## Differentiation: Connected Rates of Change - Edexcel Past Exam Questions

1. The volume of a spherical balloon of radius $r \mathrm{~cm}$ is $V \mathrm{~cm}^{3}$, where $V=\frac{4}{3} \pi r^{3}$.
(a) Find $\frac{\mathrm{d} V}{\mathrm{~d} r}$.

The volume of the balloon increases with time $t$ seconds according to the formula

$$
\frac{\mathrm{d} V}{\mathrm{~d} t}=\frac{1000}{(2 t+1)^{2}}, \quad t \geq 0
$$

(b) Using the chain rule, or otherwise, find an expression in terms of $r$ and $t$ for $\frac{\mathrm{d} r}{\mathrm{~d} t}$.

Jan 06 Q7 (edited)
3. At time $t$ seconds the length of the side of a cube is $x \mathrm{~cm}$, the surface area of the cube is $S \mathrm{~cm}^{2}$, and the volume of the cube is $V \mathrm{~cm}^{3}$.

The surface area of the cube is increasing at a constant rate of $8 \mathrm{~cm}^{2} \mathrm{~s}^{-1}$.
Show that
(a) $\frac{\mathrm{d} x}{\mathrm{~d} t}=\frac{k}{x}$, where $k$ is a constant to be found,
(b) $\frac{\mathrm{d} V}{\mathrm{~d} t}=2 V^{\frac{1}{3}}$.
(4)

June 06 Q7(edited)
4.


Figure 2
Figure 2 shows a right circular cylindrical metal rod which is expanding as it is heated. After $t$ seconds the radius of the rod is $x \mathrm{~cm}$ and the length of the rod is $5 x \mathrm{~cm}$.

The cross-sectional area of the rod is increasing at the constant rate of $0.032 \mathrm{~cm}^{2} \mathrm{~s}^{-1}$.
(a) Find $\frac{\mathrm{d} x}{\mathrm{~d} t}$ when the radius of the rod is 2 cm , giving your answer to 3 significant figures.
(b) Find the rate of increase of the volume of the rod when $x=2$.
5.


Figure 2

A container is made in the shape of a hollow inverted right circular cone. The height of the container is 24 cm and the radius is 16 cm , as shown in Figure 2. Water is flowing into the container. When the height of water is $h \mathrm{~cm}$, the surface of the water has radius $r \mathrm{~cm}$ and the volume of water is $V \mathrm{~cm}^{3}$.
(a) Show that $V=\frac{4 \pi h^{3}}{27}$.
[The volume V of a right circular cone with vertical height $h$ and base radius $r$ is given by the formula $V=\frac{1}{3} \pi r^{2} h$.]

Water flows into the container at a rate of $8 \mathrm{~cm}^{3} \mathrm{~s}^{-1}$.
(b) Find, in terms of $\pi$, the rate of change of $h$ when $h=12$.

Jan 09 Q5
6. The area $A$ of a circle is increasing at a constant rate of $1.5 \mathrm{~cm}^{2} \mathrm{~s}^{-1}$. Find, to 3 significant figures, the rate at which the radius $r$ of the circle is increasing when the area of the circle is $2 \mathrm{~cm}^{2}$.

Jan 10 Q6
7.


Figure 1
A hollow hemispherical bowl is shown in Figure 1. Water is flowing into the bowl.
When the depth of the water is $h \mathrm{~m}$, the volume $V \mathrm{~m}^{3}$ is given by

$$
V=\frac{1}{12} \pi h^{2}(3-4 h), \quad 0 \leq h \leq 0.25 .
$$

(a) Find, in terms of $\pi, \frac{\mathrm{d} V}{\mathrm{~d} h}$ when $h=0.1$.

Water flows into the bowl at a rate of $\frac{\pi}{800} \mathrm{~m}^{3} \mathrm{~s}^{-1}$.
(b) Find the rate of change of $h$, in $\mathrm{m} \mathrm{s}^{-1}$, when $h=0.1$.

