Connected Rates of Change - Edexcel Past Exam Questions MARK SCHEME

Question 1: Jan 06 Q7

(a)	$\frac{dV}{dr} = 4\pi r^2$	B1 (1)
(b)	Uses $\frac{dr}{dt} = \frac{dV}{dt} \cdot \frac{dr}{dV}$ in any form, $= \frac{1000}{4\pi r^2 (2t+1)^2}$	M1,A1 (2)

Question 2: Jan 06 Q7

Question Number	Scheme		Marks
(a)	From question, $\frac{dS}{dt} = 8$	$\frac{dS}{dt} = 8$	B1
	$S = 6x^2 \Rightarrow \frac{dS}{dx} = 12x$	$\frac{dS}{dx} = 12x$	B1
	$\frac{dx}{dt} = \frac{dS}{dt} \div \frac{dS}{dx} = \frac{8}{12x}; = \frac{\frac{2}{3}}{x} \Rightarrow (k = \frac{2}{3})$	Candidate's $\frac{dS}{dt} \div \frac{dS}{dx}; \frac{8}{12x}$	M1; A1oe
			[4]
(b)	$V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$	$\frac{dV}{dx} = 3x^2$	B1
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 3x^2 \cdot \left(\frac{2}{3x}\right); = 2x$	Candidate's $\frac{dV}{dx} \times \frac{dx}{dt}; \lambda x$	M1; A1√
	As $x = V^{\frac{1}{3}}$, then $\frac{dV}{dt} = 2V^{\frac{1}{3}}$ **AG**	Use of $x = V^{\frac{1}{3}}$, to give $\frac{dV}{dt} = 2V^{\frac{1}{3}}$	A1
			[4]

Question 3: June 06 Q7

Question Number	Scheme		Marks
(a)	From question, $\frac{dS}{dt} = 8$	$\frac{dS}{dt} = 8$	B1
	$S = 6x^2 \Rightarrow \frac{dS}{dx} = 12x$	$\frac{dS}{dx} = 12x$	B1
	$\frac{dx}{dt} = \frac{dS}{dt} \div \frac{dS}{dx} = \frac{8}{12x}; = \frac{\frac{2}{3}}{x} \Rightarrow \left(k = \frac{2}{3}\right)$	Candidate's $\frac{dS}{dt} \div \frac{dS}{dx}; \frac{8}{12x}$	M1; A1oe
			[4]
(b)	$V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$	$\frac{dV}{dx} = 3x^2$	B1
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 3x^2.\left(\frac{2}{3x}\right); = 2x$	Candidate's $\frac{dV}{dx} \times \frac{dx}{dt}; \lambda x$	M1; A1√
	As $x = V^{\frac{1}{3}}$, then $\frac{dV}{dt} = 2V^{\frac{1}{3}}$ **AG**	Use of $x = V^{\frac{1}{3}}$, to give $\frac{dV}{dt} = 2V^{\frac{1}{3}}$	A1
			[4]

Question Number	Scheme		Marks
Aliter (b) **Way 2**	$x = V^{\frac{1}{3}}$ & $S = 6x^2 \Rightarrow S = 6V^{\frac{2}{3}}$	$S = 6V^{\frac{2}{3}}$	B1√
	$\frac{dS}{dV} = 4V^{-\frac{1}{3}}$ or $\frac{dV}{dS} = \frac{1}{4}V^{\frac{1}{3}}$	$\frac{dS}{dV} = 4V^{-\frac{1}{3}}$ or $\frac{dV}{dS} = \frac{1}{4}V^{\frac{1}{3}}$	B1
	$\frac{dV}{dt} = \frac{dS}{dt} \times \frac{dV}{dS} = 8.\left(\frac{1}{4V^{-\frac{1}{3}}}\right); = \frac{2}{V^{-\frac{1}{3}}} = 2V^{\frac{1}{3}}$ **AG**	Candidate's $\frac{dS}{dt} \times \frac{dV}{dS}; 2V^{\frac{1}{3}}$	M1; A1
		In ePEN, award Marks for Way 2 in the order they appear on this mark scheme.	
			[4]

Question Number	Scheme		Marks
Aliter (b) **Way 3**	*similar to way 1.*		
	$V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$	$\frac{dV}{dx} = 3x^2$	B1
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dS}{dt} \times \frac{dx}{dS} = 3x^2.8.\left(\frac{1}{12x}\right); = 2x$	Candidate's $\frac{dV}{dx} \times \frac{dS}{dt} \times \frac{dx}{dS}; \lambda x$	M1; A1√
	As $x = V^{\frac{1}{3}}$, then $\frac{dV}{dt} = 2V^{\frac{1}{3}}$ **AG**	Use of $x = V^{\frac{1}{3}}$, to give $\frac{dV}{dt} = 2V^{\frac{1}{3}}$	A1
			[4]

