Trigonometry (Addition, Double Angle & R Formulae) - Edexcel Past Exam Questions

1. (*a*) Using the identity $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that

$$\cos 2A \equiv 1 - 2\sin^2 A. \qquad \textbf{(2)}$$

(*b*) Show that

$$2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv \sin\theta(4\cos\theta + 6\sin\theta - 3). \qquad \textbf{(4)}$$

(*c*) Express $4\cos\theta + 6\sin\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

(4)

(*d*) Hence, for $0 \le \theta < \pi$, solve

$$2\sin 2\theta = 3(\cos 2\theta + \sin\theta - 1),$$

giving your answers in radians to 3 significant figures, where appropriate. **(5)**

June 05 Q5

2.

$$f(x) = 12\cos x - 4\sin x.$$

Given that $f(x) = R\cos(x + \alpha)$, where $R \ge 0$ and $0 \le \alpha \le 90°$,

(*a*) find the value of R and the value of α. **(4)**

(*b*) Hence solve the equation

$$12\cos x - 4\sin x = 7$$

for $0 \le x < 360°$, giving your answers to one decimal place. **(5)**

(*c*) (i) Write down the minimum value of $12\cos x - 4\sin x$. **(1)**

(ii) Find, to 2 decimal places, the smallest positive value of x for which this minimum value occurs. **(2)**

Jan 06 Q6

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3. (*a*) Show that

(i) $\dfrac{\cos 2x}{\cos x + \sin x} \equiv \cos x - \sin x, \quad x \neq (n - \tfrac{1}{4})\pi,\ n \in \mathbb{Z},$ **(2)**

(ii) $\tfrac{1}{2}(\cos 2x - \sin 2x) \equiv \cos^2 x - \cos x \sin x - \tfrac{1}{2}.$ **(3)**

(*b*) Hence, or otherwise, show that the equation

$$\cos\theta\left(\frac{\cos 2\theta}{\cos\theta + \sin\theta}\right) = \frac{1}{2}$$

can be written as

$$\sin 2\theta = \cos 2\theta.$$ **(3)**

(*c*) Solve, for $0 \leq \theta < 2\pi$,

$$\sin 2\theta = \cos 2\theta,$$

giving your answers in terms of π. **(4)**

Jan 06 Q7

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4. (*a*) Given that $\cos A = \tfrac{3}{4}$, where $270° < A < 360°$, find the exact value of $\sin 2A$.

(5)

(*b*) Show that $\cos\left(2x + \dfrac{\pi}{3}\right) + \cos\left(2x - \dfrac{\pi}{3}\right) \equiv \cos 2x.$ **(3)**

June 06 Q8*(edited)*

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5. (*a*) By writing $\sin 3\theta$ as $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$ **(5)**

(*b*) Given that $\sin\theta = \dfrac{\sqrt{3}}{4}$, find the exact value of $\sin 3\theta$. **(2)**

Jan 07 Q1

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6. **Figure 1**

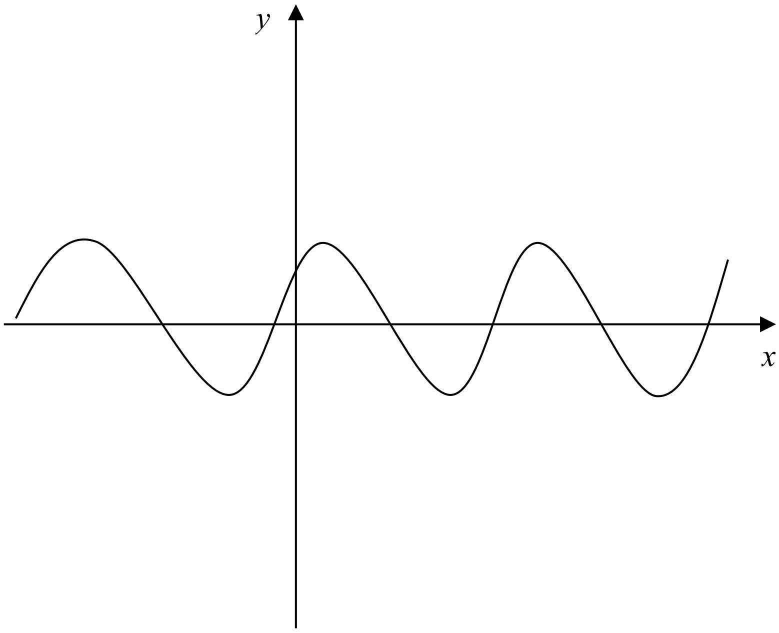

Figure 1 shows an oscilloscope screen.

The curve on the screen satisfies the equation $y = \sqrt{3} \cos x + \sin x$.

