Parametric Differentiation - Edexcel Past Exam Questions MARK SCHEME

Question 1: June 05 Q6

	(a) $\frac{dx}{dt} = -2\operatorname{cosec}^2 t,\ \frac{dy}{dt} = 4\sin t\cos t$ both	M1 A1	
	$\frac{dy}{dx} = \frac{-2\sin t\cos t}{\operatorname{cosec}^2 t} \quad \left(= -2\sin^3 t\cos t\right)$	M1 A1	
			(4)
	(b) At $t = \frac{\pi}{4}$, $x = 2$, $y = 1$ both x and y	B1	
	Substitutes $t = \frac{\pi}{4}$ into an attempt at $\frac{dy}{dx}$ to obtain gradient $\left(-\frac{1}{2}\right)$	M1	
	Equation of tangent is $y - 1 = -\frac{1}{2}(x - 2)$	M1 A1	
	Accept $x + 2y = 4$ or any correct equivalent		**(4)**
	(c) Uses $1 + \cot^2 t = \operatorname{cosec}^2 t$, or equivalent, to eliminate t	M1	
	$1 + \left(\frac{x}{2}\right)^2 = \frac{2}{y}$ correctly eliminates t	A1	
	$y = \frac{8}{4 + x^2}$ cao	A1	
	The domain is $x \ldots 0$	B1	**(4)** **[12]**
	An alternative in (c)		
	$\sin t = \left(\frac{y}{2}\right)^{\frac{1}{2}}$; $\cos t = \frac{x}{2}\sin t = \frac{x}{2}\left(\frac{y}{2}\right)^{\frac{1}{2}}$		
	$\sin^2 t + \cos^2 t = 1 \quad \Rightarrow \quad \frac{y}{2} + \frac{x^2}{4} \times \frac{y}{2} = 1$	M1 A1	
	Leading to $y = \frac{8}{4 + x^2}$	A1	

Question 2: June 06 Q4

Question Number	Scheme		Marks
(a)	$x = \sin t, \quad y = \sin\left(t + \frac{\pi}{6}\right)$		
	$\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = \cos\left(t + \frac{\pi}{6}\right)$	Attempt to differentiate both x and y wrt t to give two terms in cos Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	M1 A1
	When $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \underline{\frac{\cos\left(\frac{\pi}{6} + \frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58}$	Divides in correct way and substitutes for t to give any of the four underlined oe: Ignore the double negative if candidate has differentiated $\sin \to -\cos$	A1
	When $t = \frac{\pi}{6}$, $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$	The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ or $\underline{\left(\frac{1}{2}, \text{awrt } 0.87\right)}$	B1
	T: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + "c"$. Correct <u>EXACT</u> equation of <u>tangent</u> oe.	dM1 <u>A1</u> oe
	or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$		
	or **T**: $\left[\underline{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}}\right]$		**[6]**
(b)	$y = \sin\left(t + \frac{\pi}{6}\right) = \sin t \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6}$	Use of compound angle formula for sine.	M1
	Nb: $\sin^2 t + \cos^2 t \equiv 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t$		
	$\therefore x = \sin t$ gives $\cos t = \sqrt{(1 - x^2)}$	Use of trig identity to find $\cos t$ in terms of x or $\cos^2 t$ in terms of x.	M1
	$\therefore y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t$		
	gives $y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1 - x^2)}$ **AG**	Substitutes for $\sin t$, $\cos\frac{\pi}{6}$, $\cos t$ and $\sin\frac{\pi}{6}$ to give y in terms of x.	A1 cso **[3]**
			9 marks

