

Parametric Equations - Edexcel Past Exam Questions MARK SCHEME

Question 1: June 05 Q6

Uses $1 + \cot^2 t = \csc^2 t$, or equivalent, to eliminate t	M1	
$1 + \left(\frac{x}{2}\right)^2 = \frac{2}{y}$ correctly eliminates t	A1	
$y = \frac{8}{4 + x^2}$ cao	A1	
The domain is $x \dots 0$	В1	(4) [12]
An alternative in (c)		
$\sin t = \left(\frac{y}{2}\right)^{\frac{1}{2}}; \cos t = \frac{x}{2}\sin t = \frac{x}{2}\left(\frac{y}{2}\right)^{\frac{1}{2}}$		
$\sin^2 t + \cos^2 t = 1 \implies \frac{y}{2} + \frac{x^2}{4} \times \frac{y}{2} = 1$	M1 A1	
Leading to $y = \frac{8}{4 + x^2}$	A1	

Question 2: Jan 06 Q8

Solves $y = 0 \implies \cos t = \frac{1}{2}$ to obtain $t = \frac{\pi}{3}$ or $\frac{5\pi}{3}$ (need both for A1)

Or substitutes **both** values of t and shows that y = 0 (2)

Question 3: June 06 Q4

$y = \sin(t + \frac{\pi}{6}) = \sin t \cos \frac{\pi}{6} + \cos t \sin \frac{\pi}{6}$	Use of compound angle formula for sine.	M1
Nb: $\sin^2 t + \cos^2 t \equiv 1 \implies \cos^2 t \equiv 1 - \sin^2 t$		
$\therefore x = \sin t \text{ gives } \cos t = \sqrt{(1-x^2)}$	Use of trig identity to find $\cos t$ in terms of x or $\cos^2 t$ in terms of x.	M1
$\therefore y = \frac{\sqrt{3}}{2} \sin t + \frac{1}{2} \cos t$		
gives $y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1-x^2)}$ AG	Substitutes for sint, $\cos \frac{\pi}{6}$, cost and $\sin \frac{\pi}{6}$ to give y in terms of x.	A1 cso
	3,	[3]



Question 4: June 07 Q6

$x = \frac{\sin^2 t}{1 - \sin^2 t}$ Uses $\cos^2 t = 1 - \sin^2 t$ $x = \frac{y^2}{1 - y^2}$ Eliminates 't' to write an equation involving x and y. $x(1 - y^2) = y^2 \Rightarrow x - xy^2 = y^2$ $x = y^2 + xy^2 \Rightarrow x = y^2(1 + x)$ Rearranging and factorising with an attempt to make y^2 the subject. $y^2 = \frac{x}{1 + x}$ $1 + \cot^2 t = \csc^2 t$ Uses $1 + \cot^2 t = \csc^2 t$ Hence, $1 + \frac{1}{x} = \frac{1}{y^2}$ Uses $1 + \cot^2 t = \cot^2 t$ U		$x = \tan^2 t = \frac{\sin^2 t}{\cos^2 t} \qquad y = \sin t$		
$x(1-y^2) = y^2 \implies x - xy^2 = y^2$ $x = y^2 + xy^2 \implies x = y^2(1+x)$ Rearranging and factorising with an attempt to make y^2 the subject. $y^2 = \frac{x}{1+x}$ A1 $1 + \cot^2 t = \csc^2 t$ Uses $1 + \cot^2 t = \csc^2 t$ Hence, $1 + \frac{1}{x} = \frac{1}{y^2}$ Uses $\cot^2 t = \cot^2 t$ Hence, $1 + \frac{1}{x} = \frac{1}{y^2}$ Eliminates 't' to write an equation involving x and y. Hence, $y^2 = 1 - \frac{1}{(1+x)}$ or $\frac{x}{1+x}$ $1 - \frac{1}{(1+x)}$ or $\frac{x}{1+x}$ A1	Way 1	$X = \frac{\sin^2 t}{1 - \sin^2 t}$	$Uses \cos^2 t = 1 - \sin^2 t$	M1
Rearranging and factorising with an attempt to make y^2 the subject. Aliter 1 + $\cot^2 t = \csc^2 t$ Way 2 $= \frac{1}{\sin^2 t}$ Uses $1 + \cot^2 t = \csc^2 t$ Uses $1 + \cot^2 t = \csc^2 t$ Uses $1 + \cot^2 t = \csc^2 t$ Hence, $1 + \frac{1}{x} = \frac{1}{y^2}$ Eliminates 't' to write an equation involving x and y . Hence, $y^2 = 1 - \frac{1}{(1+x)}$ or $\frac{x}{1+x}$ $1 - \frac{1}{(1+x)}$ or $\frac{x}{1+x}$ A1 A1		$X = \frac{y^2}{1 - y^2}$	2 ^ 전 10 전 2 전 10 전 10 전 10 전 2 전 2 전 2 전 2 전 2 전 2 전 2 전 2 전 2 전	M1
Aliter Way 2 $x = y^2 + xy^2 \implies x = y^2(1+x)$ an attempt to make y^2 the subject. $y^2 = \frac{x}{1+x}$ $1 + \cot^2 t = \csc^2 t$ $= \frac{1}{\sin^2 t}$ Uses $1 + \cot^2 t = \cot^2 t = \cot^2 t$ Uses $1 + \cot^2 t = \cot^2 t$ $= \frac{1}{\sin^2 t}$ Uses $1 + \cot^2 t = \cot^2 t$ Uses $1 + \cot^2 t = \cot^2 t$ $= \frac{1}{\sin^2 t}$ Hence, $1 + \frac{1}{x} = \frac{1}{y^2}$ Eliminates 't' to write an equation involving x and y. Hence, $y^2 = 1 - \frac{1}{(1+x)}$ or $\frac{x}{1+x}$ $1 - \frac{1}{(1+x)}$ or $\frac{x}{1+x}$ A1		$X(1-y^2) = y^2 \implies X - Xy^2 = y^2$		
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127 (1.1.1.1.1.1.1.1.1.1.1.1.1.1.1.1.1.1.1.		Hence, $1 + \frac{1}{x} = \frac{1}{y^2}$	0.0 DECEMBER 100	ddM1
[4]		Hence, $y^2 = 1 - \frac{1}{(1+x)}$ or $\frac{x}{1+x}$	$1 - \frac{1}{(1+x)} \text{or} \frac{x}{1+x}$	55.500.000.000
			¥11	[4]

