4} & \underbrace{17}_{+4} & \underbrace{21}_{+4} & \Rightarrow 2n^2 \end{array}$$

$$\begin{array}{ccccc} 2n^2 & 2 & 8 & 18 & 32 & 50 \\ +5 \downarrow & +8 \downarrow & +11 \downarrow & +14 \downarrow & +17 \downarrow \\ 7 & 16 & 29 & 46 & 67 \end{array}$$

$$\begin{array}{ccccc} 5 & 8 & 11 & 14 & 17 \\ +3 \downarrow & +3 \downarrow & +3 \downarrow & +3 \downarrow & \Rightarrow 3n - 2 \end{array}$$

$\therefore N^{th}$  term is  $2n^2 + 3n + 2$

$$2n^2 + 3n + 2$$

(4 marks)

(b) Hence find the term that has value 862

$$2n^2 + 3n + 2 = 862$$

$$2n^2 + 3n - 860 = 0$$

$$\text{Using } n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \therefore 20^{th} \text{ term}$$

$$n = \frac{-3 \pm \sqrt{3^2 - 4(2)(-86)}}{2(2)}$$

$$n = 20 \text{ or } -21.5$$

$\equiv$  NOT VALID

20<sup>th</sup>

(2 marks)



13. Here are the first 5 terms of a sequence.

$$\begin{array}{ccccccc} 3 & 9 & 17 & 27 & 39 \\ \underbrace{+6}_{+2} & \underbrace{+8}_{+2} & \underbrace{+10}_{+2} & \underbrace{+12}_{+2} & \Rightarrow n^2 \end{array}$$

Claire says that 161 is a term of this sequence

(a) Is Claire correct? Give a reason for your answer

$$\begin{array}{ccccccc} n^2 & 1 & 4 & 9 & 16 & 25 \\ +2 \downarrow & +5 \downarrow & +8 \downarrow & +11 \downarrow & +14 \downarrow \\ 3 & 9 & 17 & 27 & 39 \end{array}$$

$$\begin{array}{ccccc} 2, & 5, & 8, & 11, & 14 \\ \underbrace{+3}_{+3} & \underbrace{+3}_{+3} & \underbrace{+3}_{+3} & \Rightarrow 3n - 1 \end{array}$$

$\therefore N^{th}$  term =  $n^2 + 3n - 1$

$$\begin{aligned} n^2 + 3n - 1 &= 161 \\ n^2 + 3n - 162 &= 0 \\ a = 1, b = 3, c = -162 \end{aligned}$$

$$n = \frac{-3 \pm \sqrt{3^2 - 4(1)(-162)}}{2(1)}$$

$$n = 11.31\dots \text{ or } n = -14.31\dots$$

Since  $n$  is NOT A WHOLE NUMBER, therefore 161 is not a term in this sequence

(4 marks)

Rob says that all the terms in the sequence are odd numbers

(b) Is Rob correct? Give a reason for your answer.

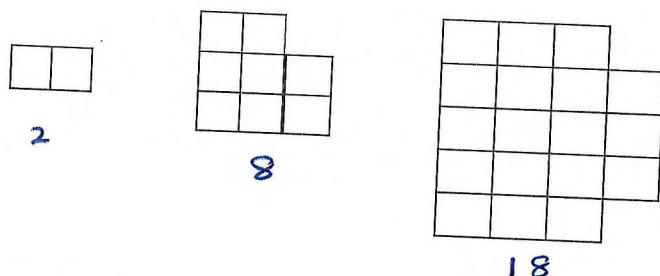
Rob is correct as all numbers are odd

$$3, 9, 17, 27, 39, 53, 69, 87, 107, 129\dots$$

(2 marks)



14. Here are some patterns made from square slabs.



(a) Write down an expression, in terms of  $n$ , for the  $n$ th term of this sequence.

$$2, 8, 18$$

$\overbrace{\quad}^{+6}$        $\overbrace{\quad}^{+10}$   
 $\overbrace{\quad}^{+4} \Rightarrow 2n^2$

$$2n^2 \quad 2 \quad 8 \quad 18$$

$\downarrow +6$        $\downarrow +8$        $\downarrow +10$   
 $2 \quad 8 \quad 18$

$\therefore N^{\text{th}}$  term is  $2n^2$

$$2n^2$$

(4 marks)

(b) Jane says that 75 is a term in the quadratic sequence.

Is Jane correct? Give a reason for your answer.

$$2n^2 = 75$$

$$n = \sqrt{\frac{75}{2}}$$

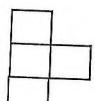
$$n = 6.12$$

NO, as  $n$  is not an integer

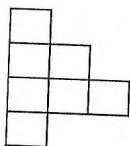
(2 marks)



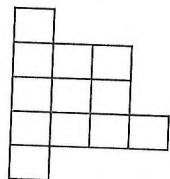
15. Here are some patterns made from tiles.