Question Number	Scheme		Marks
Aliter (a) **Way 2**	$x = \sin t, \quad y = \sin\left(t + \frac{\pi}{6}\right) = \sin t\cos\frac{\pi}{6} + \cos t\sin\frac{\pi}{6}$	(Do not give this for part (b)) Attempt to differentiate x and y wrt t to give $\frac{dx}{dt}$ in terms of cos and $\frac{dy}{dt}$ in the form $\pm a\cos t \pm b\sin t$	M1
	$\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = \cos t\cos\frac{\pi}{6} - \sin t\sin\frac{\pi}{6}$	Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1
	When $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \underline{\frac{\cos\frac{\pi}{6}\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\sin\frac{\pi}{6}}{\cos\left(\frac{\pi}{6}\right)}}$ $= \underline{\frac{\frac{3}{4}-\frac{1}{4}}{\frac{\sqrt{3}}{2}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58}$	Divides in correct way and substitutes for t to give any of the four underlined oe:	A1
	When $t = \frac{\pi}{6}$, $\quad x = \frac{1}{2}, \quad y = \frac{\sqrt{3}}{2}$	The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ or $\underline{\left(\frac{1}{2}, \text{awrt } 0.87\right)}$	B1
	T: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + "c"$.	dM1
	or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$	Correct EXACT equation of <u>tangent</u> oe.	<u>A1</u> oe
	or **T**: $\left[\underline{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}}\right]$		**[6]**

Question Number	Scheme		Marks
Aliter (a) **Way 3**	$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1-x^2)}$		
	$\frac{dy}{dx} = \frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1-x^2)^{-\frac{1}{2}}(-2x)$	Attempt to differentiate two terms using the chain rule for the second term. Correct $\frac{dy}{dx}$	M1 A1
	$\frac{dy}{dx} = \frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1-(0.5)^2)^{-\frac{1}{2}}(-2(0.5)) = \underline{\frac{1}{\sqrt{3}}}$	Correct substitution of $x = \frac{1}{2}$ into a correct $\frac{dy}{dx}$	A1
	When $t = \frac{\pi}{6}$, $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$	The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ or $\underline{\left(\frac{1}{2}, \text{awrt } 0.87\right)}$	B1
	T: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + "c"$. Correct <u>EXACT</u> equation of <u>tangent</u> oe.	dM1 <u>A1</u> oe
Aliter (b) **Way 2**	$x = \sin t$ gives $y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\sqrt{(1-\sin^2 t)}$	Substitutes $x = \sin t$ into the equation give in y.	M1
	Nb: $\sin^2 t + \cos^2 t \equiv 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t$		
	$\cos t = \sqrt{(1-\sin^2 t)}$	Use of trig identity to deduce that $\cos t = \sqrt{(1-\sin^2 t)}$.	M1
	gives $y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t$		
	Hence $y = \sin t \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6} = \sin\left(t + \frac{\pi}{6}\right)$	Using the compound angle formula to prove $y = \sin\left(t + \frac{\pi}{6}\right)$	A1 cso **[3]**
			9 marks

Question 3: Jan 07 Q3

Question Number	Scheme		Marks
(a)	$x = 7\cos t - \cos 7t,\quad y = 7\sin t - \sin 7t,$		
	$\frac{dx}{dt} = -7\sin t + 7\sin 7t,\quad \frac{dy}{dt} = 7\cos t - 7\cos 7t$	Attempt to differentiate x **and** y with respect to t to give $\frac{dx}{dt}$ in the form $\pm A\sin t \pm B\sin 7t$; $\frac{dy}{dt}$ in the form $\pm C\cos t \pm D\cos 7t$	M1
		Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1
	$\therefore \frac{dy}{dx} = \frac{7\cos t - 7\cos 7t}{-7\sin t + 7\sin 7t}$	Candidate's $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	B1 $\sqrt{}$
			[3]
(b)	When $t = \frac{\pi}{6}$, $m(\mathbf{T}) = \frac{dy}{dx} = \frac{7\cos\frac{\pi}{6} - 7\cos\frac{7\pi}{6}}{-7\sin\frac{\pi}{6} + 7\sin\frac{7\pi}{6}}$;	Substitutes $t = \frac{\pi}{6}$ or $30°$ into their $\frac{dy}{dx}$ expression;	M1
	$= \underline{\frac{\frac{7\sqrt{3}}{2} - \left(-\frac{7\sqrt{3}}{2}\right)}{-\frac{7}{2} - \frac{7}{2}}} = \underline{\frac{7\sqrt{3}}{-7}} = \underline{-\sqrt{3}} = \underline{\text{awrt } -1.73}$	to give any of the four underlined expressions oe **(must be correct solution only)**	A1 **cso**
	Hence $m(\mathbf{N}) = \frac{-1}{-\sqrt{3}}$ or $\frac{1}{\sqrt{3}}$ = awrt 0.58	Uses $m(\mathbf{T})$ to 'correctly' find $m(\mathbf{N})$. Can be ft from "their tangent gradient".	A1$\sqrt{}$ oe.
	When $t = \frac{\pi}{6}$, $x = 7\cos\frac{\pi}{6} - \cos\frac{7\pi}{6} = \frac{7\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right) = \frac{8\sqrt{3}}{2} = 4\sqrt{3}$; $y = 7\sin\frac{\pi}{6} - \sin\frac{7\pi}{6} = \frac{7}{2} - \left(-\frac{1}{2}\right) = \frac{8}{2} = 4$	The point $\underline{\left(4\sqrt{3},\ 4\right)}$ or $\underline{\left(\text{awrt } 6.9,\ 4\right)}$	B1
	N: $y - 4 = \frac{1}{\sqrt{3}}\left(x - 4\sqrt{3}\right)$	Finding an equation of a normal with their point and their normal gradient or finds c by using $y = \text{(their gradient)}x + \text{"c"}$.	M1
	N: $\underline{y = \frac{1}{\sqrt{3}}x}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$	Correct simplified EXACT equation of <u>normal</u>. This is dependent on candidate using correct $\left(4\sqrt{3},\ 4\right)$	<u>A1</u> oe
	or $4 = \frac{1}{\sqrt{3}}\left(4\sqrt{3}\right) + c \Rightarrow c = 4 - 4 = 0$		
	Hence **N:** $\underline{y = \frac{1}{\sqrt{3}}x}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$		**[6]** **9 marks**