 $\frac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.



Parametric Equations

Hence, $y^2 = 1 - \frac{1}{(1+x)}$ or $\frac{x}{1+x}$ $1 - \frac{1}{(1+x)}$ or $\frac{x}{1+x}$ A1 Aliter Way 4 $y^2 = \sin^2 t = 1 - \cos^2 t$ Uses $\sin^2 t = 1 - \cos^2 t$ M1 $= 1 - \frac{1}{\sec^2 t}$ Uses $\cos^2 t = \frac{1}{\sec^2 t}$ M1 $= 1 - \frac{1}{(1+\tan^2 t)}$ then uses $\sec^2 t = 1 + \tan^2 t$ ddM	W. Y.		·
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(17 sair 17	way 4	Vo.	Uses $\cos^2 t = \frac{1}{\sec^2 t}$ M1
1 0		$= 1 - \frac{1}{(1 + \tan^2 t)}$	then uses $\sec^2 t = 1 + \tan^2 t$ ddM1
Attaste (0.000)		Hence, $y^2 = 1 - \frac{1}{(1+x)}$ or $\frac{x}{1+x}$	$1 - \frac{1}{(1+x)}$ or $\frac{x}{1+x}$ A1
			[4]

 $\frac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

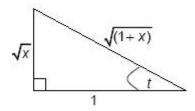
Parametric Equations

Aliter

Way 5

$$x = \tan^2 t$$
 $y = \sin t$

$$x = \tan^2 t \implies \tan t = \sqrt{x}$$



Hence,
$$y = \sin t = \frac{\sqrt{x}}{\sqrt{1+x}}$$

Hence,
$$y^2 = \frac{x}{1+x}$$

Draws a right-angled triangle and places both \sqrt{x} and 1 on the triangle

Uses Pythagoras to deduce the hypotenuse M1

Eliminates 't' to write an equation involving x and y.

ind y. ddM1

 $\frac{x}{+x}$ A1

[4]

12 marks

 $\frac{1}{1+\frac{1}{2}}$ is an acceptable response for the final accuracy A1 mark.

There are so many ways that a candidate can proceed with part (c). If a candidate produces a correct solution then please award all four marks. If they use a method commensurate with the five ways as detailed on the mark scheme then award the marks appropriately. If you are unsure of how to apply the scheme please escalate your response up to your team leader.



Question 5: Jan 08 Q7

Question Number	Scheme		Marks
	$x = \ln(t+2), \qquad y = \frac{1}{t+1}$		
(a)	$e^x = t + 2 \implies t = e^x - 2$	Attempt to make $t =$ the subject giving $t = e^x - 2$	M1 A1
	$y = \frac{1}{e^x - 2 + 1} \implies y = \frac{1}{e^x - 1}$	Eliminates t by substituting in y giving $y = \frac{1}{e^x - 1}$	dM1 A1 [4]
Aliter	$t+1=\frac{1}{y} \implies t=\frac{1}{y}-1 \text{ or } t=\frac{1-y}{y}$	Attempt to make $t =$ the subject	M1
Way 2	$y(t+1) = 1 \implies yt + y = 1 \implies yt = 1 - y \implies t = \frac{1-y}{y}$	Giving either $t = \frac{1}{y} - 1$ or $t = \frac{1 - y}{y}$	A1
	$x = \ln\left(\frac{1}{y} - 1 + 2\right)$ or $x = \ln\left(\frac{1 - y}{y} + 2\right)$	Eliminates t by substituting in x	dM1
	$x = \ln\left(\frac{1}{y} + 1\right)$		
	$e^x = \frac{1}{y} + 1$		
	$e^x - 1 = \frac{1}{y}$		
	$y = \frac{1}{e^x - 1}$	giving $y = \frac{1}{e^x - 1}$	A1
(b)	Domain: $x > 0$	$\underline{x > 0}$ or just > 0	[4] B1
			[1] 15 marks



