Name:

Total Marks:

Pure Mathematics 1

Advanced Subsidiary

Practice Paper J9

Time: 2 hours

Information for Candidates

- This practice paper is an adapted legacy old paper for the Edexcel GCE AS Level Specifications
- There are 12 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Question 1

The equation $kx^2 + 4x + (5 - k) = 0$, where k is a constant, has 2 different real solutions for x.

(a) Show that k satisfies

$$k^2 - 5k + 4 > 0.$$ (3)

(b) Hence find the set of possible values of k. (4)

(Total 7 marks)

Question 2

Given that $\dfrac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $2x^p - x^q$,

(a) write down the value of p and the value of q. (2)

Given that $y = 5x^4 - 3 + \dfrac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$,

(b) find $\dfrac{dy}{dx}$, simplifying the coefficient of each term. (4)

(Total 6 marks)

Question 3

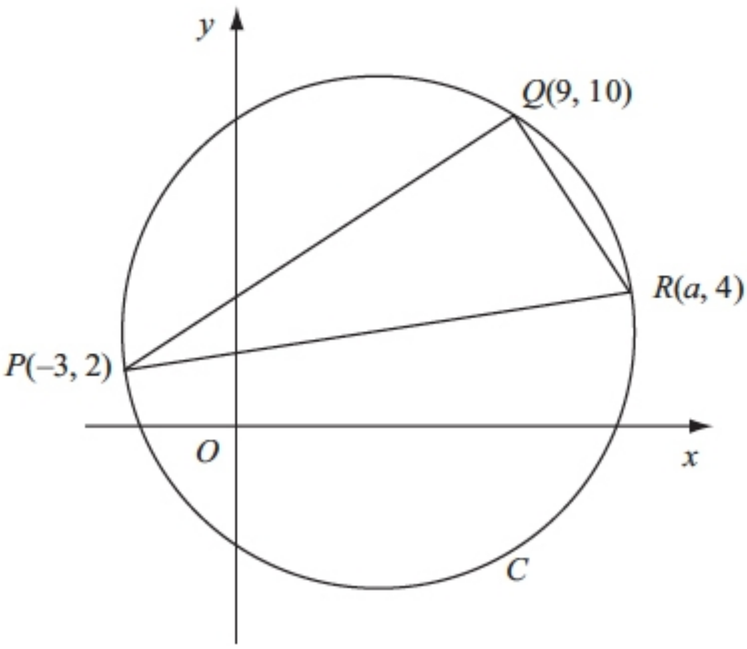


Figure 2

The points P(−3, 2), $Q(9, 10)$ and $R(a, 4)$ lie on the circle C, as shown in Figure 2. Given that PR is a diameter of C,

(a) show that $a = 13$, **(3)**

(b) find an equation for C. **(5)**

(Total 8 marks)

Question 4

The line l_1 passes through the point A (2, 5) and has gradient $-\frac{1}{2}$.

(a) Find an equation of l_1, giving your answer in the form $y = mx + c$. **(3)**

The point B has coordinates (−2, 7).

(b) Show that B lies on l_1. **(1)**

(c) Find the length of AB, giving your answer in the form $k\sqrt{5}$, where k is an integer. **(3)**

The point C lies on l_1 and has x-coordinate equal to p.

The length of AC is 5 units.

(d) Show that p satisfies

$$p^2 - 4p - 16 = 0.$$ **(4)**

(Total 11 marks)

Question 5

The curve C has equation

$$y = 9 - 4x - \frac{8}{x}, \qquad x > 0$$

The point P on C has x-coordinate equal to 2.

(a) Show that the equation of the tangent to C at the point P is $y = 1 - 2x$. **(6)**

(b) Find an equation of the normal to C at the point P. **(3)**

The tangent at P meets the x-axis at A and the normal at P meets the x-axis at B.

(c) Find the area of triangle APB. **(4)**

(Total 13 marks)

Question 6

A solid right circular cylinder has radius *r* cm and height *h* cm.
The total surface area of the cylinder is 800 cm^2.