Question 4: June 07 Q6

Question Number	Scheme		Marks
(a)	$x = \tan^2 t\,,\qquad y = \sin t$		
	$\frac{dx}{dt} = 2(\tan t)\sec^2 t\,,\quad \frac{dy}{dt} = \cos t$	Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	B1
	$\therefore \frac{dy}{dx} = \frac{\cos t}{2\tan t \sec^2 t}\quad \left(= \frac{\cos^4 t}{2\sin t}\right)$	$\frac{\pm\cos t}{\text{their } \frac{dx}{dt}}$	M1
		$\frac{+\cos t}{\text{their } \frac{dx}{dt}}$	A1 $\sqrt{}$
			[3]
(b)	When $t = \frac{\pi}{4}$, $\quad x = 1\,,\ y = \frac{1}{\sqrt{2}}$ (need values)	The point $\underline{\left(1, \frac{1}{\sqrt{2}}\right)}$ or $\underline{(1, \text{awrt } 0.71)}$ **These coordinates can be implied.** ($y = \sin\left(\frac{\pi}{4}\right)$ is not sufficient for B1)	B1, B1
	When $t = \frac{\pi}{4}$, $\ m(\mathbf{T}) = \frac{dy}{dx} = \frac{\cos\frac{\pi}{4}}{2\tan\frac{\pi}{4}\sec^2\frac{\pi}{4}}$		
	$= \underline{\frac{\frac{1}{\sqrt{2}}}{2.(1)\left(\frac{1}{\frac{1}{\sqrt{2}}}\right)^2}} = \underline{\frac{\frac{1}{\sqrt{2}}}{2.(1)\left(\frac{1}{\frac{1}{2}}\right)}} = \underline{\frac{\frac{1}{\sqrt{2}}}{2.(1)(2)}} = \underline{\frac{1}{4\sqrt{2}}} = \underline{\frac{\sqrt{2}}{8}}$	any of the five underlined expressions or awrt 0.18	B1 aef
	$\mathbf{T}$: $y - \frac{1}{\sqrt{2}} = \frac{1}{4\sqrt{2}}(x - 1)$	Finding an equation of a tangent with ***their point*** and ***their tangent gradient*** or finds c by using $y = \underline{(\text{their gradient})}x + "\underline{c}"$.	M1 $\sqrt{}$ aef
	$\mathbf{T}$: $\underline{y = \frac{1}{4\sqrt{2}}x + \frac{3}{4\sqrt{2}}}$ or $\underline{y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}}$	Correct simplified EXACT equation of $\underline{\text{tangent}}$	$\underline{\text{A1}}$ aef **cso**
	or $\frac{1}{\sqrt{2}} = \frac{1}{4\sqrt{2}}(1) + c \Rightarrow c = \frac{1}{\sqrt{2}} - \frac{1}{4\sqrt{2}} = \frac{3}{4\sqrt{2}}$		
	Hence $\mathbf{T}$: $\underline{y = \frac{1}{4\sqrt{2}}x + \frac{3}{4\sqrt{2}}}$ or $\underline{y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}}$		
			[5]

Note: The x and y coordinates must be the right way round.

