Parametric Equations - Edexcel Past Exam Questions MARK SCHEME

Question 1: June 05 Q6

	Uses $1+\cot^2 t = \text{cosec}^2 t$, or equivalent, to eliminate t	M1	
	$1+\left(\frac{x}{2}\right)^2 = \frac{2}{y}$ correctly eliminates t	A1	
	$y = \frac{8}{4+x^2}$ cao	A1	
	The domain is $x \ldots 0$	B1	(4) [12]
	An alternative in (c)		
	$\sin t = \left(\frac{y}{2}\right)^{\frac{1}{2}}$; $\cos t = \frac{x}{2}\sin t = \frac{x}{2}\left(\frac{y}{2}\right)^{\frac{1}{2}}$		
	$\sin^2 t + \cos^2 t = 1 \Rightarrow \frac{y}{2} + \frac{x^2}{4} \times \frac{y}{2} = 1$	M1 A1	
	Leading to $y = \frac{8}{4+x^2}$	A1	

Question 2: Jan 06 Q8

	Solves $y = 0 \Rightarrow \cos t = \frac{1}{2}$ to obtain $t = \frac{\pi}{3}$ *or* $\frac{5\pi}{3}$ (need both for A1) Or substitutes **both** values of t and shows that $y = 0$	M1 A1	(2)

Question 3: June 06 Q4

	$y = \sin(t + \frac{\pi}{6}) = \sin t \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6}$	Use of compound angle formula for sine.	M1
	Nb: $\sin^2 t + \cos^2 t \equiv 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t$		
	$\therefore x = \sin t$ gives $\cos t = \sqrt{(1-x^2)}$	Use of trig identity to find $\cos t$ in terms of x or $\cos^2 t$ in terms of x.	M1
	$\therefore y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t$		
	gives $y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1-x^2)}$ **AG**	Substitutes for $\sin t$, $\cos\frac{\pi}{6}$, $\cos t$ and $\sin\frac{\pi}{6}$ to give y in terms of x.	A1 cso [3]

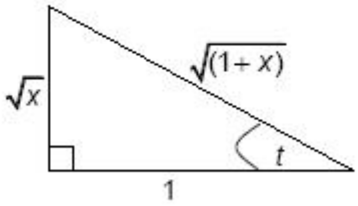
Question 4: June 07 Q6

Way 1	$x = \tan^2 t = \dfrac{\sin^2 t}{\cos^2 t}$ $\quad y = \sin t$		
	$x = \dfrac{\sin^2 t}{1-\sin^2 t}$	Uses $\cos^2 t = 1-\sin^2 t$	M1
	$x = \dfrac{y^2}{1-y^2}$	Eliminates 't' to write an equation involving x and y.	M1
	$x(1-y^2) = y^2 \Rightarrow x - xy^2 = y^2$		
	$x = y^2 + xy^2 \Rightarrow x = y^2(1+x)$	Rearranging and factorising with an attempt to make y^2 the subject.	ddM1
	$y^2 = \dfrac{x}{1+x}$	$\dfrac{x}{1+x}$	A1
			[4]
Aliter **Way 2**	$1+\cot^2 t = \operatorname{cosec}^2 t$	Uses $1+\cot^2 t = \operatorname{cosec}^2 t$	M1
	$= \dfrac{1}{\sin^2 t}$	Uses $\operatorname{cosec}^2 t = \dfrac{1}{\sin^2 t}$	M1 implied
	Hence, $1+\dfrac{1}{x} = \dfrac{1}{y^2}$	Eliminates 't' to write an equation involving x and y.	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	$1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	A1
			[4]