(a) Show that the volume, *V* cm^3 , of the cylinder is given by

$$V = 400r - \pi r^3.$$ **(4)**

Given that *r* varies,

(b) use calculus to find the maximum value of *V*, to the nearest cm^3. **(6)**

(c) Justify that the value of *V* you have found is a maximum. **(2)**

(Total 12 marks)

Question 7

Given that $0 < x < 4$ and

$$\log_5 (4 - x) - 2\log_5 x = 1,$$

find the value of *x*. **(6)**

(Total 6 marks)

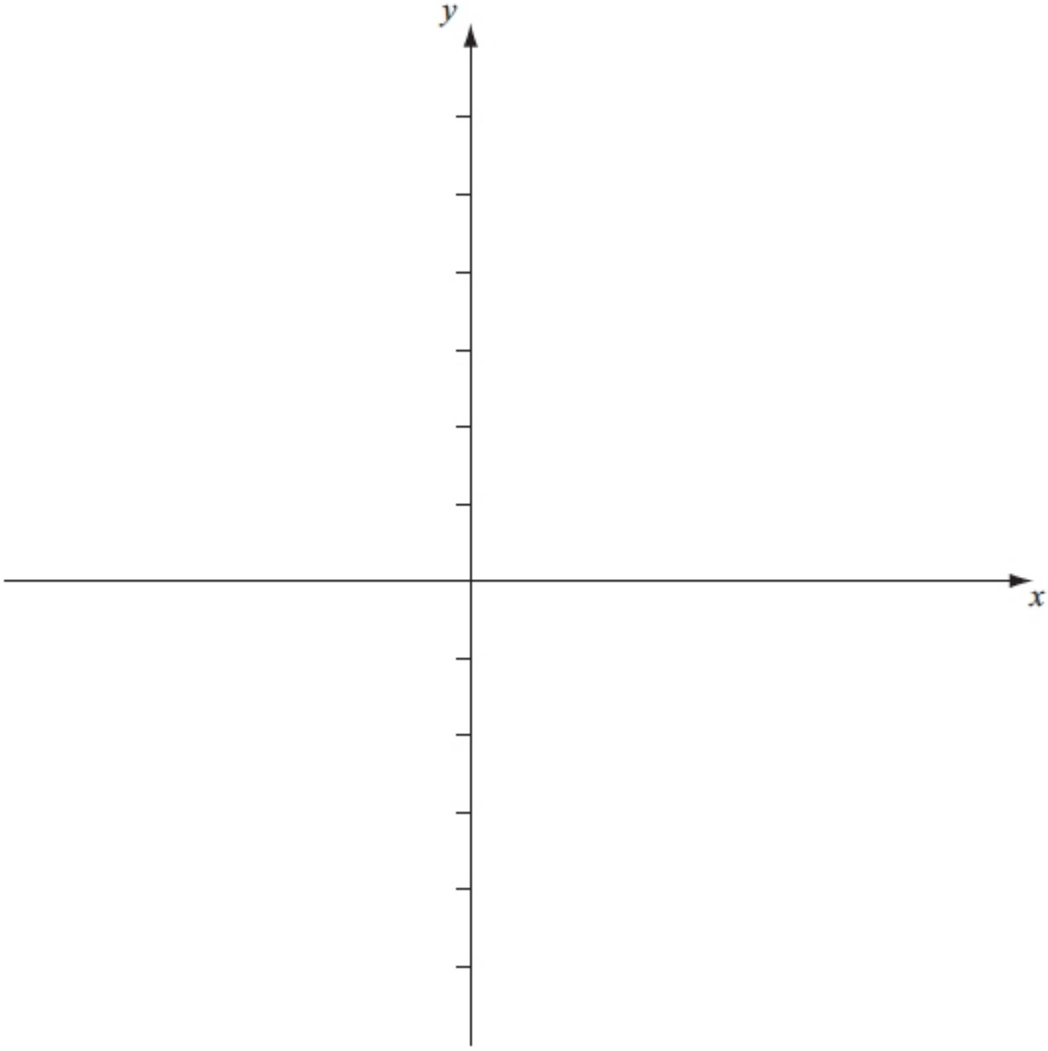
Question 8

(a) Sketch the graph of $y = x(x-1)(x+2)^2$, stating clearly the points of intersection with the axes **(3)**

(b) The point with coordinates (-3,0) lies on the curve with equation

$y = (x+a)(x+a-1)(x+a+2)^2$ where a is a constant.