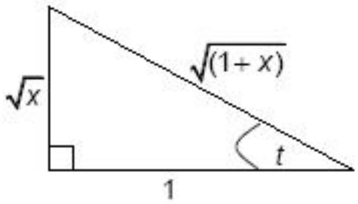
A candidate who incorrectly differentiates $\tan^2 t$ to give $\frac{dx}{dt} = 2\sec^2 t$ or $\frac{dx}{dt} = \sec^4 t$ is then able to fluke the correct answer in part (b). Such candidates can potentially get: (a) B0M1A1 $\sqrt{}$ (b) B1B1B1M1A0 **cso**.
Note: cso means "correct solution only".
Note: part (a) not fully correct implies candidate can achieve a maximum of 4 out of 5 marks in part (b).

(c) **Way 1**	$x = \tan^2 t = \dfrac{\sin^2 t}{\cos^2 t}$ $\quad y = \sin t$		
	$x = \dfrac{\sin^2 t}{1-\sin^2 t}$	Uses $\cos^2 t = 1-\sin^2 t$	M1
	$x = \dfrac{y^2}{1-y^2}$	Eliminates 't' to write an equation involving x and y.	M1
	$x(1-y^2) = y^2 \Rightarrow x - xy^2 = y^2$		
	$x = y^2 + xy^2 \Rightarrow x = y^2(1+x)$	Rearranging and factorising with an attempt to make y^2 the subject.	ddM1
	$y^2 = \dfrac{x}{1+x}$	$\dfrac{x}{1+x}$	A1
			[4]
Aliter (c) **Way 2**	$1+\cot^2 t = \operatorname{cosec}^2 t$	Uses $1+\cot^2 t = \operatorname{cosec}^2 t$	M1
	$= \dfrac{1}{\sin^2 t}$	Uses $\operatorname{cosec}^2 t = \dfrac{1}{\sin^2 t}$	M1 implied
	Hence, $1+\dfrac{1}{x} = \dfrac{1}{y^2}$	Eliminates 't' to write an equation involving x and y.	ddM1
	Hence, $y^2 = 1-\dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	$1-\dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	A1
			[4]

$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

Aliter (c) **Way 3**	$x = \tan^2 t \qquad y = \sin t$		
	$1 + \tan^2 t = \sec^2 t$	Uses $1 + \tan^2 t = \sec^2 t$	M1
	$= \dfrac{1}{\cos^2 t}$	Uses $\sec^2 t = \dfrac{1}{\cos^2 t}$	M1
	$= \dfrac{1}{1 - \sin^2 t}$		
	Hence, $1 + x = \dfrac{1}{1 - y^2}$	Eliminates 't' to write an equation involving x and y.	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	$1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	A1
			[4]
Aliter (c) **Way 4**	$y^2 = \sin^2 t = 1 - \cos^2 t$	Uses $\sin^2 t = 1 - \cos^2 t$	M1
	$= 1 - \dfrac{1}{\sec^2 t}$	Uses $\cos^2 t = \dfrac{1}{\sec^2 t}$	M1
	$= 1 - \dfrac{1}{(1 + \tan^2 t)}$	then uses $\sec^2 t = 1 + \tan^2 t$	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	$1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	A1
			[4]

$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

Aliter (c) **Way 5**	$x = \tan^2 t \qquad y = \sin t$		
	$x = \tan^2 t \;\Rightarrow\; \tan t = \sqrt{x}$		
		Draws a right-angled triangle and places both $\sqrt{x}$ and 1 on the triangle	M1
		Uses Pythagoras to deduce the hypotenuse	M1
	Hence, $y = \sin t = \dfrac{\sqrt{x}}{\sqrt{1+x}}$	Eliminates 't' *to* write an equation involving x and y.	ddM1
	Hence, $y^2 = \dfrac{x}{1+x}$	$\dfrac{x}{1+x}$	A1 **[4]**
			12 marks

$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

There are so many ways that a candidate can proceed with part (c). If a candidate produces a correct solution then please award all four marks. If they use a method commensurate with the five ways as detailed on the mark scheme then award the marks appropriately. If you are unsure of how to apply the scheme please escalate your response up to your team leader.