$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

Aliter **Way 3**	$x = \tan^2 t \qquad y = \sin t$		
	$1 + \tan^2 t = \sec^2 t$	Uses $1 + \tan^2 t = \sec^2 t$	M1
	$= \dfrac{1}{\cos^2 t}$	Uses $\sec^2 t = \dfrac{1}{\cos^2 t}$	M1
	$= \dfrac{1}{1 - \sin^2 t}$		
	Hence, $1 + x = \dfrac{1}{1 - y^2}$	Eliminates 't' to write an equation involving x and y.	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	$1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	A1
			[4]
Aliter **Way 4**	$y^2 = \sin^2 t = 1 - \cos^2 t$	Uses $\sin^2 t = 1 - \cos^2 t$	M1
	$= 1 - \dfrac{1}{\sec^2 t}$	Uses $\cos^2 t = \dfrac{1}{\sec^2 t}$	M1
	$= 1 - \dfrac{1}{(1 + \tan^2 t)}$	then uses $\sec^2 t = 1 + \tan^2 t$	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	$1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	A1
			[4]

$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

Aliter **Way 5**	$x = \tan^2 t \qquad y = \sin t$ $x = \tan^2 t \Rightarrow \tan t = \sqrt{x}$ 	Draws a right-angled triangle and places both $\sqrt{x}$ and 1 on the triangle Uses Pythagoras to deduce the hypotenuse	M1 M1
	Hence, $y = \sin t = \dfrac{\sqrt{x}}{\sqrt{1+x}}$	Eliminates 't' to write an equation involving x and y.	ddM1
	Hence, $y^2 = \dfrac{x}{1+x}$	$\dfrac{x}{1+x}$	A1 **[4]**
			12 marks

$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

There are so many ways that a candidate can proceed with part (c). If a candidate produces a correct solution then please award all four marks. If they use a method commensurate with the five ways as detailed on the mark scheme then award the marks appropriately. If you are unsure of how to apply the scheme please escalate your response up to your team leader.

Question 5: Jan 08 Q7

Question Number	Scheme		Marks
	$x = \ln(t+2), \quad y = \frac{1}{t+1}$		
(a)	$e^x = t+2 \Rightarrow t = e^x - 2$	Attempt to make $t = \ldots$ the subject giving $t = e^x - 2$	M1 A1
	$y = \frac{1}{e^x - 2 + 1} \Rightarrow y = \frac{1}{e^x - 1}$	Eliminates t by substituting in y giving $y = \frac{1}{e^x - 1}$	dM1 A1 [4]
Aliter **Way 2**	$t+1 = \frac{1}{y} \Rightarrow t = \frac{1}{y} - 1$ or $t = \frac{1-y}{y}$ $y(t+1) = 1 \Rightarrow yt + y = 1 \Rightarrow yt = 1 - y \Rightarrow t = \frac{1-y}{y}$	Attempt to make $t = \ldots$ the subject Giving either $t = \frac{1}{y} - 1$ or $t = \frac{1-y}{y}$	M1 A1
	$x = \ln\left(\frac{1}{y} - 1 + 2\right)$ or $x = \ln\left(\frac{1-y}{y} + 2\right)$	Eliminates t by substituting in x	dM1
	$x = \ln\left(\frac{1}{y} + 1\right)$		
	$e^x = \frac{1}{y} + 1$		
	$e^x - 1 = \frac{1}{y}$		
	$y = \frac{1}{e^x - 1}$	giving $y = \frac{1}{e^x - 1}$	A1 [4]
(b)	Domain : $\underline{x > 0}$	$\underline{x > 0}$ or just > 0	B1 [1]
			15 marks

