## Parametric Differentiation - Edexcel Past Exam Questions

1. A curve has parametric equations

$$
x=2 \cot t, \quad y=2 \sin ^{2} t, \quad 0<t \leq \frac{\pi}{2} .
$$

(a) Find an expression for $\frac{\mathrm{d} y}{\mathrm{~d} x}$ in terms of the parameter $t$.
(b) Find an equation of the tangent to the curve at the point where $t=\frac{\pi}{4}$.
(c) Find a cartesian equation of the curve in the form $y=\mathrm{f}(x)$. State the domain on which the curve is defined.
2.


Figure 2

The curve shown in Figure 2 has parametric equations

$$
x=\sin t, \quad y=\sin \left(t+\frac{\pi}{6}\right), \quad-\frac{\pi}{2}<t<\frac{\pi}{2} .
$$

(a) Find an equation of the tangent to the curve at the point where $t=\frac{\pi}{6}$.
(b) Show that a cartesian equation of the curve is

$$
\begin{equation*}
y=\frac{\sqrt{ } 3}{2} x+\frac{1}{2} \sqrt{ }\left(1-x^{2}\right), \quad-1<x<1 . \tag{3}
\end{equation*}
$$

3. A curve has parametric equations

$$
x=7 \cos t-\cos 7 t, \quad y=7 \sin t-\sin 7 t, \quad \frac{\pi}{8}<t<\frac{\pi}{3} .
$$

(a) Find an expression for $\frac{\mathrm{d} y}{\mathrm{~d} x}$ in terms of $t$. You need not simplify your answer.
(b) Find an equation of the normal to the curve at the point where $t=\frac{\pi}{6}$.

Give your answer in its simplest exact form.
Jan 07 Q3
4. A curve has parametric equations

$$
x=\tan ^{2} t, \quad y=\sin t, \quad 0<t<\frac{\pi}{2}
$$

(a) Find an expression for $\frac{\mathrm{d} y}{\mathrm{~d} x}$ in terms of $t$. You need not simplify your answer.
(b) Find an equation of the tangent to the curve at the point where $t=\frac{\pi}{4}$.

Give your answer in the form $y=a x+b$, where $a$ and $b$ are constants to be determined.
(c) Find a cartesian equation of the curve in the form $y^{2}=\mathrm{f}(x)$.
5.


Figure 3
The curve $C$ shown in Figure 3 has parametric equations

$$
x=t^{3}-8 t, \quad y=t^{2}
$$

where $t$ is a parameter. Given that the point $A$ has parameter $t=-1$,
(a) find the coordinates of $A$.

The line $l$ is the tangent to $C$ at $A$.
(b) Show that an equation for $l$ is $2 x-5 y-9=0$.

The line $l$ also intersects the curve at the point $B$.
(c) Find the coordinates of $B$.
6.


Figure 2 shows a sketch of the curve with parametric equations

$$
x=2 \cos 2 t, \quad y=6 \sin t, \quad 0 \leq t \leq \frac{\pi}{2} .
$$

(a) Find the gradient of the curve at the point where $t=\frac{\pi}{3}$.
(b) Find a cartesian equation of the curve in the form

$$
y=\mathrm{f}(x), \quad-k \leq x \leq k,
$$

stating the value of the constant $k$.
(c) Write down the range of $\mathrm{f}(x)$.

June 09 Q5
7. A curve $C$ has parametric equations

$$
x=\sin ^{2} t, y=2 \tan t, 0 \leq t<\frac{\pi}{2} .
$$

(a) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$ in terms of $t$.

The tangent to $C$ at the point where $t=\frac{\pi}{3}$ cuts the $x$-axis at the point $P$.
(b) Find the $x$-coordinate of $P$.
8. The curve $C$ has parametric equations

$$
x=\ln t, \quad y=t^{2}-2, \quad t>0 .
$$

Find
(a) an equation of the normal to $C$ at the point where $t=3$,
(b) a cartesian equation of $C$.
9.


Figure 3
Figure 3 shows part of the curve $C$ with parametric equations

$$
x=\tan \theta, \quad y=\sin \theta, \quad 0 \leq \theta<\frac{\pi}{2} .
$$

The point $P$ lies on $C$ and has coordinates $\left(\sqrt{ } 3, \frac{1}{2} \sqrt{ } 3\right)$.
(a) Find the value of $\theta$ at the point $P$.

The line $l$ is a normal to $C$ at $P$. The normal cuts the $x$-axis at the point $Q$.
(b) Show that $Q$ has coordinates ( $k \sqrt{ } 3,0$ ), giving the value of the constant $k$.