Takes out brackets.

Writing down $\dfrac{1}{(t+1)(t+2)} = \dfrac{1}{(t+1)} + \dfrac{1}{(t+2)}$ means first M1A0 in (b).

Writing down $\dfrac{1}{(t+1)(t+2)} = \dfrac{1}{(t+1)} - \dfrac{1}{(t+2)}$ means first M1A1 in (b).

Question 5: Jan 09 Q7

Question Number	Scheme		Marks
(a)	At A, $x=-1+8=7$ & $y=(-1)^2=1 \Rightarrow A(7,1)$	$A(7,1)$	B1 (1)
(b)	$x=t^3-8t$, $y=t^2$,		
	$\frac{dx}{dt}=3t^2-8$, $\frac{dy}{dt}=2t$		
	$\therefore \frac{dy}{dx}=\frac{2t}{3t^2-8}$	Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$	M1
		Correct $\frac{dy}{dx}$	A1
	At A, $m(\mathbf{T})=\frac{2(-1)}{\underline{3(-1)^2-8}}=\frac{-2}{\underline{3-8}}=\frac{-2}{\underline{-5}}=\frac{2}{\underline{5}}$	Substitutes for t to give any of the four underlined oe:	
	$\mathbf{T}$: $y-(\text{their } 1)=m_T(x-(\text{their } 7))$ or $1=\frac{2}{5}(7)+c \Rightarrow c=1-\frac{14}{5}=-\frac{9}{5}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y=(\text{their gradient})x+\text{"}c\text{"}$.	dM1
	Hence $\mathbf{T}$: $y=\frac{2}{5}x-\frac{9}{5}$		
	gives $\mathbf{T}$: $\underline{2x-5y-9=0}$ **AG**	$\underline{2x-5y-9=0}$	A1 cso (5)
(c)	$2(t^3-8t)-5t^2-9=0$	Substitution of both $x=t^3-8t$ and $y=t^2$ into $\mathbf{T}$	M1
	$2t^3-5t^2-16t-9=0$		
	$(t+1)\{(2t^2-7t-9)=0\}$ $(t+1)\{(t+1)(2t-9)=0\}$	A realisation that $(t+1)$ is a factor.	dM1
	$\{t=-1 \text{ (at } A)\}$ $t=\frac{9}{2}$ at B	$t=\frac{9}{2}$	A1
	$x=\left(\frac{9}{2}\right)^2-8\left(\frac{9}{2}\right)=\frac{729}{8}-36=\frac{441}{8}=55.125$ or awrt 55.1	Candidate uses their value of t to find either the x or y coordinate	ddM1
	$y=\left(\frac{9}{2}\right)^2=\frac{81}{4}=20.25$ or awrt 20.3	One of either x or y correct.	A1
		Both x and y correct.	A1
	Hence $B\left(\frac{441}{8},\frac{81}{4}\right)$	awrt	(6)
			[12]