Question Number	Scheme		Marks
Aliter **Way 3**	$e^x = t+2 \Rightarrow t+1 = e^x - 1$	Attempt to make $t+1 = \dots$ the subject giving $t+1 = e^x - 1$	M1 A1
	$y = \frac{1}{t+1} \Rightarrow y = \frac{1}{e^x - 1}$	Eliminates t by substituting in y giving $y = \frac{1}{e^x - 1}$	dM1 A1 **[4]**
Aliter **Way 4**	$t+1 = \frac{1}{y} \Rightarrow t+2 = \frac{1}{y}+1$ or $t+2 = \frac{1+y}{y}$	Attempt to make $t+2 = \dots$ the subject Either $t+2 = \frac{1}{y}+1$ or $t+2 = \frac{1+y}{y}$	M1 A1
	$x = \ln\left(\frac{1}{y}+1\right)$ or $x = \ln\left(\frac{1+y}{y}\right)$	Eliminates t by substituting in x	dM1
	$x = \ln\left(\frac{1}{y}+1\right)$		
	$e^x = \frac{1}{y}+1 \Rightarrow e^x - 1 = \frac{1}{y}$		
	$y = \frac{1}{e^x - 1}$	giving $y = \frac{1}{e^x - 1}$	A1 **[4]**

Question 6: Jan 09 Q7

Question Number	Scheme		Marks
(a)	At A, $x=-1+8=7$ & $y=(-1)^2=1 \Rightarrow A(7,1)$	$A(7,1)$	B1 (1)
(b)	$2(t^3-8t)-5t^2-9=0$	Substitution of both $x=t^3-8t$ and $y=t^2$ into I	M1
	$2t^3-5t^2-16t-9=0$		
	$(t+1)\{(2t^2-7t-9)=0\}$ $(t+1)\{(t+1)(2t-9)=0\}$	A realisation that $(t+1)$ is a factor.	dM1
	$\{t=-1 \text{ (at } A)\}$ $t=\frac{9}{2}$ at B	$t=\frac{9}{2}$	A1
	$x=\left(\frac{9}{2}\right)^3-8\left(\frac{9}{2}\right)=\frac{729}{8}-36=\frac{441}{8}$ = 55.125 or awrt 55.1	Candidate uses their value of t to find either the x or y coordinate	ddM1
	$y=\left(\frac{9}{2}\right)^2=\frac{81}{4}=20.25$ or awrt 20.3	One of either x or y correct.	A1
		Both x and y correct.	A1
	Hence $B\left(\frac{441}{8},\frac{81}{4}\right)$	awrt	(6)
			[12]

Question 7: June 09 Q5

Question Number	Scheme		Marks
(a)	Use of $\cos 2t = 1-2\sin^2 t$		M1
	$\cos 2t = \frac{x}{2}$, $\sin t = \frac{y}{6}$		
	$\frac{x}{2}=1-2\left(\frac{y}{6}\right)^2$		M1
	Leading to $y=\sqrt{(18-9x)}$ $\left(=3\sqrt{(2-x)}\right)$	cao	A1
	$-2\leq x\leq 2$	$k=2$	B1 (4)
(b)	$0\leq \mathrm{f}(x)\leq 6$	either $0\leq \mathrm{f}(x)$ or $\mathrm{f}(x)\leq 6$	B1
		Fully correct. Accept $0\leq y\leq 6$, $[0,6]$	B1 (2)
			[10]

Question 8: Jan 10 Q7

Question Number	Scheme		Marks	
	$y = 0 \Rightarrow t(9 - t^2) = t(3 - t)(3 + t) = 0$			
	$t = 0, 3, -3$	Any one correct value	B1	
	At $t = 0$, $x = 5(0)^2 - 4 = -4$	Method for finding one value of x	M1	
	At $t = 3$, $x = 5(3)^2 - 4 = 41$			
	$\left(\text{At } t = -3,\ x = 5(-3)^2 - 4 = 41\right)$			
	At A, $x = -4$; at B, $x = 41$	Both	A1	(3)

Question 9: Jan 11 Q6

Question Number	Scheme	Marks	
	$x = \ln t \Rightarrow t = e^x$	B1	
	$y = e^{2x} - 2$	M1 A1	(3)

Question 10: June 11 Q7

Question Number	Scheme		Marks	
	(a) $\tan\theta = \sqrt{3}$ *or* $\sin\theta = \frac{\sqrt{3}}{2}$		M1	
	$\theta = \frac{\pi}{3}$	awrt 1.05	A1	(2)