Find the possible values of a **(3)**



(Total 7 marks)

Question 9

A curve has equation $y = f(x)$ and passes through the point (4, 22).

Given that

$$f'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7,$$

use integration to find $f(x)$, giving each term in its simplest form. **(5)**

(Total 5 marks)

Question 10

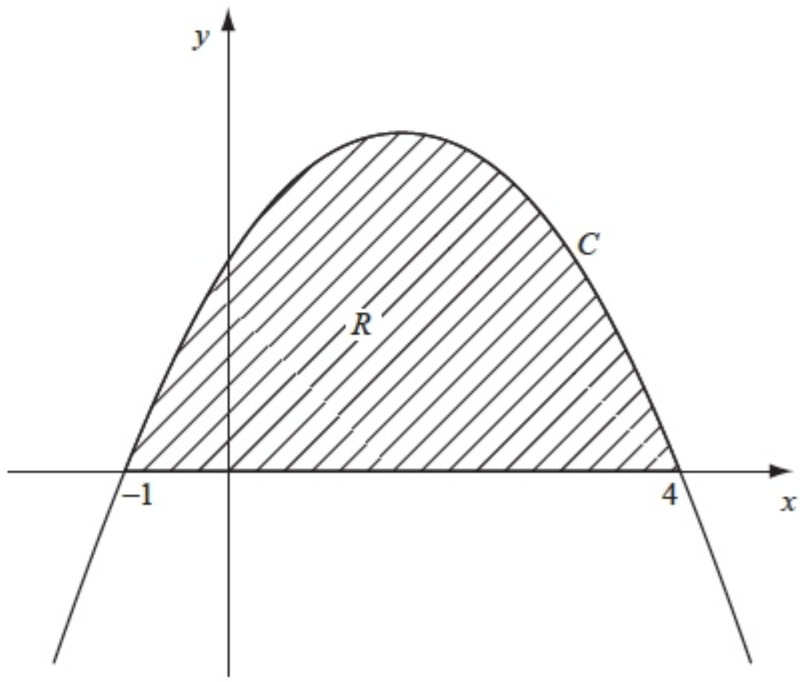


Figure 1

Figure 1 shows part of the curve C with equation $y = (1 + x)(4 - x)$.

The curve intersects the x-axis at $x = -1$ and $x = 4$. The region R, shown shaded in Figure 1, is bounded by C and the x-axis.

Use calculus to find the exact area of R. **(5)**

(Total 5 marks)

Question 11

(a) Show that the equation

$$4 \sin^2 x + 9 \cos x - 6 = 0$$

can be written as

$$4 \cos^2 x - 9 \cos x + 2 = 0. \qquad (2)$$

(b) Hence solve, for $0 \leqslant x < 720°$,

$$4 \sin^2 x + 9 \cos x - 6 = 0,$$

giving your answers to 1 decimal place. (6)

(Total 8 marks)

Question 12

(a) Prove that for any positive numbers *a* and *b*

$$a + b > \sqrt{4ab} \qquad (3)$$

(b) Show, by means of a counter example, that this inequality is not true when *a* and *b* are both negative (2)

(Total 5 marks)

Question 13

A ball is dropped from the top of a tower. The height, in metres, of the ball above the ground after *t* seconds is modelled by the function:

$H(t) = 15.25 + 17.8\,t - 4.5t^2$, where $t \geq 0$

(a) After how many seconds does the ball hit the ground (3)

(b) Write down $h(t)$ in the form $A - B(t - C)^2$, where A, B and C are constants to be found (3)

(c) Using your answer to part (b) or otherwise, find the maximum height of the ball above the ground, and the time at which this maximum height is reached. (2)

(Total 8 marks)

TOTAL FOR PAPER IS 100 MARKS