Question 6: June 09 Q5

Question Number	Scheme	Marks
Q (a)	$\frac{dx}{dt} = -4\sin 2t,\quad \frac{dy}{dt} = 6\cos t$	B1, B1
	$\frac{dy}{dx} = -\frac{6\cos t}{4\sin 2t} \quad \left(= -\frac{3}{4\sin t}\right)$	M1
	At $t = \frac{\pi}{3}$, $\quad m = -\frac{3}{4\times\frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{2}$ accept equivalents, awrt -0.87	A1 (4)
(b)	Use of $\cos 2t = 1 - 2\sin^2 t$	M1
	$\cos 2t = \frac{x}{2},\ \sin t = \frac{y}{6}$	
	$\frac{x}{2} = 1 - 2\left(\frac{y}{6}\right)^2$	M1
	Leading to $\quad y = \sqrt{(18-9x)} \quad \left(= 3\sqrt{(2-x)}\right)$ cao	A1
	$-2 \le x \le 2$ $\quad k = 2$	B1 (4)
(c)	$0 \le f(x) \le 6$ $\quad$ either $0 \le f(x)$ or $f(x) \le 6$	B1
	Fully correct. Accept $0 \le y \le 6$, $[0, 6]$	B1 (2)
		[10]
	Alternatives to (a) where the parameter is eliminated	
	① $\quad y = (18-9x)^{\frac{1}{2}}$	
	$\frac{dy}{dx} = \frac{1}{2}(18-9x)^{-\frac{1}{2}} \times (-9)$	B1
	At $t = \frac{\pi}{3}$, $x = \cos\frac{2\pi}{3} = -1$	B1
	$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{\sqrt{(27)}} \times -9 = -\frac{\sqrt{3}}{2}$	M1 A1 (4)
	② $\quad y^2 = 18 - 9x$	
	$2y\frac{dy}{dx} = -9$	B1
	At $t = \frac{\pi}{3}$, $\quad y = 6\sin\frac{\pi}{3} = 3\sqrt{3}$	B1
	$\frac{dy}{dx} = -\frac{9}{2\times 3\sqrt{3}} = -\frac{\sqrt{3}}{2}$	M1 A1 (4)

Question 7: June 10 Q4

Question Number	Scheme	Marks
	(a) $\frac{dx}{dt} = 2\sin t\cos t,\ \frac{dy}{dt} = 2\sec^2 t$	B1 B1
	$\frac{dy}{dx} = \frac{\sec^2 t}{\sin t\cos t}\left(= \frac{1}{\sin t\cos^3 t}\right)$ or equivalent	M1 A1 (4)
	(b) At $t = \frac{\pi}{3},\ x = \frac{3}{4},\ y = 2\sqrt{3}$	B1
	$\frac{dy}{dx} = \frac{\sec^2 \frac{\pi}{3}}{\sin\frac{\pi}{3}\cos\frac{\pi}{3}} = \frac{16}{\sqrt{3}}$	M1 A1
	$y - 2\sqrt{3} = \frac{16}{\sqrt{3}}\left(x - \frac{3}{4}\right)$	M1
	$y = 0 \ \Rightarrow \ x = \frac{3}{8}$	M1 A1 (6)
		[10]

Question 8: Jan 11 Q6

Question Number	Scheme	Marks
(a)	$\frac{dx}{dt} = \frac{1}{t},\quad \frac{dy}{dt} = 2t$	
	$\frac{dy}{dx} = 2t^2$	M1 A1
	Using $mm' = -1$, at $t = 3$	
	$m' = -\frac{1}{18}$	M1 A1
	$y - 7 = -\frac{1}{18}(x - \ln 3)$	M1 A1 (6)
(b)	$x = \ln t \ \Rightarrow \ t = e^x$	B1
	$y = e^{2x} - 2$	M1 A1 (3)

Question 9: June 11 Q7

Question Number	Scheme		Marks	
	(a) $\tan\theta = \sqrt{3}$ *or* $\sin\theta = \frac{\sqrt{3}}{2}$		M1	
	$\theta = \frac{\pi}{3}$	awrt 1.05	A1	(2)
	(b) $\frac{dx}{d\theta} = \sec^2\theta,\ \frac{dy}{d\theta} = \cos\theta$			
	$\frac{dy}{dx} = \frac{\cos\theta}{\sec^2\theta}\ \left(= \cos^3\theta\right)$		M1 A1	
	At P, $m = \cos^3\left(\frac{\pi}{3}\right) = \frac{1}{8}$	Can be implied	A1	
	Using $mm' = -1$, $m' = -8$		M1	
	For normal $y - \frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$		M1	
	At Q, $y = 0$ $-\frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$			
	leading to $x = \frac{17}{16}\sqrt{3}$ $\left(k = \frac{17}{16}\right)$	1.0625	A1	(6)