4



7



12

Find an expression, in terms of  $n$ , for the  $n$ th term of this sequence.

$$\begin{array}{c} 4, 7, 12 \\ \underbrace{\quad}_{+3} \quad \underbrace{\quad}_{+5} \\ \qquad \qquad \qquad \underbrace{\quad}_{+2} \Rightarrow n^2 \end{array}$$

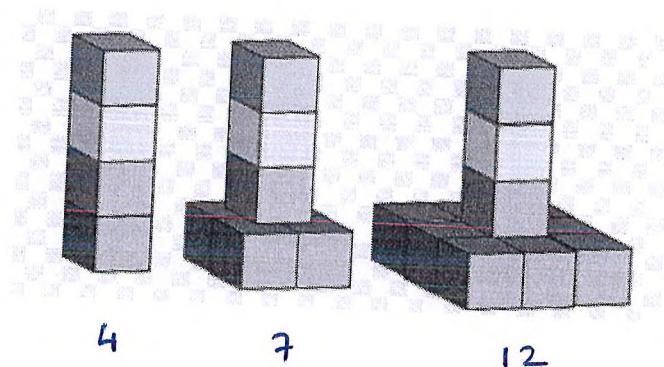
$$\begin{array}{cccc} n^2 & 1 & 4 & 9 \\ +3 \downarrow & +3 \downarrow & +3 \downarrow \\ 4 & 7 & 12 \end{array}$$

$$\therefore N^{\text{th}} \text{ term is } n^2 + 3$$

$$n^2 + 3$$

(4 marks)

16. Here are some patterns made from cubes.



Find an expression, in terms of  $n$ , for the  $n$ th term of this sequence.

$$\begin{array}{c} 4, \quad 7, \quad 12 \\ \underbrace{\quad}_{+3} \quad \underbrace{\quad}_{+5} \\ +2 \end{array} \Rightarrow n^2$$

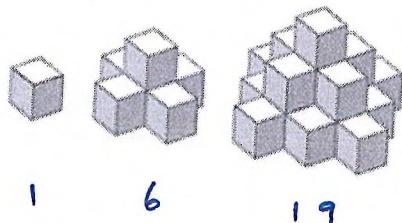
$$\begin{array}{c} n^2 \\ 1 \quad 4 \quad 9 \\ \downarrow +3 \quad \downarrow +3 \quad \downarrow +3 \\ 4 \quad 7 \quad 12 \end{array}$$

$$\therefore \text{N}^{\text{th}} \text{ term is } \underline{\underline{n^2 + 3}}$$

$$n^2 + 3$$

(4 marks)

17. Here are some patterns made from cubes.



(a) Work out the number of cubes needed to make the next pattern.

$$\begin{array}{cccc} 1, & 6, & 19, & 40 \\ \underbrace{+5}_{+8}, & \underbrace{+13}_{+8}, & \underbrace{+21}_{+8}, & \end{array}$$

40

(2 marks)

(b) Find an expression, in terms of  $n$ , for the  $n$ th term of this sequence.

$$\begin{array}{cccc} 1, & 6, & 19, & 40 \\ \underbrace{+5}_{+8}, & \underbrace{+13}_{+8}, & \underbrace{+21}_{+8}, & \\ & +8 & +8 & \Rightarrow 4n^2 \end{array}$$

$$\begin{array}{cccc} 4n^2 & 4, & 16, & 36, & 64 \\ -3\downarrow & -12\downarrow & -17\downarrow & -24\downarrow \\ 1, & 6, & 19, & 40 \end{array}$$

$$\therefore \text{N}^{\text{th}} \text{ term} = 4n^2 + 7n - 4$$

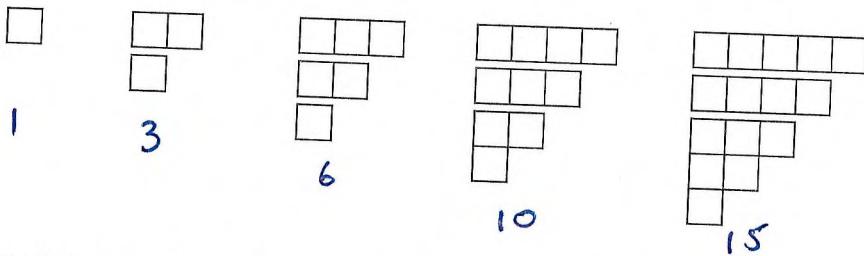
$$\begin{array}{cccc} 3, & 10, & 17, & 24 \\ \underbrace{+7}_{+7}, & \underbrace{+7}_{+7}, & \underbrace{+7}_{+7}, & \Rightarrow 7n - 4 \end{array}$$

$$4n^2 + 7n - 4$$

(4 marks)



18. Here is a sequence of patterns made from centimetre squares.



Find the number of centimetre squares in the 100th pattern.