(a) Express the equation of the curve in the form $y = R \sin (x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **(4)**

(b) Find the values of x, $0 \leq x < 2\pi$, for which $y = 1$. **(4)**

Jan 07 Q5

7. (a) Express $3 \sin x + 2 \cos x$ in the form $R \sin(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.
(4)

(b) Hence find the greatest value of $(3 \sin x + 2 \cos x)^4$. **(2)**

(c) Solve, for $0 < x < 2\pi$, the equation

$$3 \sin x + 2 \cos x = 1,$$

giving your answers to 3 decimal places. **(5)**

June 07 Q6

8. (*a*) Prove that

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2\text{ cosec }2\theta, \quad \theta \neq 90n°.$$ **(4)**

(*b*) Sketch the graph of $y = 2\text{ cosec }2\theta$ for $0° < \theta < 360°$.
(2)

(*c*) Solve, for $0° < \theta < 360°$, the equation

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 3$$

giving your answers to 1 decimal place. **(6)**

June 07 Q7

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9. (*a*) Use the double angle formulae and the identity

$$\cos(A + B) \equiv \cos A\cos B - \sin A\sin B$$

to obtain an expression for $\cos 3x$ in terms of powers of $\cos x$ only. **(4)**

(*b*) (i) Prove that

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} \equiv 2\sec x, \qquad x \neq (2n+1)\frac{\pi}{2}.$$ **(4)**

(ii) Hence find, for $0 < x < 2\pi$, all the solutions of

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 4.$$ **(3)**

Jan 08 Q6

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10. $$\mathrm{f}(x) = 5\cos x + 12\sin x.$$

Given that $\mathrm{f}(x) = R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$,

(*a*) find the value of R and the value of α to 3 decimal places. **(4)**

(*b*) Hence solve the equation

$$5\cos x + 12\sin x = 6$$

for $0 \le x < 2\pi$. **(5)**

(*c*) (i) Write down the maximum value of $5\cos x + 12\sin x$. **(1)**

(ii) Find the smallest positive value of x for which this maximum value occurs. **(2)**

June 08 Q2

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11. (*a*) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$ **(4)**

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8\sin^3\theta - 6\sin\theta + 1 = 0.$$

Give your answers in terms of π. **(5)**

(*b*) Using $\sin(\theta - \alpha) = \sin\theta\cos\alpha - \cos\theta\sin\alpha$, or otherwise, show that

$$\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2}).$$ **(4)**

Jan 09 Q6

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12. (*a*) Use the identity $\cos(A+B) = \cos A \cos B - \sin A \sin B$, to show that

$$\cos 2A = 1 - 2\sin^2 A$$ **(2)**

The curves C_1 and C_2 have equations

$$C_1: y = 3\sin 2x$$

$$C_2: y = 4\sin^2 x - 2\cos 2x$$

(*b*) Show that the x-coordinates of the points where C_1 and C_2 intersect satisfy the equation

$$4\cos 2x + 3\sin 2x = 2$$ **(3)**

(*c*) Express $4\cos 2x + 3\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places. **(3)**

(*d*) Hence find, for $0 \le x < 180°$, all the solutions of

$$4\cos 2x + 3\sin 2x = 2,$$

giving your answers to 1 decimal place. **(4)**

June 09 Q6

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13. (*a*) Express $5\cos x - 3\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. **(4)**

(*b*) Hence, or otherwise, solve the equation

$$5\cos x - 3\sin x = 4$$

for $0 \le x < 2\pi$, giving your answers to 2 decimal places. **(5)**

Jan 10 Q3

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14. Solve

$$\operatorname{cosec}^2 2x - \cot 2x = 1$$

for $0 \le x \le 180°$. **(7)**

Jan 10 Q8

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15. (*a*) Show that

$$\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta. \qquad \textbf{(2)}$$

(*b*) Hence find, for $-180° \le \theta < 180°$, all the solutions of

$$\frac{2\sin 2\theta}{1+\cos 2\theta} = 1.$$

Give your answers to 1 decimal place. **(3)**

June 10 Q1

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16. (*a*) Express $7\cos x - 24\sin x$ in the form $R\cos(x+\alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places. **(3)**

(*b*) Hence write down the minimum value of $7\cos x - 24\sin x$. **(1)**

(*c*) Solve, for $0 \le x < 2\pi$, the equation

$$7\cos x - 24\sin x = 10,$$

giving your answers to 2 decimal places. **(5)**

Jan 11 Q1

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17. Find all the solutions of

$$2\cos 2\theta = 1 - 2\sin\theta$$

in the interval $0 \le \theta < 360°$. **(6)**

Jan 11 Q3

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18. (*a*) Prove that

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \tan\theta, \qquad \theta \ne 90n°, \quad n \in \mathbb{Z}. \qquad \textbf{(4)}$$

(*b*) Hence, or otherwise,

(i) show that $\tan 15° = 2 - \sqrt{3}$, **(3)**

(ii) solve, for $0 < x < 360°$,

$$\operatorname{cosec} 4x - \cot 4x = 1. \qquad \textbf{(5)}$$

June 11 Q6

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19. (a) Express $2 \cos 3x - 3 \sin 3x$ in the form $R \cos (3x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your answers to 3 significant figures. **(4)**

June 11 Q8*(edited)*