Parametric Equations

Question Number	Scheme		Mar	ks
Aliter Way 3	$e^x = t + 2 \implies t + 1 = e^x - 1$	Attempt to make $t+1 =$ the subject giving $t+1 = e^x -1$	M1 A1	
	$y = \frac{1}{t+1} \implies y = \frac{1}{e^x - 1}$	Eliminates <i>t</i> by substituting in <i>y</i> giving $y = \frac{1}{e^x - 1}$	dM1 A1	[4]
Aliter Way 4	$t+1=\frac{1}{y} \implies t+2=\frac{1}{y}+1 \text{ or } t+2=\frac{1+y}{y}$	Attempt to make $t+2=$ the subject Either $t+2=\frac{1}{y}+1$ or $t+2=\frac{1+y}{y}$	M1 A1	
	$x = \ln\left(\frac{1}{y} + 1\right)$ or $x = \ln\left(\frac{1+y}{y}\right)$	Eliminates t by substituting in x	dM1	
	$x = \ln\left(\frac{1}{y} + 1\right)$			
	$e^x = \frac{1}{y} + 1 \implies e^x - 1 = \frac{1}{y}$			
	$y = \frac{1}{e^x - 1}$	giving $y = \frac{1}{e^x - 1}$	A1	[4]



Question 6: Jan 09 Q7

Question Number	Scheme		Marks
(a)	At A , $x = -1 + 8 = 7$ & $y = (-1)^2 = 1 \Rightarrow A(7,1)$	A(7,1)	B1 (1)
(b)	$2(t^3 - 8t) - 5t^2 - 9 = 0$	Substitution of both $x = t^3 - 8t$ and $y = t^2$ into T	M1
	$2t^3 - 5t^2 - 16t - 9 = 0$		
	$(t+1)\left\{(2t^2-7t-9)=0\right\}$	A realisation that	-m-4
	$(t+1)\{(t+1)(2t-9)=0\}$	(t+1) is a factor.	dM1
	$\left\{t = -1 \ (\text{at } A)\right\} \ t = \frac{9}{2} \ \text{at } B$	$t = \frac{9}{2}$	A1
	$x = \left(\frac{9}{2}\right)^2 - 8\left(\frac{9}{2}\right) = \frac{739}{8} - 36 = \frac{441}{8} = 55.125 \text{ or awrt } 55.1$	Candidate uses their value of t to find either the x or y coordinate	ddM1
	$y = \left(\frac{9}{2}\right)^2 = \frac{81}{4} = 20.25$ or awrt 20.3	One of either x or y correct. Both x and y correct.	A1 A1
	Hence $B(\frac{441}{8}, \frac{81}{4})$	awrt	(6)
			[12]

Question 7: June 09 Q5

Question Number		Sch	eme		Ма	rks
(a)	Use of	$\cos 2t = 1 - 2\sin t$ $\cos 2t = \frac{x}{2}, \sin t = \frac{y}{6}$	A		М1	
		$\frac{x}{2} = 1 - 2\left(\frac{y}{6}\right)$)2		M1	
	Leading to	$y = \sqrt{(18-9)}$	$(x) = 3\sqrt{2-x}$	cao	A1	
		$-2 \le x \le 2$		k = 2	В1	(4)
(b)		$0 \le f(x) \le 6$	either $0 \le f(x)$ or	$f(x) \le 6$	В1	
		Fully o	correct. Accept $0 \le y \le$	6, [0, 6]	В1	(2)
						[10]



Question 8: Jan 10 Q7

Question Number	Scheme			(S
	$y = 0 \implies t(9-t^2) = t(3-t)(3-t)(3-t) = 0, 3, -3$ At $t = 0$, $x = 5(0)^2 - 4 = -4$ At $t = 3$, $x = 5(3)^2 - 4 = 41$ $(At t = -3, x = 5(-3)^2 - 4 = 41)$	(3+t)=0 Any one correct value Method for finding one value of x	B1 M1	
	At A, $x = -4$; at B, $x = 41$	Both	A1	(3)

Question 9: Jan 11 Q6

Question Number	Scheme	Marks
	$x = \ln t \implies t = e^x$ $y = e^{2x} - 2$	B1 M1 A1 (3)

Question 10: June 11 Q7

Question Number	Scheme			Ma	Marks	
	(a) to	$an \theta = \sqrt{3}$ or $sin \theta = \frac{\sqrt{3}}{2}$		M1		
		$\theta = \frac{\pi}{3}$	awrt 1.05	A1	(2)	