Question 4: June 08 Q3

Question Number	Scheme	Marks
(a)	From question, $\frac{dA}{dt} = 0.032$	B1
	$\left\{A = \pi x^2 \Rightarrow \frac{dA}{dx} =\right\} 2\pi x$	B1
	$\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = (0.032)\frac{1}{2\pi x}; \left\{= \frac{0.016}{\pi x}\right\}$	M1
	When $x = 2\text{cm}$, $\frac{dx}{dt} = \frac{0.016}{2\pi}$	
	Hence, $\frac{dx}{dt} = 0.002546479...$ (cm s^{-1})	A1 cso (4)
(b)	$V = \underline{\pi x^2(5x)} = \underline{5\pi x^3}$	B1
	$\frac{dV}{dx} = 15\pi x^2$	B1 ft
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 15\pi x^2.\left(\frac{0.016}{\pi x}\right); \{= 0.24x\}$	M1
	When $x = 2\text{cm}$, $\frac{dV}{dt} = 0.24(2) = \underline{0.48}$ $(\text{cm}^3\,\text{s}^{-1})$	A1 (4)
		(8 marks)

Question 5: Jan 09 Q5

Question Number	Scheme		Marks
(a)	Similar triangles $\Rightarrow \frac{r}{h}=\frac{16}{24} \Rightarrow r=\frac{2h}{3}$	Uses similar triangles, ratios or trigonometry to find either one of these two expressions oe.	M1
	$V=\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{2h}{3}\right)^2 h = \frac{4\pi h^3}{27}$ **AG**	Substitutes $r=\frac{2h}{3}$ into the formula for the volume of water V.	A1 (2)
(b)	From the question, $\frac{dV}{dt}=8$	$\frac{dV}{dt}=8$	B1
	$\frac{dV}{dh}=\frac{12\pi h^2}{27}=\frac{4\pi h^2}{9}$	$\frac{dV}{dh}=\frac{12\pi h^2}{27}$ or $\frac{4\pi h^2}{9}$	B1
	$\frac{dh}{dt}=\frac{dV}{dt}\div\frac{dV}{dh} = 8\times\frac{9}{4\pi h^2}=\frac{18}{\pi h^2}$	Candidate's $\frac{dV}{dt}\div\frac{dV}{dh}$; $8\div\left(\frac{12\pi h^2}{27}\right)$ or $8\times\frac{9}{4\pi h^2}$ or $\frac{18}{\pi h^2}$ oe	M1; A1
	When $h=12$, $\frac{dh}{dt}=\frac{18}{144\pi}=\frac{1}{8\pi}$	$\frac{18}{144\pi}$ or $\frac{1}{8\pi}$	A1 oe isw
	Note the answer must be a one term exact value. Note, also you can ignore subsequent working after $\frac{18}{144\pi}$.		(5) [7]

Question 6: Jan 10 Q6

Question Number	Scheme	Marks
	$\frac{dA}{dt} = 1.5$	B1
	$A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r$	B1
	When $A = 2$	
	$2 = \pi r^2 \Rightarrow r = \sqrt{\frac{2}{\pi}}$ $(= 0.797\,884\ldots)$	M1
	$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$	
	$1.5 = 2\pi r \frac{dr}{dt}$	M1
	$\frac{dr}{dt} = \frac{1.5}{2\pi\sqrt{\frac{2}{\pi}}} \approx 0.299$ awrt 0.299	A1
		[5]

Question 7: June 11 Q3

Question Number	Scheme	Marks
	(a) $\frac{dV}{dh} = \frac{1}{2}\pi h - \pi h^2$ or equivalent	M1 A1
	At $h = 0.1$, $\frac{dV}{dh} = \frac{1}{2}\pi(0.1) - \pi(0.1)^2 = 0.04\pi$ $\frac{\pi}{25}$	M1 A1 **(4)**
	(b) $\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} = \frac{\pi}{800} \times \frac{1}{\frac{1}{2}\pi h - \pi h^2}$ or $\frac{\pi}{800} \div$ their (a)	M1
	At $h = 0.1$, $\frac{dh}{dt} = \frac{\pi}{800} \times \frac{25}{\pi} = \frac{1}{32}$ awrt 0.031	A1 **(2)**
		[6]