$$1 \quad 3 \quad 6 \quad 10 \quad 15$$

+2      +3      +4      +5  
 +1      +1      +1      +1       $\Rightarrow \frac{1}{2}n^2$

$$\frac{1}{2}n^2 \quad 0.5 \quad 2 \quad 4.5 \quad 8 \quad 12.5$$

+0.5 ↓    +1 ↓    +1.5 ↓    +2 ↓    +2.5 ↓  
 1        3        6        10        15

$$0.5, 1, 1.5, 2, 2.5$$

+0.5      +0.5      +0.5      +0.5       $\Rightarrow 0.5n - 0$

$$\therefore N^{th} \text{ term} = \frac{1}{2}n^2 + \frac{1}{2}n$$

or

$$= 0.5n^2 + 0.5n$$

$$100^{\text{th}} \text{ pattern} \Rightarrow n = 100$$

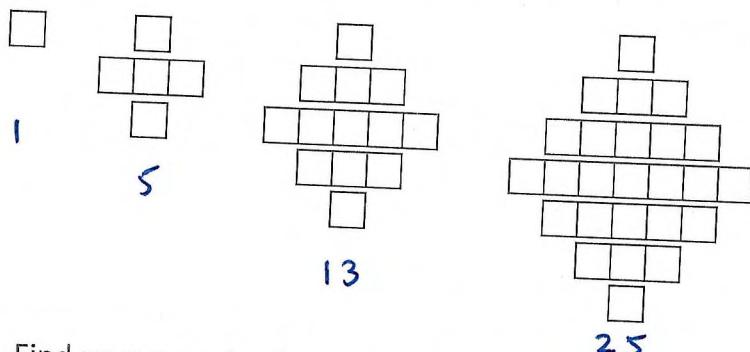
$$\therefore 0.5(100)^2 + 0.5(100) = 5050 \quad \underline{\underline{\underline{\quad}}}$$

$$\frac{1}{2}n^2 + \frac{1}{2}n$$

(5 marks)

5050

19. Here is a sequence of patterns made from centimetre squares.



Find an expression in terms of  $n$  for the number of centimetre squares in the  $n$ th pattern.

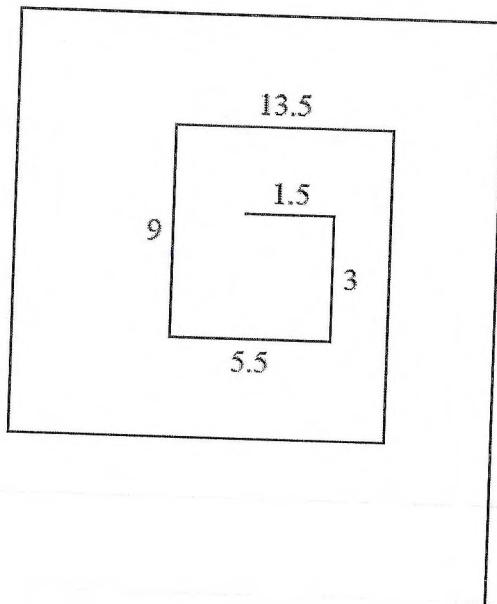
$$\begin{array}{cccc} 1 & 5 & 13 & 25 \\ \underbrace{\quad}_{+4} \quad \underbrace{\quad}_{+8} \quad \underbrace{\quad}_{+12} & & & \\ +4 & +4 & +4 & \Rightarrow 2n^2 \end{array}$$

$$\begin{array}{cccccc} 2n^2 & 2 & 8 & 18 & 32 \\ -1 \downarrow & -3 \downarrow & -5 \downarrow & -7 \downarrow & & \\ 1 & 5 & 13 & 25 & & \\ -1, -3, -5, -7 \\ \underbrace{-2}_{-2} \quad \underbrace{-2}_{-2} \quad \underbrace{-2}_{-2} & \Rightarrow -2n + 1 \\ \therefore n^{\text{th}} \text{ term} = 2n^2 - 2n + 1 \end{array}$$

$$2n^2 - 2n + 1$$

(5 marks)

20. The diagram shows the first 10 sides of a spiral pattern.  
It also gives the lengths, in cm, of the first 5 sides.



The lengths, in cm, of the sides of the spiral form a sequence.

Find an expression in terms of  $n$  for the length, in cm, of the  $n$ th side.

$$1.5, 3, 5.5, 9, 13.5$$

$$\begin{array}{c} +1.5 \\ \underbrace{\quad}_{+1} \quad \underbrace{+2.5}_{+1} \quad \underbrace{+3.5}_{+1} \quad \underbrace{+4.5}_{+1} \end{array} \Rightarrow \frac{1}{2}n^2$$

$$\frac{1}{2}n^2 \quad \frac{1}{2}, \quad 2, \quad \frac{9}{2}, \quad 8, \quad \frac{25}{2}$$

$$\begin{array}{c} +1 \downarrow \\ 1.5, \quad 3, \quad 5.5, \quad 9, \quad 13.5 \end{array} \Rightarrow +1$$

$$\therefore N^{th} \text{ term} = \cancel{\frac{1}{2}n^2 + 1}$$

$$\frac{1}{2}n^2 + 1$$

(3 marks)

**TOTAL FOR PAPER IS 85 MARKS**